Active Vibration Control of a Magnetorheological Sandwich Beam

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ABSTRACT

Sandwich beam structures constructed with MR fluids can be implemented as distributed vibration absorbers to suppress unwanted vibrations. This paper introduces an analytical model for MR structures based on the Kelvin-Voigt model and Hamilton principle. The relationship between the magnetic field and the complex shear modulus of MR sandwich beam in the pre-yield regime is presented. The governing partial differential equations describing the dynamics of MR sandwich beam are derived and a model analysis is performed. An active vibration controller based on Lyapunov stability theory is designed. Simulations show the stable response and improved transient performance provided by the control system.

INTRODUCTION

Flexible beam elements constructed with fabrics, composite, polymers, and light metals are increasingly employed in a variety of large structures in automotive, marine, aerospace, robotics, and machinery industries. On one hand, the advance of computer aided design has meant that designers can reduce traditional over-design necessary to produce a reliable structure. This is often achieved by reducing design safety factors to a minimum, while a traditional structure may be unnecessarily heavy to give the required stiffness and damping to eliminate vibrations. On the other hand, through synthetising technique, it is possible to use light materials to achieve a high strength-to-weight ratio under stringent constraints of energy consumption. The use of smart structural components that are able to change their stiffness and damping coefficients will enable lighter designs realisable.

These lighter structures, however, are physically characterised by low structural damping, low stiffness, and low natural frequencies. Consequently, the structures readily experience high-amplitude resonant vibrations under external disturbances, such as forces produced by unbalanced rotating machines, reciprocating machines, or shock impacts. High-amplitude resonant vibrations will degrade the reliability and safety of the structures. Fatigue failure or even collapse of structures is an awesome possibility under resonant vibrations. Therefore, the development of strategies for reducing low-frequency resonant vibrations has been a key area of interest of vibration control for flexible structures.

In the context of low-frequency resonance control of structures, there are several challenging Design Questions or requirements that are prevalent over different applications, namely:

1. how to design a real-time control system to suppress vibrations under a nonstationary excitation, i.e., an excitation whose frequency and/or amplitude vary at a finite rate with time?

2. how to integrate sensors/actuators into the structure to optimise the interaction between the control system and the structure without negating the benefits provided by the use of light materials?

3. how to design a distributed control system that can suppress both transient and steady-state multiple harmonic resonances and/or multiple-mode free vibrations without causing control spillover problem (A problem is referring to the interactions between the controller and the uncontrolled modes of the system)? and

4. how to implement distributed actuators with high control authority to achieve effective vibration control?

For resonance suppression, currently employed techniques are based primarily on passive methods such as adding dampers and springs between the disturbing source and the structure, or attaching to the structure with another spring-mass-damper system, known as a dynamic vibration absorber (DVA) [Den Hartog 1947]. Additional improvement in desired performance may be obtained by employing active measures by means of external servo like mechanism, such as using hydraulic or electro-mechanical actuators [Hansen and Snyder 1997].

Recent developments in the technology of smart structures have shown some promising solutions for vibration control of structures [Srinivasan 2001]. The use of smart or intelligent materials will enable construction of structures whose mechanical properties can be altered to avoid anticipated resonances. Varied smart materials such as piezoelectric ceramics, shape memory alloys, and magnetostrictive materials, have been widely used in smart structures [Preumont 2002].

The use of MR fluids, one of the most versatile fluids in current range of smart materials, has been suggested and tried for the construction of smart structural components by many researchers [Sims 1999]. MR fluids, which experience reversible changes in rheological properties such as viscosity, plasticity, and elasticity when subjected to a magnetic field, was first discovered by Jacob Rabinow in the late 1940s [Rabinow 1948, 1951]. Since then, publications describing the mechanism and applications of MR fluids have abounded. The fluids generally consist of micron or submicron ferromagnetic particles suspended in hydrocarbon or silicon oil. A key to the magnetorheological response of MR fluids lies in the fact that the polarisation induced in the suspended particles by application of an external magnetic field. The interaction between the resulting induced dipoles causes the particles to form columnar structures, parallel to the applied field, as shown in Figure 1.

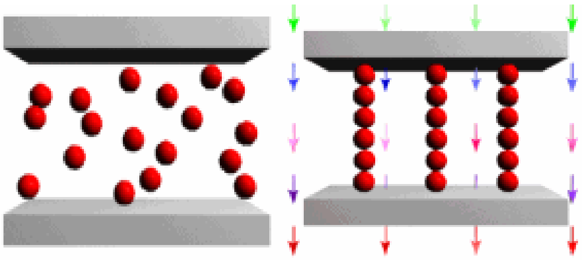

Figure 1. Micro-structures of MR fluids without/with external magnetic field.

These chain-like structures restrict the motion of the fluid hence increase the viscous characteristics of the suspension. In essence, MR fluid behaviour transforms from that of a liquid to that of a solid-like gel when an external magnetic field is applied. The dramatic transformation of MR fluids can be quite fast, on the order of 10^{-3} and 10^{-4} seconds [Jansen 2002], therefore, the MR fluids can be used as actuators in various damping schemes. The ability to change the yield strength of MR fluids according to the magnetic field enables MR fluids to alter the structural damping and stiffness coefficients under nonstationary excitations as described in the above Design Question 1.

A number of MR fluids and various MR fluid-based systems have recently been commercialised, such as a controllable MR fluid damper for use in truck seat suspensions [Carlson 1996], an MR fluid brake for use in the exercise industry [Anon 1995], and an MR fluid shock absorber for automobile racing track [Lord 1997]. MR fluid dampers can be controlled with a low power (e.g., less than 50 W), low voltage (e.g., 24V batteries) control source. They are capable of generating large control forces for industry applications [Dyke 1996a, 1996b]. Compared with other smart materials, such as piezoelectric ceramics, shape memory alloys, and another famous smart fluid electrorheological (ER) fluid, MR fluids can provide much higher control authorities. For example, MR fluids usually exhibit dynamic yield strengths of 50~100kPa for applied magnetic fields of 150~250 kA/m and off-state viscosity of 0.20~0.30 Pa-s at 25°C. Therefore they are much desirable as required by the Design Question 4.

Currently, all the developments about MR fluids have been focused on using them as lumped-parameter mass dampers or vibration isolators, which can be only attached to the primary structure point-wisely. For flexible structures, in order to control the multiple resonances simultaneously, multiple such MR dampers are required [Akhiev 2002]. This implementation will annihilate the benefits provided by the use of light materials, as a result of the attached masses. Therefore, it is necessary to develop a technology of distributed MR fluid absorbers for flexible structures so that this technology will satisfy the Design Question 2 and 3.

Although there are some existing research and applications on the technology of distributed ER fluid absorbers [Rahn 1994, Berg 1996, Yalcintas 1998, Park 1991], and there are some similarities between ER fluids and MR fluids, the technology of distributed MR fluid absorber will be quite of difference in that of ER fluid one. MR fluids compared with ER fluids provide much higher dynamic yield strengths, wider temperature range, greater insensitivity to temperature variation and contaminants. Another important factor in the growing acceptance by industry of MR fluid devices is the fact that an MR device is powered by a low voltage source. Many industries, notably aerospace and automotive, have all but ruled out ER fluid devices owing to a reluctance to provide electric fields of up to 4 kV/mm to excite ER fluids. However, to many potential users of smart fluids, the need to provide an electromagnetic circuit to excite the MR fluid is a small price to pay to obviate the requirement for high voltages. It is believed that MR fluids will have a significant impact on hydraulic and pneumatic equipment, and will be utilised in the aerospace, automotive, marine, and robotics industries.

To investigate issues of analytical modelling and control of MR structures, a MR sandwith cantilever beam is chosen here as the research vehicle for this work because it is a convenient structure for exhibiting such complicated phenomena in a controlled laboratory setting. This paper introduces an analytical model for MR structures based on the Kelvin-Voigt model and Hamilton principle. The governing partial differential equations describing the dynamics of a MR sandwich beam are derived and a model analysis is performed. The relationship between the magnetic field and the complex shear modulus of MR sandwich beam in the pre-yield regime is presented. An active vibration controller based on Lyapunov stability theory is designed. Simulations of a cantilevered MR beam show the stable response and improved transient performance provided by the control system.

The remainder of this paper is organized as follows. First, the MR sandwich cantilever beam configuration is introduced. Second, the viscoelastic behaviour of the MR fluids in the pre-yield regime is analysed. Then, the governing partial differential equations describing the dynamics of the MR sandwich beam are derived. Based on the derived model, an active vibration controller using Lyapunov stability theory is designed. The effectiveness of the active control system is shown through simulations of beam's impose response. Finally, some concluding remarks are given.

SYSTEM DYNAMIC MODEL

Model for pre-yield MR fluids

The relationship between shear stress and shear strain of MR fluids is schematically shown in Figure 2.

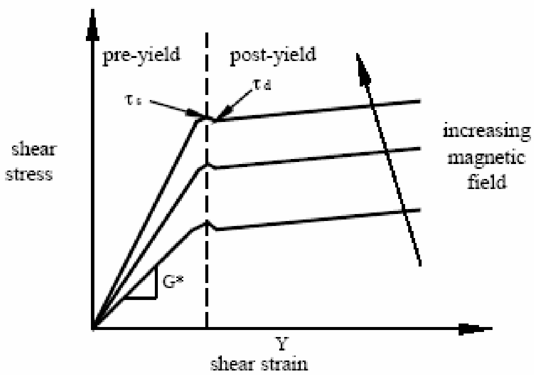

Figure 2. MR fluids shear stress versus shear strain under different levels of strength of magnetic field.

In the pre-yield regime, MR fluids can be represented by the Kelvin-Voigt model. The relationship between the shear stress τ and shear strain γ is written as,

$$\tau = G\gamma + \kappa\dot{\gamma}\,, \tag{1}$$

where G and κ are the complex shear modulus and viscous damping ratio of the MR material, respectively, both of which are functions of magnetic flied.

Dynamic model for MR sandwich beam

A simple sandwich cantilever beam system is selected as a research vehicle to implement the MR fluid technique. The cantilever beam system may effectively represent a simple model for various transport-vehicle structures, such as an aircraft wing, a helicopter blade, a solar panel of a solar vehicle, etc. A schematic diagram illustrating the configuration of the beam with MR fluids between two aluminium plates is shown in Figure 3.

For simplicity, planar motion and isotropic beams of constant cross-section are assumed here. In addition, the case of a long slender beam experiencing small strains and moderate deformations are considered. Furthermore, it is assumed that:

1. no extension of the beam's neutral axis occurs;
2. no slipping occurs between the aluminium layers and the MR layer;
3. all three layers have the same transverse displacement; and
4. no shear strains exist inside the aluminium layers.

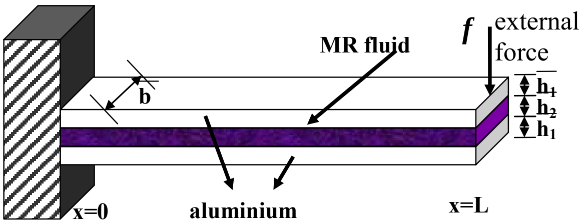

Figure 3. Configuration of the MR sandwich beam.

When the beam is subject to external disturbance, the displacements, rotation, and shear strain of the beam are shown in Figure 4.

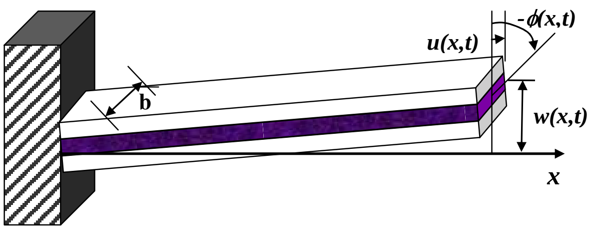

Figure 4. Variables of the MR sandwich beam.

The longitudinal extension of the midsurface of the top plates, transverse displacement, cross section rotation, and shear strain of the MR layer are represented by $u(x,t)$, $w(x,t)$, $\phi(x,t)$, and $\gamma(x,t)$, respectively.

To find out the governing partial differential equations of the sandwich beam, the Hamilton principle is applied and the form can be expressed as [Meirovitch 1990]

$$\delta\int_0^t \left(T - V_e - V_b - V_s + W_f + W_v\right)dt = 0\,, \tag{2}$$

where T is the kinetic energy of the beam; V_e, V_b, and V_s are the potential energies owing to extensional, bending, and shear motions, respectively; W_f and W_v are work energies of the applied external force and viscous damping work, respectively.

The kinetic energy owing to transverse and rotational motions can be written as:

$$T = \frac{1}{2}\int_0^L \left(\rho\left[\frac{\partial w}{\partial t}\right]^2 + J\left[\frac{\partial \phi}{\partial t}\right]^2\right)dt, \tag{3}$$

where ρ and J are transverse and rotary inertia of the beam, respectively. L is the length of the beam. The extension energy in the two face plates is

$$V_e = h_1 b E_P \int_0^L \left(\frac{\partial u}{\partial x}\right)^2 dx\,, \tag{4}$$

where h_1 and b are the thickness of each face plates and the width, respectively. E_p is the Young's modulus of the face plates.

The bending energy in the face plates can be written as

$$V_b = E_P I_P \int_0^L \left(\frac{\partial^2 w}{\partial x^2}\right)^2 dx, \tag{5}$$

where I_p is the centroidal moment of inertia in the face plates. The shear energy in the MR layer is

$$V_s = \frac{1}{2} G h_2 b \int_0^L \gamma^2(x,t)dx. \tag{6}$$

The work energy owing to the external force f can be written as:

$$W = \int_0^L f(x,t)w(x,t)dx. \tag{7}$$

The work energy owing to the viscous damping can be written as:

$$\delta W_v = -bh_2\kappa\int_0^L \dot{\gamma}(x,t)dx\delta\gamma \tag{8}$$

The boundary conditions of the cantilever beam can be arranged as:

$$w = \frac{\partial w}{\partial x} = \phi = 0, \text{at} \quad x = 0 \;\text{ and}$$

$$\frac{\partial^2 w}{\partial x^2} = \frac{\partial \phi}{\partial x} = G\left(\frac{\partial w}{\partial x} - \phi\right) - \frac{\partial^3 w}{\partial x^3} = 0, \text{at} \quad x = L.$$

Substitution of Eqs. (3-7) into Hamiliton's equation (2) yields the governing partial differential equations of the beam:

$$\left\{\begin{aligned} &\rho\frac{\partial^2 w}{\partial t^2}-\frac{\kappa b(h_1+h_2)^2}{h_2}\frac{\partial^2}{\partial x\partial t}\left(\frac{\partial w}{\partial x}-\phi\right)\\ &\quad+2E_pI_p\frac{\partial^4 w}{\partial x^4}-\frac{Gb(h_1+h_2)^2}{h_2}\frac{\partial}{\partial x}\left(\frac{\partial w}{\partial x}-\phi\right)=f,\\ &J\frac{\partial^2\phi}{\partial t^2}-\frac{\kappa b(h_1+h_2)^2}{h_2}\frac{\partial}{\partial t}\left(\frac{\partial w}{\partial x}-\phi\right)\\ &-\frac{bh_1(h_1+h_2)^2E_f}{2}\frac{\partial^2\phi}{\partial x^2}-\frac{Gb(h_1+h_2)^2}{h_2}\left(\frac{\partial w}{\partial x}-\phi\right)=0.\end{aligned}\right. \tag{9}$$

In order to simplify Eq. (8), a set of nondimensional variables is defined as

$$x^*=\frac{x}{L},\; w^*=\frac{w}{L},\; \phi^*=\phi,\; t^*=\frac{t}{L^2}\sqrt{\frac{2E_pI_p}{\rho}},$$

$$G^*=\frac{bL^2(h_1+h_2)^2}{2h_2E_pI_p}G,\; I_p^*=\frac{bh_1(h_1+h_2)^2}{4I_p},$$

$$\kappa^*=\frac{b(h_1+h_2)^2}{h_2\sqrt{2\rho E_pI_p}}\kappa,\qquad J^*=\frac{J}{\rho L^2},\qquad \text{and}$$

$$f^*=\frac{L^3}{2E_pI_p}f.$$

Substitution of these variables into (8) yields the nondimensionalised equations of motion

$$\left\{\begin{aligned} &w_{tt}-\kappa(w_x-\phi)_{xt}+w_{xxxx}-G(w_x-\phi)_x=f\\ &J\phi_{tt}-\kappa(w_x-\phi)_t-I_p\phi_{xx}-G(w_x-\phi)=0\end{aligned}\right., \tag{10}$$

where all the superscript asterisks are omitted for notational simplicity and the subscript letters mean the partial differentiations.

For the case of cantilever beam, the rotary dynamics has much higher resonant frequency than the transverse vibration dynamics. Hence, the second equation of (9) can be further simplified by neglecting its first term

$$\left\{\begin{aligned} &w_{tt}-\kappa(w_x-\phi)_{xt}+w_{xxxx}-G(w_x-\phi)_x=f\\ &\kappa(w_x-\phi)_t+I_p\phi_{xx}+G(w_x-\phi)=0\end{aligned}\right.. \tag{11}$$

From the principle of modal analysis [Inman 2001], it is known that the complete dynamic behaviour of the cantilever beam can be discretised as a set of individual modes of vibration, each having a characteristic natural frequency, damping factor, and mode shape. By using these modal parameters to represent the system model, the governing equations at specific resonances can be examined and subsequently solved. The solutions for (11) can be expressed as

$$w(x,t)=\sum_{n=1}^{\infty}W_n(x)q_n(t), \tag{12}$$

$$\phi(x,t)=\sum_{n=1}^{\infty}\Phi_n(x)q_n(t), \tag{13}$$

where, $W_n(x)$ and $\Phi_n(x)$ are the mode shape functions for the n^{th} transverse and rotational modes, respectively, and $q_n(t)$ is the n^{th} generalised coordinate. Then, the modal dynamic equations in nondimensional form can be obtained

$$\ddot{q}_n+\kappa\left[\frac{\lambda^4 I_p}{\lambda^2 I_p+G}\right]\dot{q}_n+\left[\lambda^4+\frac{G\lambda^4 I_p}{\lambda^2 I_p+G}\right]q_n=0, \tag{14}$$

where λ is the solution from the characteristic equation

$$I_p\lambda^6-G(1+I_p)\lambda^4-\omega_n^2I_p\lambda^2+\omega_n^2G=0. \tag{15}$$

Here ω_n is the beam's n^{th} mode angular frequency.

ACTIVE CONTROLLER DESIGN

In this section, an active vibration control law is developed for the MR sandwich beam described in Equation (11). The purpose of the controller design is to suppress all the mode's vibration energy from the beam upon which external forces are imposed. To achieve this objective, the Lyapunov stability theory for distributed systems [Leipholz, 1980] is used. The design methodology is summarised below.

The total energy of the MR sandwich beam can be chosen as a Lyapunov function candidate:

$$L=\int_0^1(T+V_e+V_b+V_s)dx. \tag{16}$$

The Lyapunov function is positive definite and bounded with respect to the norm

$$\|[w,\phi]\|^2=\frac{1}{2}\int_0^1\left(w_t^{\ 2}+\phi_t^{\ 2}+w_{xx}^{\ \ 2}+\phi_x^{\ 2}\right)dx. \tag{17}$$

The Lyapunov function's first derivative of time can be found as

$$\dot{L}=-\kappa\int_0^1(w_{xt}-\phi_t)^2dx-\frac{\beta G}{2}\int_0^1\left[(w_x-\phi)^2\right]_t dx, \tag{18}$$

where the shear modulus $G_s(B)=G(1+\beta(B))$ with $|\beta|\ll 1$. Here B is the magnetic induction.

According to the Lyapunov theory, system (11) is asymptotic stabile if the first time derivative of the Lyapunov function is negative, i.e.,

$$\dot{L}\le 0. \tag{19}$$

The first term in (18) is always negative as the viscous damping ratio $\kappa(B)>0$. The second term in (18) is also negative if

$$\beta=-g(y), \tag{20}$$

where

$$y = -\frac{1}{2}1\int_0^1 \left[(w_x - \phi)^2\right]_t dx, \tag{21}$$

and g(.) can be designed as a continuous function with

$$\begin{cases} g(y) > 0 & \text{if } y > 0 \\ g(y) < 0 & \text{if } y < 0 \end{cases}. \tag{22}$$

The control law (20) can be rewritten in terms of the magnetic induction. As explained in equation (1), the MR fluids can be modelled as the Kelvin- Voigt material. The functional dependence of G and κ can be approximated as:

$$\begin{cases} G(B) = \alpha_1 B + \alpha_2 \\ \kappa(B) = \varepsilon_1 B + \varepsilon_2 \end{cases}, \tag{23}$$

where α_1, α_2, ε_1, and ε_2 are experimentally determined parameters. The control law is then designed as

$$B = B_0 - \frac{\beta G}{\alpha_1} = B_0 - \frac{g(y)G}{\alpha_1}, \tag{24}$$

where

$$B_0 = \frac{G - \alpha_2}{\alpha_1}. \tag{25}$$

SIMULATION FOR THE MR CANTILVER BEAM

To demonstrate the effectiveness of the control law derived above, a MR cantilever beam is simulated and the corresponding experimental study is currently under way. Table 1 shows all the parameters used for the simulation.

Table 1. MR sandwich beam parameters

Beam dimensions	
L=254mm	b=25.4mm
h_1=1mm	h_2=1mm
Aluminium layer	
ρ=2710kg/m^3	E_p=70 GPa
MR layer (Lord MRX-336)	
G=1.75kPa/T	κ=0.75 Pa-s
α_1=3.56 Ξ 10^{-4}	ε_1=3.85 Ξ 10^{-6}
α_2=5.78Ξ 10^{-1}	ε_2=6.31Ξ 10^{-3}

The control law (24) is implemented with a saturation feedback function:

$$g(y) = \begin{cases} 1 & \text{if } y > 1/k \\ ky & \text{if } |y| < 1/k \\ -1 & \text{if } y < -1/k \end{cases}, \tag{26}$$

where *k* is a controller design gain.

The schematic diagram of the active vibration control system is shown in Figure 5. Here a single point sensor is used to measure the longitudinal motion of the beam. The output of the controller is used to change the magnitude of the magnetic field or the magnetic induction.

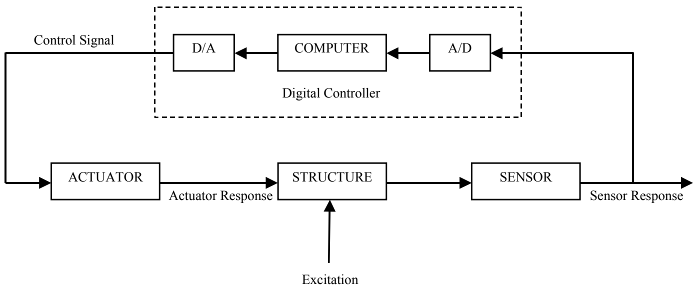

Figure 5. Schematic diagram of the active vibration control system.

Figure 6 and 7 show the simulated time responses of three modes of the MR sandwich beam to a unit impulse applied at the free end. The case without control, i.e., no external magnetic field is applied, is shown in Figure 6. A long decay time can be observed. This case shows that the MR fluid has litter effect on the system's damping when no magnetic field present.

The case under control is shown in Figure 7. It can be seen that the response decays exponentially. This case shows the effectiveness of increased system damping owing to the controlled MR fluid.

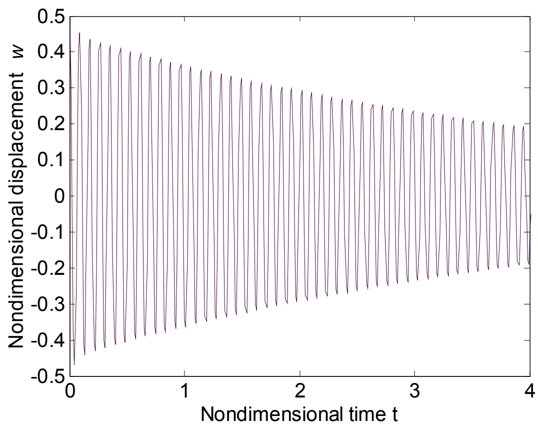

Figure 6. Simulated response of the MR sandwich beam to a tip impluse when no control is applied.

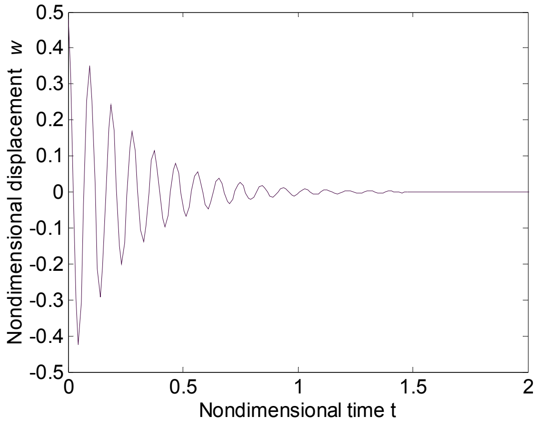

Figure 7. se of the MR sandwich beam to a tip impulse when control is applied.

CONCLUSIONS

The use of magnetorheological (MR) fluids, one of the most versatile fluids in current range of smart materials, has been suggested and tried for construction of smart structural components by many researchers. In essence, MR fluid behaviour transforms from that of a liquid to that of a solid-like gel when an external magnetic field is applied, therefore, the MR fluids can be used as actuators in various damping schemes. In this paper, a sandwich beam structure constructed with MR fluids is studied and implemented as distributed vibration absorbers to adapt to a changing environment, such as variable loadings or uncertain disturbances.

In this paper, an analytic model for the MR sandwich beam is derived. The MR fluids can be modelled as Kelvin-Voigt material in the pre-yield regime. The governing differential equation of motion is developed using Halmilton's equation. Based on the derived model, an active vibration controller is designed using Lyapunov stability theory which enures stability for all modes of the beam. The controller will change the magnitude of the magnetic field, therefore, alter the damping ratio of the MR fluids and damp out the vibrations in the beam. Simulated responses of the MR cantilever beam show the effectiveness of the proposed MR structure and controller.

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