A new cost function algorithm for adaptive-passive vibration and acoustic resonators

Carl Q. Howard

School of Mechanical Engineering, The University of Adelaide, Adelaide, S.A. Australia

ABSTRACT

Adaptive-passive vibration and acoustic control systems can provide significant attenuation without requiring significant power compared to an active control system. The control system includes an algorithm to evaluate a cost-function to be minimised, which is typically a time-averaged multiplication of the vibration or noise on the resonator and at a point in the system where the disturbance is to be minimised. This conventional algorithm of cost-function evaluation is suitable for simple systems. A new cost-function evaluation algorithm is proposed based on the sliding-Goertzel algorithm, that is tolerant of multiple harmonics and high background broadband noise. The algorithm was demonstrated on an experimental rig of an adaptive tuned vibration neutraliser and a two-stage vibration isolation system.

INTRODUCTION

Adaptive-passive tuned vibration and acoustic absorbers are devices that are attached to a primary system to reduce tonal vibration or noise in a primary system. These tuneable devices can alter their resonance frequency so that they can be tuned to a tonal excitation vibration or noise in the primary system.

An adaptive-passive control system, shown in Figure 1, comprises the adaptive-passive device, an actuator that alters the dynamic characteristics of the device, sensors such as accelerometers or microphones, a controller (such as a digital signal processor), and the adaptive algorithm software that operates on the controller. The sensors measure the vibration or acoustic response of the system, which is digitised and used to evaluate a metric of the system response. For vibration systems, the metric might be the acceleration, velocity, displacement at a point on the system, or more complex metrics such as kinetic energy, power flow, or energy density. For example, often the vibration of a system is to be minimised, and hence an appropriate cost function to be minimised is the velocity measured by an accelerometer. The adaptive algorithm determines an appropriate signal to send to actuators that will cause the cost function to be minimised.

This paper describes a new method of determining the relative phase angle and signal amplitude in an adaptive-passive vibration or acoustic system, which can be used as the cost-function to be minimised. The method is based on the sliding-Goertzel algorithm for determining the Fourier coefficients at a single frequency of interest.

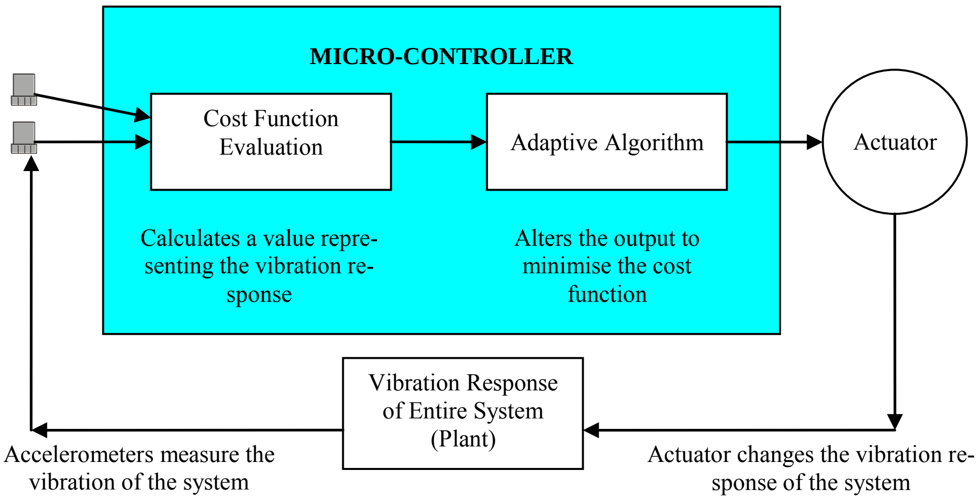

Figure 1: Adaptive-passive control system.

This paper is organised as follows - the *Background* section provides an overview of the intrinsic difficulties with tuning an adaptive-passive device to attenuate a primary tonal excitation. The *Cost Function* section describes commonly used cost-functions and the problems with these methods in realistic vibration or acoustic environments with high background noise levels and harmonics, followed by a description of the proposed sliding-Goertzel algorithm. The *Experimental* section describes a laboratory test of the application of the sliding-Goertzel with an adaptive-passive Tuned Vibration Neutraliser (TVN) attached to a two-stage vibration isolation system.

BACKGROUND

A schematic of a tuned vibration neutraliser (mass m_2, variable stiffness k_2, and damping c_2) attached to a vibrating primary structure m_1, is shown in Figure 2. The variable stiffness element in the tuned vibration neutraliser k_2, can be altered so that the resonance frequency of the device can be tuned to coincide with the frequency of the vibration excitation F_1. The equations of motion of this system are described in Howard (2009). The phase of the frequency response function between the vibration of m_2 and m_1 is shown in Figure 3. At driving frequencies less than the resonance frequency of the TVN, the masses vibrate in phase, and at driving frequencies greater than the resonance frequency, the masses vibrate out of phase. The TVN provides vibration attenuation when its resonance frequency coincides with the excitation frequency, which has the characteristic that the phase angle is -90 degrees. Hence, the goal of the control system is to tune the adaptive TVN such that the phase angle is -90 degrees.

Three phase states can be defined where the phase angle is:

- close to 0 degrees = State 1,
- close to -180 degrees = State 3, and
- in the region of the (near) step change between 0 to -180 degrees = State 2.

The goal of an effective tuning algorithm is to alter the dynamics of the TVN so that it is resonant, which occurs when the phase angle is -90 degrees (in State 2). The sharp swing in the phase response presents atypical challenges for designing a stable and practical control system. The response of most systems (plant) involves relatively gradual changes in response due to changes in the input conditions, and textbook control algorithms such as PID, H-infinity and others are suitable. Unfortunately, due to the step-response in the cost-function (phase angle) for this problem, many of these textbook control algorithms are not suitable.

COST FUNCTION ALGORITHMS

Researchers have used several methods for calculating an appropriate cost function that are described in the following section. The practical limitations of these algorithms are described, followed by the proposed algorithm.

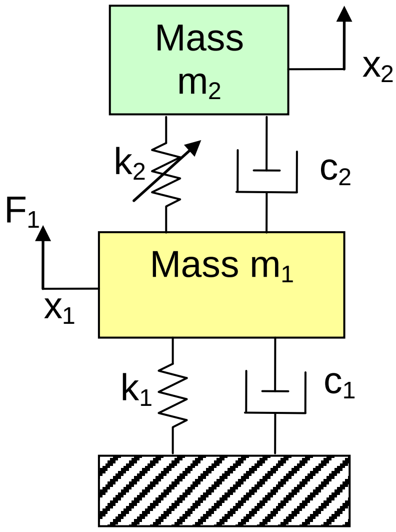

Figure 2: Model of an adaptive tuned vibration neutraliser attached to a vibrating primary structure.

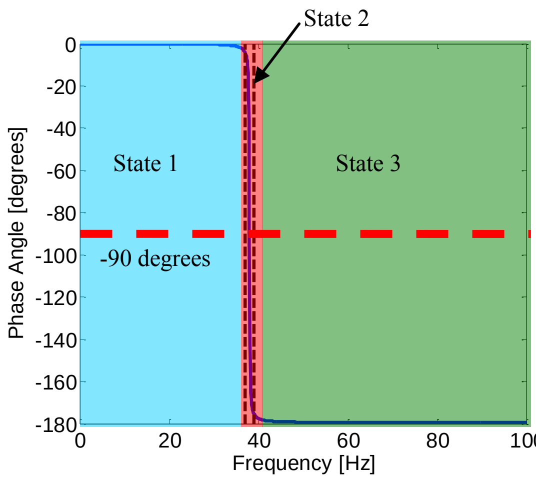

Figure 3: Definition of system states based on the phase angle of the TVN.

Previous

As described in the previous section, when adaptive-tuned neutralisers are tuned to be resonant, they attenuate vibration or noise in the primary system. These traits have led to two main classes of cost functions: (1) minimisation of cost functions based on the amplitude of vibration or noise of the primary system; (2) a cost function that is based on a frequency response function between the resonant part of the adaptive tunable device and at a point in the primary system that is to be attenuated.

Consider the first class where a cost function based on vibration or noise amplitude is to be minimised. For systems where the excitation level is known and stable, this first method may be suitable. However, this method does not account for changes in excitation level, so that it is not possible to determine if the vibration or noise in the system has been minimised.

The second class of cost functions based on a frequency response function overcomes the limitation of the first class by normalising the response due to the excitation level. This class can be further sub-divided into algorithms that use the amplitude or phase of the frequency response function. Adaptive algorithms that use a cost function based on the amplitude of the FRF attempt to maximise it, which will cause the adaptive-passive device to be resonant. Provided that the peak amplitude of the FRF of the resonant adaptive-passive device is stable, then this method can work well. An alternative is to control the system based on the phase angle of the FRF - by altering the dynamics of the adaptive-passive device until the relative phase angle is -90 degrees.

A common method used to calculate phase angle of a TVN (or similar acoustic system) is the multiplication of the acceleration signals of the primary structure and the acceleration signal from the mass of the TVN, and then calculate a moving time average. For the case where the acceleration of the primary structure $\ddot{x}_1$ and the acceleration of the mass of the vibration absorber $\ddot{x}_2$ are given by

$$\ddot{x}_1 = \ddot{X}_1 \cos(\omega t) \tag{1}$$

$$\ddot{x}_2 = \ddot{X}_2 \cos(\omega t - \theta) \tag{2}$$

where $\ddot{X}_1$ and $\ddot{X}_2$ are the acceleration amplitudes, ω is the frequency, and θ is the relative phase angle of the acceleration between the masses, then the time average product of these two *tonal* signals is given by (Brennan et.al. 1996)

$$\overline{\ddot{x}_1\, \ddot{x}_2} = \frac{1}{T}\int_0^T \ddot{x}_1 \ddot{x}_2 dt = \frac{\ddot{X}_1 \ddot{X}_2}{2}\cos\theta \tag{3}$$

This calculation results in a signal that is "DC" meaning that it is offset from zero and does not have sinusoidal components. It can be seen from Eq. (16) that the calculation results in a signal that is proportional to the cosine of the phase angle. When the TVN is optimally tuned, the phase angle will be θ=90 degrees, so cos(90)=0, and hence the time average product will equal zero. This method has been used by several researchers (for example, see Brennan *et al.* 1996, Long *et al.* 1995, Johnson *et al.* 2005).

One of the attractive features of using this method to calculate the phase angle, is that it is computationally simple and hence fast to compute using digital processors, or using analog circuits (Johnson etal, 2005). However, all these method described above have requirements on the signals to enable the methods to work.

- The signals can only contain single tones at the specific frequency of interest. If the signals contain unrelated tones then these methods will not work, unless the unwanted signals are filtered to leave only the vibration signals at the frequency of interest. This can require the use of tracking narrow band-pass filters, which adds complexity to the system. All filters have the potential to cause amplitude and phase shifts on the original signal and care has to be taken not to unintentionally alter the signals. A tracking narrow band filter can be created using an adaptive feed-forward Least Mean Squares filter, however this also require calibration of the system to ensure that the adaptive filter will always be stable.
- When transient vibration (or noise) occurs, broadband vibrational energy is injected into the sensors that can be unrelated to the vibration frequency of interest. Again, it is necessary to remove these spurious vibration signals. If an adaptive feedforward filter is used, the system must remain stable when subjected to transients.

An alternative algorithm is proposed in the following section that addresses these limitations, and is deterministic and stable to transient signals.

Proposed Sliding-Goertzel Algorithm

To overcome these limitations, a new method of determining the phase-angle was developed. The Goertzel algorithm (Mitra, 1998) is a computationally fast method of calculating the complex Fourier transform at a single frequency or bin. The Fast Fourier Transform (FFT) method requires the calculation of all frequency "bins" and hence takes longer to calculate. Another very important feature of using the Goertzel algorithm is that the frequency of interest can be selected precisely, whereas the FFT can only calculate the Fourier transform at frequencies that are multiple of the frequency spacing. The frequency spacing for the FFT method is the sampling frequency divided by half the number of analysis "lines" or bins. The number of bins must be a power of 2, such as 2^9 = 512 lines. For example, if the sampling frequency is 500 Hz and (N=) 512 bins are used, then frequency spacing is 500 / (512/2) = 1.95 Hz. However with the Goertzel algorithm, the number of lines, or frequency resolution, is selected (N), and the frequency of analysis (k) is selected that can be a non-integer within the range from 1-N. Hence it is possible to calculate the Fourier transform at the excitation frequency.

A further improvement to the measurement of the phase angle is to replace the standard Goertzel algorithm with the sliding Goertzel algorithm (Chicaro etal, 1996; Jacobsen etal, 2003; Jacobsen etal, 2004). The latter has better accuracy when there is a poor signal-to-noise ratio, such as when the signal amplitudes are small, or when there is a second tonal signal with a similar frequency. The "sliding" nature of the algorithm enables the calculation of the Fourier coefficients in less than one signal period, which is advantageous for providing rapid updates of the value of the cost-function.

The sliding Goertzel algorithm implements the z-domain transfer function of (Jacobsen etal, 2003; Garcia-Retegui etal, 2007)

$$H_{SG}(z) = \frac{\left(1 - e^{-j2\pi k/N} z^{-1}\right)\left(1 - z^{-N}\right)}{1 - 2\cos\left(2\pi k/N\right)z^{-1} + z^{-2}} \tag{4}$$

where N is the N-point Discrete Fourier transform (DFT), and k is the frequency variable. Hence the single analysis frequency is kf_s / N Hz where f_s is the sampling frequency.

The sliding Goerztel algorithm was implemented in the Matlab-Simulink software and a schematic of the model is shown in Figure 4. Starting from the top left of the figure, the two vibration signals are inputs to the sub-system and are multiplexed together. They are passed through a zero-order hold block and then buffered. The buffering operation groups the individual samples into a "frame" of 512 samples, to enable the calculation of the Fourier Transform using the Goertzel method. Before calculating the Fourier transform, a Hanning window is applied to the "frame" of data to truncate the data set and to ensure that there is no spectral "leakage". The Goertzel algorithm is used to calculate the complex Fourier transform coefficients of two input signals at the reference frequency. These two data leave the sub-system in a vector and separated into two elements. Next, the transfer function is calculated, which is the division of the two Fourier transformed signals. To prevent "division by zero" errors from occurring, a small bias is added to the denominator. The transfer function is exponentially averaged, and then the magnitude and phase of the transfer function are calculated. The phase angle is converted from radians to degrees by multiplying by 180/π. The phase angle is the output from this sub-system block.

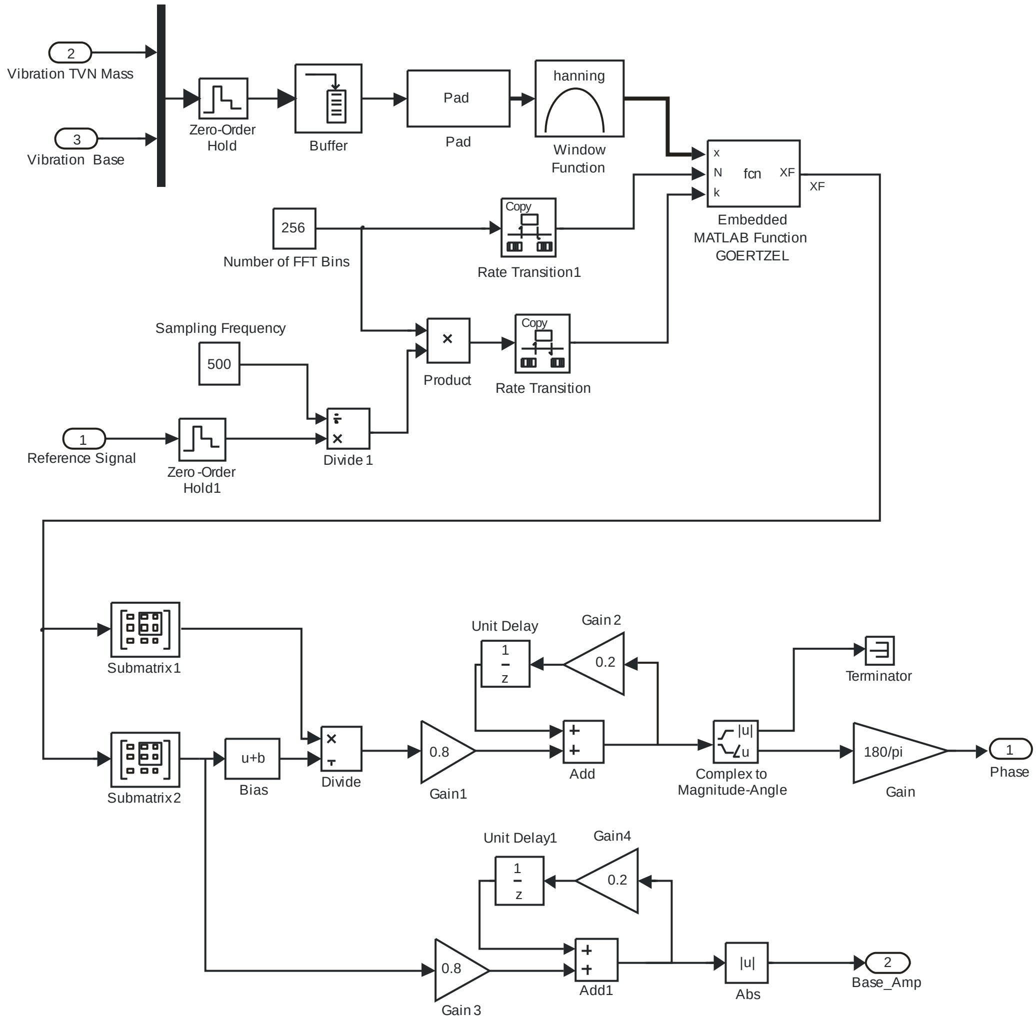

Figure 4: Simulink model to calculate the 'cost function' that uses the Goertzel algorithm.

EXPERIMENTAL DEMONSTRATION

An experiment was conducted to demonstrate the use of the sliding Goertzel algorithm as a cost function for tuning adaptive passive vibration (and potentially acoustic) resonator systems. Note that the experimental program was not intended to be a comparison between the proposed and alternative methods. The commonly used method of evaluating the time averaged multiplication of two vibration signals, as used by Brennan et.al. (1996) and others, cannot be used when there are multiple tones in the source excitation, without additional filtering.

An adaptive tuned vibration neutraliser was constructed, as illustrated in Figure 5, that comprised masses on cantilever arms. The resonance frequency of the device is adjusted by moving the masses towards or away from the base support, which alters the effective stiffness in the system, and hence alters its resonance frequency. A motor actuator rotates a screw that is used to position the masses on the cantilever arms.

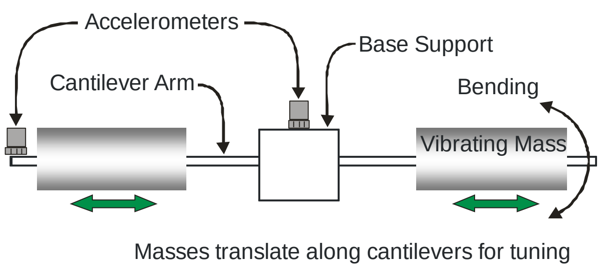

Figure 5: Adaptive tuned vibration neutraliser comprising masses on cantilever arms.

The adaptive algorithm, shown in Figure 1, uses the results from cost-function algorithm described here, to determine an appropriate signal for the motor actuator, that attempts to minimise the cost function. For these experiments, the control algorithm comprised coarse and fine tuning algorithms. The coarse tuning algorithm attempts to adjust the phase angle to within +/- 20 degrees of -90 degrees and then a fine tuning algorithm is used. The fine tuning algorithm measures the phase angle and then performs small adjustments to the positions of the mass.

The adaptive tuned vibration neutraliser was attached to a laboratory setup of a two-stage vibration isolation system. Figure 6 shows the experimental setup with the instrumentation. An electrodynamic shaker provided a vibration excitation source and was attached to the upper mass using a stinger. The force exerted by the shaker on the upper mass was measured by a force transducer, which enables the normalising of the results by the driving force. Two adaptive tuned vibration neutralisers were attached to the intermediate mass. Rubber vibration isolators separated the upper mass, intermediate mass and ground plate. A Bruel and Kjaer Pulse signal analyser was used to record the vibration spectra from the force transducer and four Bruel and Kjaer accelerometers attached to the intermediate mass. Accelerometers were attached to the end of the cantilever arms and the base support. The base support of the TVN was attached to the intermediate mass, hence by minimising the vibration at the base support of the TVN also minimised the vibration of the intermediate mass.

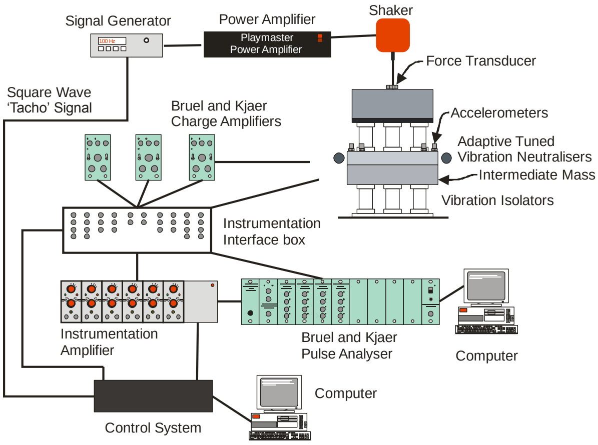

Figure 6: Experimental setup.

The hardware used to implement the control system was a DSpace 1104, which implemented the control algorithms that were programmed using the Matlab-Simulink software.

An experiment was conducted where the excitation frequency was set at 30Hz and then step changed to 40Hz. The B&K Pulse analyser was set to record only the normalised acceleration at 40Hz over time. The normalised acceleration was calculated as the acceleration at the base of the TVN divided by the force from the excitation shaker.

Results

The purpose of conducting this experiment was to demonstrate that the proposed Goertzel algorithm was suitable for use with an adaptive tuned vibration neutraliser. The focus was not the efficacy of the vibration isolation system, nor the speed of adaptation, which is a function of the mechanical design, signal processing, and speed of the actuator.

Figure 7 shows the reduction in vibration level of the system over time. At the start of the experiment the shaker was set to 30Hz, whereas the analyser was set to record at 40Hz, hence the initial data is random. After 10s the excitation shaker was set to 40Hz and the control system commenced adapting. It can be seen that the normalized acceleration initially increases, which is expected, and then decreases as the TVN enters state 2 and remains stable with a vibration reduction of about 35dB. It can be seen in Figure 7 that the time to tune the TVN from 30Hz to 40Hz took 50 seconds, which was due to the slow actuator used to position the masses. As the actuator and tuning was slow and the driving frequency was constant, no instability of the control system occurred during the experiments.

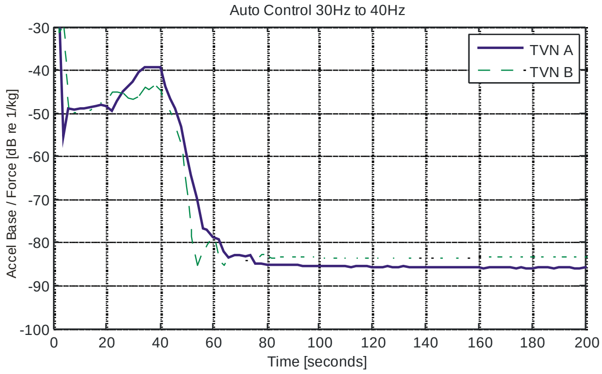

Figure 7: Experimental results showing the reduction in vibration amplitude.

The main point from this experiment is confirmation that both the sliding Goertzel algorithm for determining the phase angle and the control algorithm can tune the TVN and remains stable.

CONCLUSIONS

A sliding Goertzel algorithm was presented for use in determining the phase angle in an adaptive tuned vibration neutralizer. This algorithm provides an effective and robust method for extracting tonal vibration signals at the frequency of interest from a complex vibration spectra, whereas other methods require tracking filters. The sliding Goertzel algorithm requires less computational resources than the standard Fast Fourier Transform and can calculate the Fourier coefficients at the exact frequency of interest.

The results confirmed that the sliding Goertzel and tuning algorithms can be used effectively in adaptive-passive vibration control systems, and could also be used in adaptive-passive acoustic control systems.

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