Optimisation of Design and Location of Acoustic and Vibration Absorbers Using a Distributed Computing Network

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ABSTRACT

A distributed computing network was created using the software called Condor and a large number of networked desktop computers. This computational tool was used to optimise the design and location of passive vibration and acoustic absorbers attached to the payload fairing of a rocket launch vehicle. This paper describes a mathematical model to calculate the coupled vibro-acoustic response of a system from the uncoupled structural and acoustic modal responses obtained from finite element analysis. A genetic algorithm was used in conjunction with the distributed computing network to optimise the parameters of the absorbers. The optimisations using the computing network could be completed in significantly less time compared to a single desktop computer.

INTRODUCTION

Many companies are interested in optimising the designs of their products to reduce noise, vibration, weight, cost, etc. Unfortunately, most computational methods for complex vibro-acoustic systems require significant computational resources to evaluate each parameter of a multi-variable cost function. Optimisation of the design parameters is often not achievable within a reasonable time frame due to the complexity of the problem. The work presented here shows how a distributed computing network can be established relatively easily to conduct an optimisation of a complex vibro-acoustic system in a reasonable time.

In the work conducted here, an optimisation of the parameters for tuned mass dampers (TMDs) and Helmholtz resonators (HRs) attached to the walls of the payload bay of a rocket used to launch satellites was conducted to reduce the noise levels inside the payload bay. The combination of a TMD and HR into a single device is called a Passive Vibro-Acoustic Device (PVAD). The cost function was the acoustic potential energy within the cavity and each evaluation took approximately 6 minutes on a 3.0GHz Pentium desktop computer (Pentium 4 with Hyperthreading, 800MHz FSB, 1Gb RAM PC3200). A Genetic Algorithm (GA) was used to optimise the parameters over 18,000 cost function evaluations. If the optimisation had been conducted on a single 3.0GHz Pentium it would have taken 75 days. By using a distributed computing network of about 180 computers of varying processor speeds from 1.8GHz to 2.4GHz, the time taken to conduct the optimisation was less than 3 days, with constant interruptions handled by the Condor system.

The contribution of the work presented here shows how a parallel genetic algorithm and a distributed computing network can be implemented relatively easily to solve vibro-acoustic optimisation problems.

MODELLING

Fahy (1994) describes equations for the coupled structural-acoustic response of a system in terms of the summation of structural and acoustic mode shapes. The equation for the coupled response of the structure is given by

$$\ddot{w}_p + \omega_p^2 w_p = \frac{S}{\Lambda_p} \sum_n p_n C_{np} + \frac{F_p}{\Lambda_p} \tag{1}$$

where w_p are the structural modal participation factors of the p^{th} mode, ω_p are the structural resonance frequencies, Λ_p are the modal masses, F_p are the modal forces applied to the structure, S is the surface area of the structure, and C_{np} is the dimensionless coupling coefficient given by the integral of the product of the structural (ϕ_p) and acoustic (ψ_n) mode shape functions over the surface of the structure, given by

$$C_{np} = \frac{1}{S} \int_S \psi_n(\mathbf{r}_S)\, \phi_p(\mathbf{r}_S)\, dS \tag{2}$$

The equation for the coupled response of the fluid is given by

$$\ddot{p}_n + \omega_n^2 p_n = -\left(\frac{\rho_0 c^2 S}{\Lambda_n}\right) \sum_p \ddot{w}_p C_{np} + \left(\frac{\rho_0 c^2}{\Lambda_n}\right) \dot{Q}_n \tag{3}$$

where p_n are the modal participation factors of the n^{th} acoustic modal responses, ω_n are resonance frequencies of the cavity, Λ_n are the modal volumes, ρ_0 is the density of the fluid, c is the speed of sound in the fluid, and Q_n is the source strength with units of volume velocity (hence $\dot{Q}_n$ has units of volume acceleration).

Tuned Mass Dampers

A Tuned Mass Damper (TMD) is a device consisting of a spring and mass, the resonance frequency of which is adjusted to coincide with the frequency of the forces driving the structure, or with a particular resonance frequency of structural vibration. The purpose of a TMD is to reduce the vibration levels on the structural boundary enclosing the cavity with the intention of reducing the sound transmitted into the cavity. However, alternative uses for such a device include tuning it to an acoustic resonance frequency of a cavity, or even attempting to shift the energy of one vibration mode into another vibration mode that has a poor acoustic radiation efficiency (or radiation ratio or index), as is used in

Active Structural Acoustic Control (ASAC) (Fuller et al. 1997).

A Tuned-Mass-Damper (TMD) can be modelled as a rigid mass connected by a flexible spring to an underlying structure whose vibration is to be altered. Figure 1 shows a TMD attached to the central node of four structural elements.

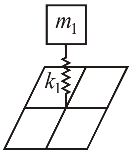

Figure 1: Model of a tuned mass damper.

The method used to model the coupling of the TMD to the structure is similar to the method described in Howard et al. (1997) and Howard (1999). This formulation can be easily extended to multiple degree of freedom TMDs to include three translational and three rotational degrees of freedom, or multi-modal TMDs, and multi-modal acoustic Helmholtz resonators.

Consider multiple single degree of freedom TMDs (J=1… N_{TMD}) attached to the structure, that have mass m_J^{TMD}, stiffness k_J^{TMD}, and are driven by a harmonic force at the attachment point of the spring to the structure. Considering the case for the structural domain only for the moment, the equations for the vibration of the structure and the TMDs can be written in matrix form as

$$\begin{bmatrix} k_J^{\mathrm{TMD}} - \omega^2 m_J^{\mathrm{TMD}} & -k_J^{\mathrm{TMD}}[\boldsymbol{\psi}_J] \\ -[\boldsymbol{\psi}_J]^{\mathrm{T}} k_J^{\mathrm{TMD}} & \begin{bmatrix}\Lambda_p\left(\omega_p^2-\omega^2\right)+ \\ [\boldsymbol{\psi}_J]^{\mathrm{T}} k_J^{\mathrm{TMD}}[\boldsymbol{\psi}_J]\end{bmatrix}\end{bmatrix} \times \begin{bmatrix} x_J^{\mathrm{TMD}} \\ w_p\end{bmatrix} = \begin{bmatrix}\mathbf{0}\\ \mathbf{F}_p\end{bmatrix} \qquad (4)$$

where $[\boldsymbol{\psi}_J]$ is the structural mode shape vector evaluated at the J^{th} connection point of the TMD to the structure and has dimensions (1 x N_s) where N_s is the number of structural modes, T is the matrix transpose operator, $\mathbf{F}_p$ is the vector of modal participation factors for the forces that drive the structure.

The equations derived thus far have not included damping terms. Damping can be included by using a hysteretic structural loss factor, so that the stiffness value for the TMD becomes a complex number. Hence the complex stiffness can be written as $k_J^{\mathrm{TMD}} = k_J^{\mathrm{TMD}}(1 + j\eta)$, where η is the structural loss factor.

Helmholtz Resonators

Helmholtz resonators are passive acoustic devices that are used to reduce the sound transmitted in a duct or change the acoustic field inside a cavity. Figure 2(a) shows a Helmholtz resonator that comprises a volume connected via neck to an acoustic system, such as a duct or a cavity, that contains an unwanted noise. The Helmholtz resonator acoustic system can be modelled as a vibrating mass in the throat of the device that is sprung mounted to a rigid support. Figure 2(b) shows the equivalent acoustic system where the acoustic spring is formed from the compliance of the volume of air in the Helmholtz resonator.

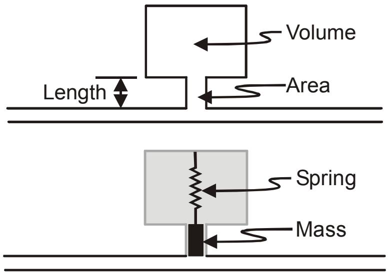

Figure 2: (a) Helmholtz resonator, (b) the equivalent spring-mass system.

In the ANSYS software a Helmholtz resonator can be modelled as a spring for the compliant volume, with the displacement fixed at one end, and the other end attached to a node that is part of four Fluid-Structure-Interaction (FSI) elements, as shown in Figure 3.

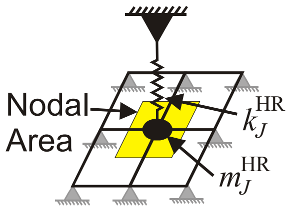

Figure 3: The connection of a model of a Helmholtz resonator to the acoustic finite element model.

The FSI elements have four degrees of freedom: displacements along the x, y, and z axes and pressure. The displacements for all the nodes on the exterior of the elements were fixed and only the central node attached to the spring was allowed to move. This effectively creates a piston that can move the fluid and generate a volume velocity sound source.

Consider multiple Helmholtz resonators (J=1…N_{HR}) attached to the acoustic cavity where k_J^{HR} is the equivalent spring stiffness of the compliant volume, and m_J^{HR} is the equivalent mass of the volume of fluid inside the neck of the resonator. The relationships between these terms and the acoustic properties are discussed in Beranek & Ver (1992). Consider the case for the acoustic domain only for the moment, by omitting the coupling term C_{np}. The equation for the coupled response of the acoustic cavity and the effect of the Helmholtz resonators can be written in matrix form as

$$\begin{bmatrix} k_J^{\mathrm{HR}} - \omega^2 m_J^{\mathrm{HR}} & -A_J^{\mathrm{HR}}[\phi_J] \\ -\omega^2 A_J^{\mathrm{HR}}[\phi_J]^{\mathrm{T}} & \dfrac{\Lambda_n}{\rho_0 c^2}\left(\omega_n^2-\omega^2\right)\end{bmatrix}\begin{bmatrix}x_J^{\mathrm{HR}}\\ p_n\end{bmatrix} = \begin{bmatrix}\mathbf{0}\\ \dot{\mathbf{Q}}_n\end{bmatrix} \qquad (5)$$

where x_J^{HR} is the particle displacement of the equivalent mass in the throat of the Helmholtz resonator, A_J^{HR} is the area associated with the node attaching the spring to the acoustic cavity, and $[\phi_J]$ is the matrix of the mode shape functions for the acoustic cavity.

Fully Coupled Model with Helmholtz Resonators and Tuned Mass Dampers

The equations for the fully coupled vibro-acoustic system, incuding the effects of the TMDs and HRs can be formed into a matrix equation as

$$\begin{bmatrix} \begin{bmatrix} \mathbf{A}_{11} & \mathbf{A}_{12} \\ \mathbf{A}_{21} & \mathbf{A}_{22} \end{bmatrix} & \begin{matrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{A}_{24} \end{matrix} \\ \begin{matrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{B}_{42} \end{matrix} & \begin{bmatrix} \mathbf{B}_{33} & \mathbf{B}_{34} \\ \mathbf{B}_{43} & \mathbf{B}_{44} \end{bmatrix} \end{bmatrix} \begin{bmatrix} \mathbf{x}^{\mathrm{TMD}} \\ \mathbf{w}_p \\ \mathbf{x}^{\mathrm{HR}} \\ \mathbf{p}_n \end{bmatrix} = \begin{bmatrix} \mathbf{0} \\ \mathbf{F}_p \\ \mathbf{0} \\ \dot{\mathbf{Q}}_n \end{bmatrix} \tag{6}$$

where

$$\mathbf{A}_{11} = \left[k_J^{\mathrm{TMD}} - m_J^{\mathrm{TMD}} \omega^2 \right] \tag{7}$$

$$\mathbf{A}_{12} = \left[-k_J^{\mathrm{TMD}} [\psi_J] \right] \tag{8}$$

$$\mathbf{A}_{21} = [\mathbf{A}_{12}]^{\mathrm{T}} \tag{9}$$

$$\mathbf{A}_{22} = \Lambda_p \left(\omega_p^2 - \omega^2 \right) + [\psi_J]^{\mathrm{T}} k_J^{\mathrm{TMD}} [\psi_J] \tag{10}$$

$$\mathbf{B}_{33} = \left[k_J^{\mathrm{HR}} - \omega^2 m_J^{\mathrm{HR}} \right] \tag{11}$$

$$\mathbf{B}_{34} = \left[-A_J^{\mathrm{HR}} [\phi_J] \right] \tag{12}$$

$$\mathbf{B}_{43} = \left[-\omega^2 A_J^{\mathrm{HR}} [\phi_J]^{\mathrm{T}} \right] \tag{13}$$

$$\mathbf{B}_{44} = \frac{\Lambda_n}{\rho_0 c^2} \left(\omega_n^2 - \omega^2 \right) \tag{14}$$

The remaining elements account for the cross coupling between the structure and the fluid and are given by

$$\mathbf{A}_{24} = -S\,\mathbf{C}_{np} \tag{15}$$

$$\mathbf{B}_{42} = -\omega^2 S \left[\mathbf{C}_{np} \right]^{\mathrm{T}} \tag{16}$$

The solution to the response of the vibro-acoustic system ($\mathbf{x}^{\mathrm{TMD}}$, $\mathbf{w}_p$, $\mathbf{x}^{\mathrm{HR}}$, and $\mathbf{p}_n$) is found by substituting the values of the parameters of the PVADs into Eq. (6), pre-multiplying each side of the matrix equation by the inverse of the matrix on the left-hand side of the equation.

Acoustic Potential Energy

The acoustic potential energy is a measure of the total acoustic energy contained within a cavity and is defined as

$$E_p = \mathbf{p}^{\mathrm{H}} \Lambda \mathbf{p} \tag{17}$$

where Λ is a diagonal matrix defined as

$$\Lambda(n,n) = \Lambda_n \Big/ \left(4\rho_0 c^2 \right) \tag{18}$$

After the solution to the vibro-acoustic system has been calculated, the acoustic modal participation factors $\mathbf{p}_n$ are used to calculate the acoustic potential energy at each frequency. The total acoustic potential energy for the frequency range of interest is calculated by the integral of the acoustic potential energy at each frequency within the frequency analysis range. The total acoustic potential energy is the cost function to be minimised by optimising the locations and parameters of the PVADs.

The mathematical model for the coupled vibro-acoustic response including the effects of the TMDs and HRs was implemented using the software package Matlab. The mathematical optimisation routines were also implemented in Matlab and the following section describes the optimisation method that was used in this investigation.

OPTIMISATION METHOD

Genetic Algorithm

A GA is a numerical optimisation method based on the principles of natural selection and genetics. GAs attempt to manipulate a set of possible solutions until an optimum solution is found. The set of possible solutions is called the population. Each candidate solution in the population is represented as a single chromosome. The chromosome is a series of numbers that are grouped together into a single string. The string can be a series of zeroes and ones, or it could be a series of integers or floating point numbers. Each one of these numbers represents a feature that is unique to the particular problem to be optimised. For example, the numbers in the string could represent the stiffness of a spring, or the mass of a tuned mass damper. The initial population of chromosomes is typically generated randomly. The chromosomes are used as input parameters to calculate a cost function, such as the acoustic potential energy inside the cavity, and are then ranked with the best performers ranked the highest (often described as the fittest). The fittest individual chromosomes are selected as 'parents' for the next generation and are combined (crossover) to form a new chromosome. This process is similar to asexual recombination that occurs in organisms. After the crossover operation, the new chromosome is mutated by randomly changing parts of the string. The mutation process introduces new genetic material into the population and causes some diversity in the population. The selection, crossover and mutation operations are conducted on the fittest individuals in the population and form the next evolution of the population. GAs follow the Darwinian evolutionary principle that good traits are retained in the population and bad traits are eliminated, which is determined by evaluating the cost function to find the fittest individuals.

The GA can be easily adapted for use in a distributed computing environment, where a *master* processor distributes jobs to *slave* processors, as shown in Figure 4.

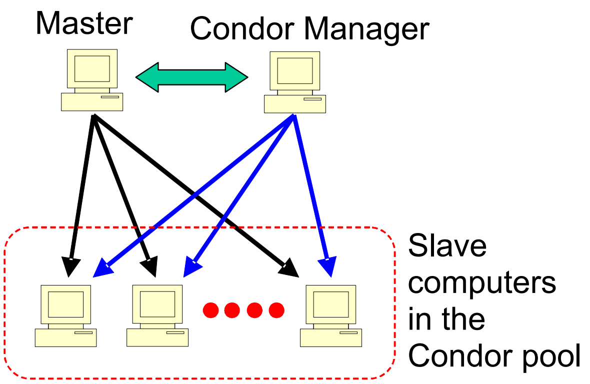

Figure 4: Organisation of the distributed computing network that is managed by the Condor software.

The master processor takes care of managing the population and the synchronisation of the slave processors. The master processor handles the selection and mating. The slave processors calculate the cost function and return the results. In the simplest form, all the cost functions are evaluated at the same time, and the master processor waits until all the results are returned from the slave processors. This scheme is called a *synchronous* master-slave GA, which is relatively easy to implement but suffers from the drawback that it requires all the slave processors to complete their tasks before the master processor can update the population. If one of the slave processors fails, then the population cannot be updated. In addition the GA can only update at the speed of the slowest slave processor. A variation of this scheme is the *semi-synchronous* master-slave, where the master process does not depend on the completion of all the slave processors, and instead selects members 'on the fly'.

Distributed Computing Network

The Condor software package is freely available and can be used for distributed computing. It is best suited to 'embarrassingly parallel' problems, where there is no inter-processor communication. This type of framework is suitable for parallel GAs where a master processor is responsible for 'managing' the population, conducting the breeding and mutation and distributing the cost function evaluations to 'slave' processors. The slave processors receive the jobs from the master processor, and evaluate the cost function, and return the value of the cost function, in this case the acoustic potential energy within the cavity.

The Condor system was installed on 180 Pentium computers in the Computer Aided Teaching Suite in the Faculty of Engineering, Computing and Mathematical Sciences at the University of Adelaide, and one computer was assigned as the master processor and distributed the jobs to the slave computers.

An attractive feature of the Condor system is that the master processor responsible for the distribution of tasks can look at the usage of the computer and tell if someone is using the machine. If the machine is being used by a student, then that computer is marked as unavailable. The system was set up so that computers that have been left idle for 15 minutes are marked as available to the Condor pool. One limitation of the present system, for computers running Microsoft Windows XP, is that if the computer is used by a student whilst it is processing a job, the computer will halt the task and find another computer to run the task, but it will re-start the job from the beginning instead of continuing from where the interruption occurred. This is not a significant problem because the students do not use the computers overnight, when they are typically used for the GA calculation.

The Matlab compiler (Mathworks 2004) was used to create a binary executable file for the cost function evaluation. The executable file reads in Matlab .mat files and extracts the parameters for the PVADs and then uses those parameters to calculate the acoustic potential energy, which is then written to an output Matlab .mat file. The genetic algorithm reads the results from the cost function evaluation and determines if there was any improvement in the results. It should be noted that compilation of the Matlab code generally does not reduce the excution time compared to using the interactive environment.

Finite Element Model

A finite element model of a vibro-acoustic system was developed for use with the computational framework described here. The system is a large cylindrical cavity enclosed by a composite material that is approximately 2.56m long, 2.46m in diameter, and has 2mm thick walls. Figure 5(a) shows a cross sectional view of the finite element model of the structure created using the ANSYS software package, and Figure 5(b) shows the corresponding acoustic model that is enclosed by the structure, and it has an average mesh density of 0.08m.

The ANSYS software package was used to extract the first 300 modes of the in-vacuo structural and acoustic mode shapes of the cylindrical vibro-acoustic system.

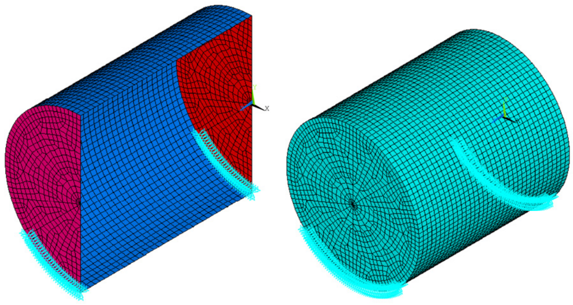

Figure 5: (a) Structural and (b) acoustic finite element models of the composite cylinder.

RESULTS

The acoustic loading on the cylindrical structure was assumed to be a plane wave incident on the cylinder at 90 degrees to the axis of the cylinder. It is recognised that it is important to accurately model the acoustic loading on the external of the fairing structure to make accurate predictions of the internal noise levels. However the focus of the work conducted here was aimed at the development of a powerful computational tool, and not to develop accurate acoustic loading conditions that are representative of actual launch conditions. Potter (1966) describes the mathematical derivation of the acoustic pressure on exterior of a cylinder from a harmonic plane wave incident on a cylinder at an oblique angle. Figure 6 shows a model of the acoustic loads on the cylinder. It is assumed that a harmonic plane wave is travelling in the $-x$ direction and strikes the cylinder. The acoustic pressure distribution on the cylinder will have a cosine distribution. The acoustic pressure can be converted to an equivalent nodal force that acts normal to the surface of the structure and is given by

$$F_{\text{node}} = P_0 A_{\text{node}} \cos(\theta) \exp(ikR\cos(\theta)) \tag{19}$$

where P_0 is the amplitude of the incident acoustic harmonic wave which is assumed to be 200Pa (=140dB re 20μPa), A_{node} is the area associated with each node that is determined during the finite element analysis of the system, θ is the angle measured to a point on the cylinder measured from the direction of the incident sound wave, i the complex number, k is the wave-number of the incident sound, and R is the radius of the cylinder. It should be noted that for simplicity of the model, diffraction of sound around the cylinder and radiation damping have not been included in this model of the acoustic loading.

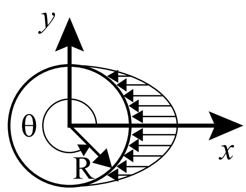

Figure 6: Mathematical model of the acoustic loads on the cylinder.

Optimisations were conducted to determine the variation of the acoustic potential energy within the cavity when varying the number and properties of PVADs attached to the walls of the cylinder. Optimisations were conducted over 18,000 cost functions. The parameters of the PVADs were permitted to vary over the ranges listed in Table 1. The masses of the TMDs and HRs were fixed at 0.45kg and 0.01kg, respectively. Hence, the greater number of PVADs, the greater the mass attached to the structure.

Table 1: Parameter ranges for the PVADs used during the optimisation of the composite cylinder.

PVAD parameter	Min	Max	No. Values
PVAD position	1	4032	4032
Mass-spring frequency (Hz)	11	510	500
Mass-spring damping (η)	0.01	0.25	10
Acoustic resonator frequency (Hz)	11	510	500
Acoustic resonator damping (η)	0.01	0.25	10

The semi-synchronous parallel genetic algorithm and the distributed computing network were used to optimise the locations and the parameters of a varying number of PVADs attached to the walls of the composite cylinder, such that the total acoustic potential energy over the frequency range from 50-300Hz was minimised.

Optimisations were conducted using 10, 20, and 30 PVADs and Figure 7 shows the variation in the acoustic potential energy versus frequency for the optimisations. Figure 8 shows the locations of the 30 PVADs on the cylinder, which have been displayed as if the cylinder had been unwrapped, with 0 degrees aligned with the indicent sound pressure wave. Figure 9 shows the total acoustic potential energy inside the cylinder from the three optimisations. The results show that the lowest acoustic potential energy occurs when 30 PVADs are attached to the fairing, which resulted in a 4dB reduction in the acoustic potential energy inside the cylinder. This result is not surprising as the use of 30 PVADs has the greatest amount of mass added to the structure.

This computational tool has also been used to conduct optimisations to evlaute the effectiveness of the PVADs for a constant added mass 'budget' and the benefit of spring-mounting the masses as compared to the blocking-mass effect of rigidly attaching the mass to the structure, which was considered by Gardonio et al. (2001).

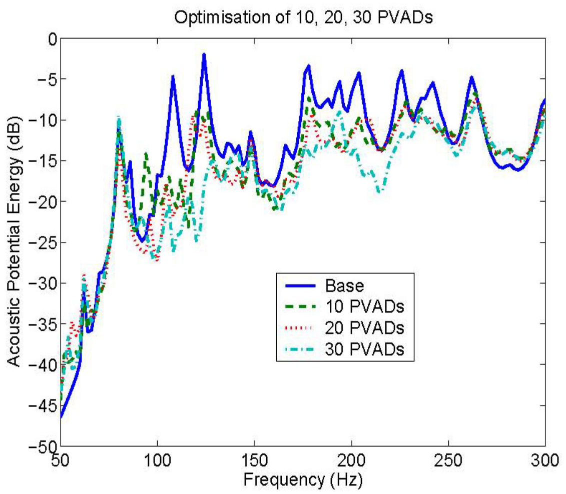

Figure 7: Acoustic potential energy versus frequency for no PVADs (Base), 10, 20, and 30 PVADs.

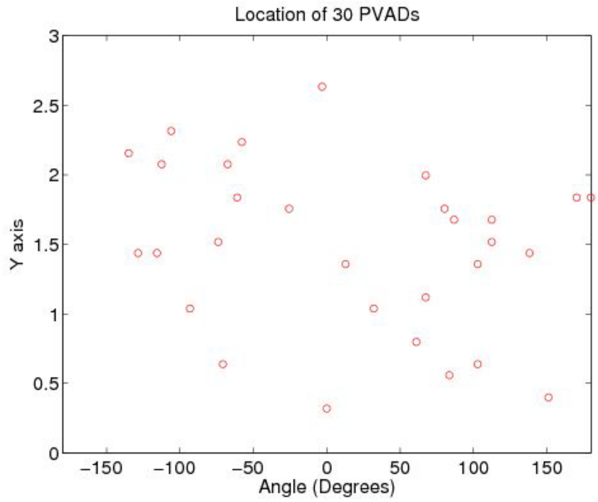

Figure 8: Location of the 30 PVADs after 18,000 cost function evaluations, displayed as if the surface of the cylinder was unwrapped.

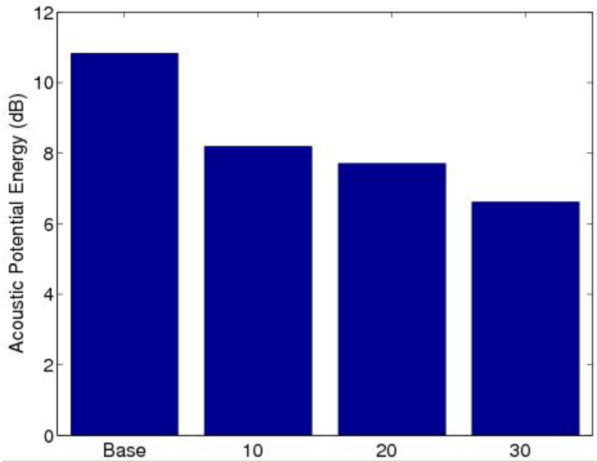

Figure 9: Total acoustic potential energy for no PVADs (Base), 10, 20, 30 PVADs.

CONCLUSIONS

The work presented in this paper has demonstrated how an asynchronous parallel genetic algorithm implemented on a distributed computing network can be used to optimise the locations and stiffnesses of tuned mass dampers and Helmholtz resonators attached to a vibro-acoustic system. The use of the large computational resources enabled the analysis and optimisation of a complex vibro-acoustic system, which would have been infeasible with a standard desktop computer.

A mathematical model was developed that can be used to calculate the acoustic potential energy inside a vibro-acoustic system. The computational tool uses the results from a finite element package, where the in-vacuo mode shapes for the structure and the acoustic space are determined, and these modal results are coupled using modal-coupling software that was implemented in Matlab. The use of the finite element software enables vibro-acoustic systems with complex shapes to be modelled such as automotive cabins and rocket fairings used to launch satellites. The mathematical model also includes the effects of passive vibration dampers and passive acoustic resonators on the response of the system.

A semi-synchronous parallel genetic algorithm was developed and was used in conjunction with a distributed computing environment on about 180 desktop computers. The genetic algorithm was used to determine the optimum locations and parameters of passive vibration and acoustic absorbers that resulted in the minimisation of the sound levels inside a composite cylindrical structure.

ACKNOWLEDGEMENTS

This work was sponsored in part by the Air Force Research Laboratory, Space Vehicles Directorate (AFRL/VS), and the Air Force Office of Scientific Research (AFOSR), under contacts F6256200M9068 and F6256299M9179. The help provided by Dr Steven A. Lane throughout this project is gratefully acknowledged.

REFERENCES

Beranek, L. & Vér, I., eds (1992), *Noise and vibration control engineering. Principles and applications*, John Wiley and Sons, U.S.A.

Cazzolato, B. (1999), *Sensing systems for active control of sound transmissino into cavities*, PhD thesis, Department of Mechanical Engineering, University of Adelaide, Adelaide, South Australia, Australia.

Condor (2005), The *Condor Project Homepage*, University of Wisconson – Madison, USA, http://www.cs.wisc.edu/condor/.

Estéve, S. & Johnson, M. (2002), *Reduction of sound transmission into a circular cylinderical shell using distributed vibration absorbers and helmholtz resonators*, Journal of the Acoustical Society of America 112(6), 2840–2848.

Fahy, F. (1994), *Sound and Structural Vibration: Radiation, Transmission and Response*, Academic Press.

Fuller C.R., Elliott, S.J. & Nelson, P.A. (1996), *Active Control of Vibration*. Academic Press.

Gardonio, P., Ferguson, N. S. & Fahy, F. J. (2001), *A modal expansion analysis of noise transmission through circular cylindrical shell structures with blocking masses*, Journal of Sound and Vibration 244(2), 259–297.

Goldberg, D. (1989), *Genetic algorithms in search, optimization, and machine learning*, Addison-Wesley.

Howard, C., Hansen, C. & Pan, J. (1997), *Power transmission from a vibrating body to a circular cylindrical shell through active elastic isolators*, Journal of the Acoustical Society of America 101(3), 1479–1491.

Howard, C. (1999*), Active vibration isolation of machinery vibration from flexible structures*, PhD thesis, Department of Mechanical Engineering, University of Adelaide, Adelaide, South Australia, Australia.

Mathworks (2004), *Matlab compiler*, Mathworks, Natick, Massachusetts, USA http://www.mathworks.com/products/compiler/.

Morse, M. & Ingard, K. U. (1986), *Theoretical Acoustics*, McGraw-Hill, New York, USA.

Potter, R. (1966), *Correlation patterns of the acoustic pressure fluctuations on the S-IC vehicle due to the exhaust noise of the test and launch stand*. Technical Report Wyle Labs Report WR 66-15, Contract NAS8-20073-1.