Active broadband control of vibrating panel structures with multiple structural sensors

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ABSTRACT

The aim of the research presented in this paper is to actively control broadband vibration on panel structures using multiple sensors. The broadband vibration profile of a panel structure is estimated by using spatial interpolation functions and vibration measurements from the surface mounted sensors. The control objective is then achieved by deriving 'spatial' error signals whose energy represents the spatially-weighted vibration energy over the structure. An optimal H-2 control design using this spatial control approach is discussed to demonstrate the effectiveness of the broadband spatial control on a panel. Numerical results show that the broadband vibration profile can be spatially controlled, not just by minimising the strength of each vibration mode, but also by controlling the relative strength of each mode.

INTRODUCTION

In many industrial acoustic applications, it is often beneficial to minimise vibration of panel-type structures such as to reduce the noise level radiated by the structures or to satisfy a particular structural performance criterion. In some cases, certain structural regions may be more prone to generating noise radiation than others. Control efforts, therefore, can be concentrated to control vibration in these regions, instead of trying to control the overall structure.

Some researchers have used multiple structural sensors for controlling vibration of structures (Meirovitch and Baruh 1982; Meirovitch 1987; Pajunen et al. 1994), although most of the research concentrated in controlling the vibration of the entire structure. Halim and Cazzolato (2005) used multiple structural sensors attached over an arbitrary panel structure to spatially control the structural vibration by employing spatial interpolation functions. Spatial vibration control can be utilised to control vibration only at particular spatial regions of a structure. The effectiveness of the control for tonal vibration control is analysed by Halim and Cazzolato (2005).

In the work presented in this paper, the work by Halim and Cazzolato (2005) is extended for active spatial broadband control using multiple structural sensors. The spatial broadband control is attractive since it can be used to spatially control vibration of an arbitrary structure without the need to obtain the material and modal properties from the structure. Only geometrical properties of the structure and the structural boundaries are needed to achieve spatial vibration control, which makes the active control method practical.

SENSING THE VIBRATION PROFILE OF A PANEL

In this section, the vibration profile of a panel structure is estimated using multiple structural sensors distributed over the panel. For instance, a set of accelerometers attached on the panel can be used to achieve the purpose. The panel's vibration profile is estimated as this information is necessary for spatially controlling its structural vibration.

Here, the approach used to estimate the vibration profile is described. Consider a panel structure of an arbitrary shape in Figure 1, where there are multiple discrete structural sensors distributed over the panel. Each sensor measures vibration w_i at location (x_i,y_i), representing a 'node'. An 'element' can be constructed from the adjacent sensors acting as nodes as shown in Figure 1. The purpose of creating these elements is so that the vibration profile can be interpolated spatially, using an implementation similar to finite element analysis (Bathe and Wilson 1976; Cheung and Leung 1991; Meirovitch 1975). In the case where the structural boundaries are known, additional nodes may be included to improve the vibration profile estimation since the vibration at the boundaries may be known to be minimal such as for rigidly clamped boundaries (Halim and Cazzolato 2005).

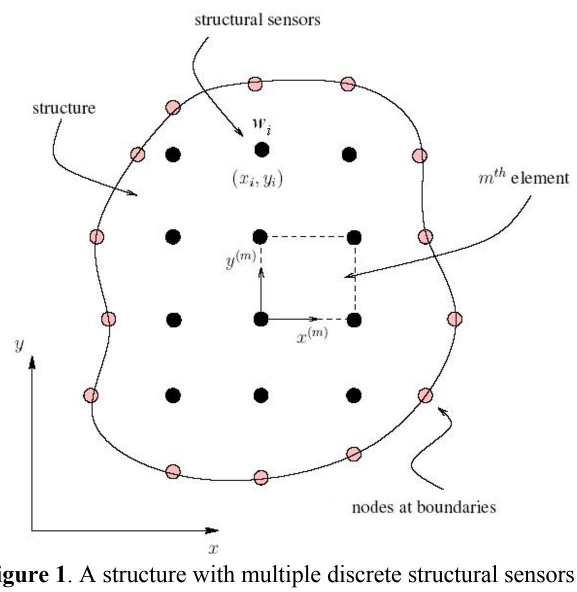

Figure 1. A structure with multiple discrete structural sensors used for vibration profile estimation.

Suppose there are M elements constructed from structural sensors and nodes at the boundaries, the vibration profile over the structure $w_{xy}(x,y,t)$ can then be estimated by (Halim and Cazzolato 2005):

$$w_{xy}(x,y,t) = M_{xy}(x,y)w(t) \quad (1)$$

where $w(t)$ is a vector containing the vibration measurements from all sensors, and $M_{xy}(x,y)$ is a spatial interpolation matrix that relates the sensor measurements to the vibration output at any point over the structure.

This vibration profile estimate $w_{xy}(x,y,t)$ can then be used for obtaining a spatial broadband control as discussed in the following section.

BROADBAND SPATIAL CONTROL

Based on the vibration profile information from the sensors, a performance/objective function to be minimised by active control means can then be defined. The objective function E is defined as:

$$E(t) = \int_S w_{xy}(x,y,t)^T Q(x,y) w_{xy}(x,y,t) dS \qquad (2)$$

where S is the region of the panel and $Q(x,y)$ represents the non-negative continuous spatial weighting to be used. The above objective function is determined based on the vibration profile estimation in Equation (1). The purpose of the spatial weighting is to emphasise the panel region whose vibration level needs to be suppressed more than others. It will be shown in the latter sections of this paper that this objective function corresponds to modifying the relative strength of the panel's vibration modes.

Substituting Equation (1) into Equation (2), the objective function can be simply re-stated as:

$$E(t) = w(t)^T A w(t) \qquad (3)$$

where

$$A = \int_S M_{xy}(x,y)^T Q(x,y) M_{xy}(x,y) dS \qquad (4)$$

Spatial signals

Matrix A can be decomposed into its eigenvalue and eigenvector matrices as

$$A = UVU^T \qquad (5)$$

where V is a diagonal eigenvalue matrix and U is a unitary eigenvector matrix. The spatial signals $v(t)$ can then be defined by substituting Equation (5) into Equation (3) (Halim and Cazzolato 2005):

$$v(t) = V^{1/2} U^T w(t) = W w(t) \qquad (6)$$

where W is the spatial filter defined as above and the objective function is now:

$$E(t) = v(t)^T v(t). \qquad (7)$$

Here, $v(t)$ are 'spatial' signals that represent the spatially-weighted vibration energy of the structure. A reduced dimension of spatial signals $v(t)$ can also be achieved by using only using several most dominant eigenvalues and eigenvectors as described by Halim and Cazzolato (2005). This dimensional reduction would be practical for control implementation since the dimension of the spatial signals could be made much less than the number of sensors used.

General approach for broadband spatial control

The significance of the above formulation can be explained from the general approach for broadband spatial control shown in Figure 2. Suppose a disturbance causes structure P to vibrate and the vibration information from the sensors are filtered by the spatial filter W to generate a reduced spatial signals $v(t)$ as described by Equation (6). For broadband control purposes, a frequency filter can be incorporated to emphasise a particular bandwidth of interest. The frequency-filtered spatial signals can then be used as error signals to the controller K for generating the required control actuation.

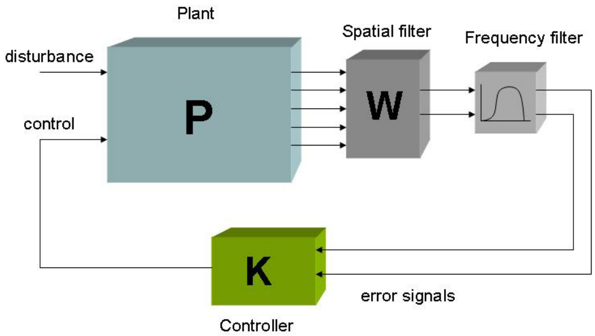

Figure 2. General approach for broadband spatial control of structures using vibration information from multiple structural sensors.

Broadband spatial feedforward control

In this section, the performance of broadband spatial control is analysed based on the optimal broadband feedforward control framework. An optimal controller that minimises the energy transfer from the disturbance input to the error signal (spatial signal) output is to be considered for control performance analysis. Figure 3 illustrates the feedforward control design where the primary P_1 and secondary P_2 path transfer functions, from the disturbance d and control u input to sensor outputs, can be represented as follows, together with their state-space representations:

$$\begin{aligned} P_1 &\equiv (A_s, B_1, C_1, 0) \\ P_2 &\equiv (A_s, B_2, C_1, 0). \end{aligned} \qquad (8)$$

In Figure 3, W is again the spatial filter, K is the desired optimal controller and a control weight matrix, c, is added to limit the amount of control gain that can be obtained for practical purposes. Note that the performance output z is now a combination of the spatial signal and the weighted control input signal.

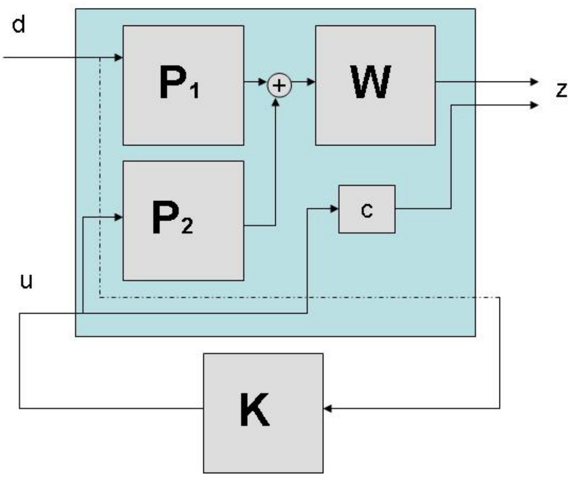

Figure 3. Optimal broadband feedforward control problem.

The task is to find an optimal broadband controller K that minimises the energy transfer from the disturbance signal d to the performance output signal z. To do this, an optimal H_2 control design is considered where the H_2 norm cost function is defined as:

$$\|T(s)\|_2^2 = \int_{-\infty}^{\infty} trace\{T(j\omega)^* T(j\omega)\} d\omega \qquad (9)$$

where T is the transfer matrix from disturbance d to the performance output z, after the controller K has been augmented to the plant. Here, T^* is the complex conjugate of matrix T and the frequency is denoted by ω, indicating the averaging process across the entire frequency range. The generalised plant shown in Figure 3, represents the transfer function from disturbance d to the performance output z, which can be described as:

$$\hat{P} \equiv (A_s, \hat{B}, \hat{C}, \hat{D}) \qquad (10)$$

where A_s is the original state matrix of the plant as shown in Equation (8) and

$$\hat{B} = \begin{bmatrix} B_1 & B_2 \end{bmatrix}$$
$$\hat{C} = \begin{bmatrix} WC_1 \\ 0 \\ 0 \end{bmatrix} \qquad (11)$$
$$\hat{D} = \begin{bmatrix} 0 & 0 \\ 0 & c \\ 1 & 0 \end{bmatrix}.$$

Standard optimal H_2 control design method can be used to solve for the optimal H_2 controller. Here, the Matlab Robust Control Toolbox has been used for this purpose. The following section will discuss the control performance of spatial broadband control of a panel structure.

BROADBAND SPATIAL CONTROL OF A PANEL STRUCTURE

A numerical analysis of a simply-supported rectangular aluminium panel has been performed to demonstrate the technique developed in the previous sections. Here, 25 discrete structural velocity sensors are distributed over the panel as shown in Figure 4. The dimensions of the panel are 500mm x 400mm x 4mm. Point sources are used as disturbance and control sources, which are located at (x,y) = (312.5mm,121.2mm) and (156.3mm,121.2mm) respectively, based on coordinates in Figure 4.

Modal analysis (de Silva 2000) is used to obtain the panel model, and the first 7 vibration modes are considered in this analysis. The natural frequencies of the modes are shown in Table 1 and the interpolation function used is a linear function as described by Halim and Cazzolato (2005). The spatial signals are reduced from 25 to just 4 signals for control purposes after consideration of the eigenvalues of matrix A given by Equation (5).

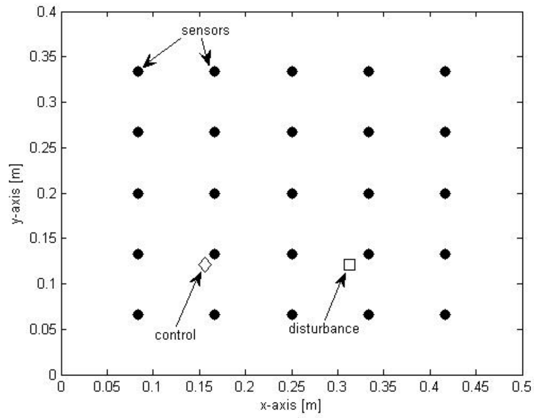

Figure 4. A rectangular plate with locations of sensors, disturbance and control sources.

Table 1. Natural frequencies of the panel.

Mode	Frequency [Hz]
(1,1)	98.3
(2,1)	213.4
(1,2)	278.2
(2,2)	393.3
(3,1)	405.3
(1,3)	578.0
(3,2)	585.2

Control analysis: Spatial weighting 1

The first analysis of broadband spatial control performance considers a particular spatial weighting function *Q(x,y)* which is shown in Figure 5. The region on the panel that is of interest is near one of the corners of the panel as reflected by the large weighting function in the figure.

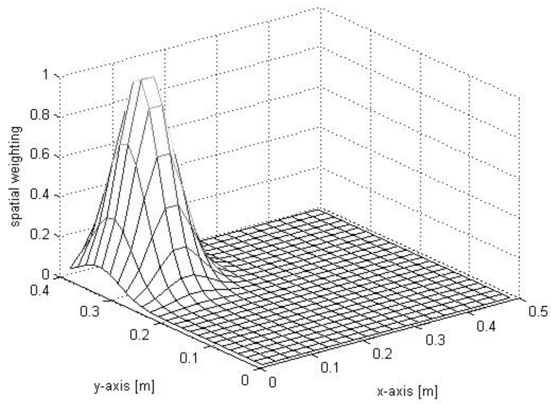

Figure 5. Spatial weighting function for the first control analysis.

To analyse the effectiveness of spatial control strategy in reducing the spatial broadband vibration energy, the following analysis is performed. The broadband vibration energy over the entire panel is analysed by plotting the broadband vibration level at any point (*x,y*) on the panel. This can be done by obtaining the H_2 norm that reflects the overall broadband energy output due to a white noise disturbance input *d*. Figures 6 and 7 show the broadband vibration profile over the panel for the uncontrolled and controlled systems, where the vibration level is measured by the H_2 norm of the systems. It can be seen that the spatial control attempts to minimise broadband vibration level more at the region that has larger weighting. It is observed that other regions over the panel also experience vibration reduction.

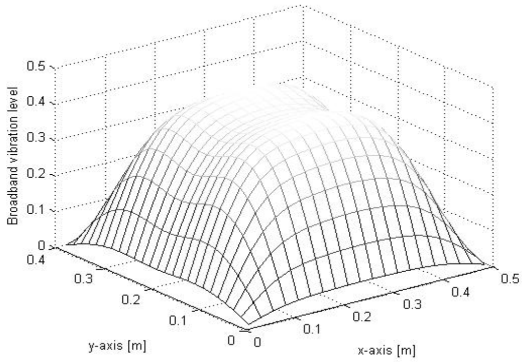

Figure 6. Broadband vibration level over the entire panel without control.

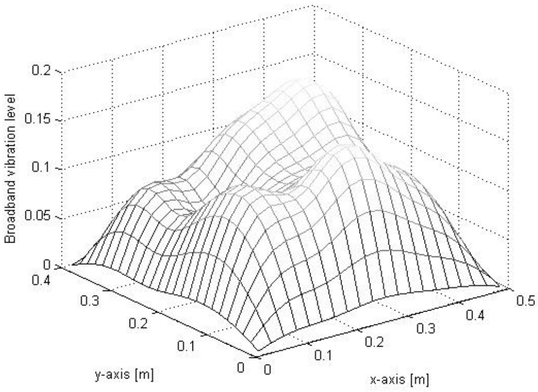

Figure 7. Broadband vibration level over the panel with control for the first spatial weighting.

The effect of spatial control to the overall broadband vibration responses can be analysed from Figure 8. The figure shows the transfer function from the disturbance to the velocity at a location of (x,y)=(83.3mm,333.3mm). It is shown that the spatial controller attempts to minimise vibration contributed by most of vibration modes. At some frequencies, however, the vibration level is amplified, although from the results it can be seen that the overall broadband energy, reflected by the area under the transfer function, is still being minimised.

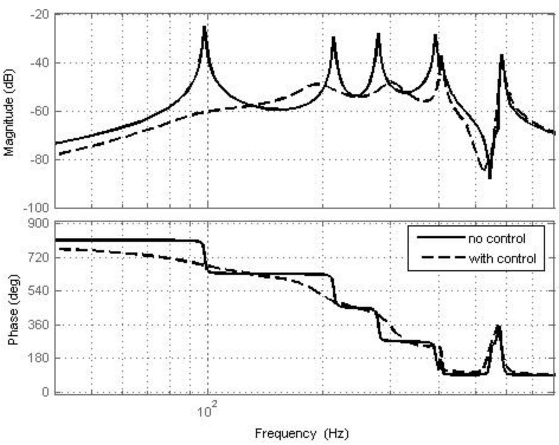

Figure 8. Two transfer functions from the disturbance to the velocity output at a location (x,y)=(83.3mm,333.3mm): with and without control.

The previous results lead to the question of how the spatial controller attempts to reduce the strength of each vibration mode to achieve its spatial broadband vibration profile. To answer the question, the relative strength of each vibration mode is plotted in Figure 9. The percentage is calculated relative to the strength of the most dominant mode for easy comparison between the uncontrolled and controlled systems. Here, the lightly-coloured bar graph is for the uncontrolled system, while the dark-coloured graph is for the controlled system. Note that the results shown are the relative strength of the modes, where the absolute strength of the controlled modes is in fact much less than that for the uncontrolled ones as expected.

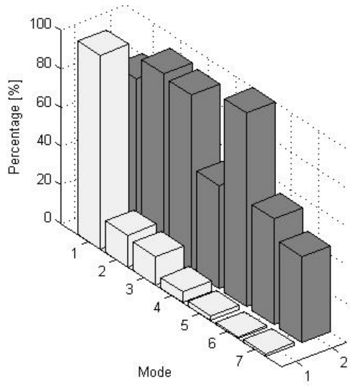

Figure 9. Relative strength of vibration modes for systems without control (lightly coloured graph) and with control (dark coloured graph).

It is shown that for the uncontrolled system, the first vibration mode (1,1) contributes most to the vibration compared to other higher wavenumber modes. However, for the controlled system, the first mode (1,1) is no longer dominant as shown from the dark coloured bar graph in Figure 9. The relative strength of modes 1-3 and 5 is now comparable, and the control makes more effort to reduce higher wavenumber modes (particularly modes 4, 6 and 7) than lower wavenumber modes. This is expected since the region that is emphasised by the spatial weighting corresponds to the corner region of the panel, where higher wavenumber modes tend to contribute more than lower wavenumber ones. However, the first mode has also been heavily controlled since the relative strength of this mode for the uncontrolled system is much larger than other modes.

Control analysis: Spatial weighting 2

The second control analysis considers the spatial weighting function shown in Figure 10 where the region close to the middle of the panel is emphasised. The broadband vibration level over the panel is illustrated in Figure 11 where the region around the centre is being minimised more than other regions on the panel as expected.

The relative strength of vibration modes is plotted in Figure 12. It can be seen that for this spatial weighting, the spatial control is expected to more heavily control the modes that contribute dominantly to the region near the centre of the panel such as mode 1. As most of the modes contributes to the broadband vibration near the centre of the panel, the results are more difficult to predict based on the relative strength of the modes, apart from the dominant first mode (1,1). This illustrates the convenience of using the spatial control approach as the judgement on how much control each mode is required, can be achieved by the spatial control algorithm itself.

It should be noted that only a single control source has been used in the previous 2 examples. When more control sources used, the attenuation of the targeted regions can be improved.

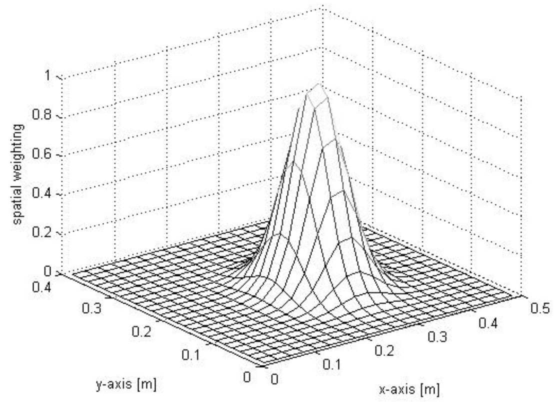

Figure 10. Spatial weighting function for the second control analysis.

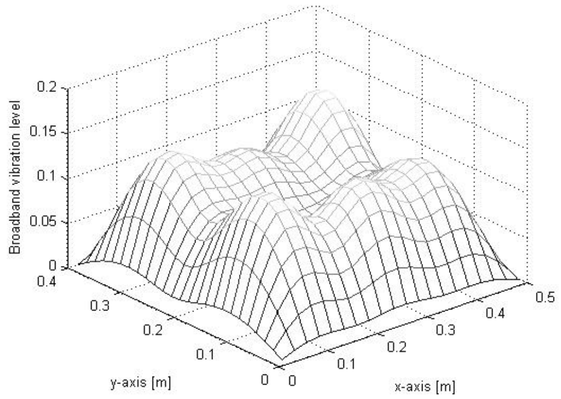

Figure 11. Broadband vibration level over the panel with control for the second spatial weighting.

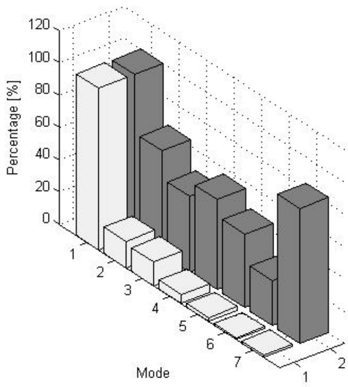

Figure 12. Relative strength of vibration modes for systems without control (lightly coloured graph) and with control (dark coloured graph).

CONCLUSIONS

The analysis of active broadband spatial control of panel structures using multiple structural sensors has been presented. It is shown that it is possible to achieve broadband vibration control where particular spatial regions of the panel receive more importance than other regions. It is also observed that the spatial control attempts not just to minimise the strength of each vibration modes, but also to consider the relative strength of each mode for achieving the spatially-weighted objective optimally.

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