Leading Element Dichotomous Coordinate Descent Exponential Recursive Least Squares Algorithm for Multichannel Active Noise Control

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ABSTRACT

In this paper, a new multichannel modified filtered-x (MFX) recursive least square (RLS) algorithm for active noise control (ANC) based on leading element dichotomous co-ordinate descent (LEDCD) iterations is proposed. It is shown that the proposed algorithm has less than half of the complexity of MFX fast transversal filter (FTF) algorithm with good performance for ideal plant models and improved robustness for noisy plant models.

INTRODUCTION

During last years many adaptive algorithms have been proposed for multichannel active noise control (ANC) applications. In ANC systems, an adaptive controller is used to optimally cancel unwanted acoustic noise (Kuo & Morgan, 1999). The use of the modified filtered-x (MFX) structure using finite impulse response (FIR) adaptive filtering (Bjarnason, 2002) will be assumed in the rest of this paper (Figure 1). Multichannel versions of the filtered-x least-mean-square (FX-LMS) and modified FX-LMS (MFX-LMS) algorithms are the benchmarks to which most adaptive filtering algorithms are compared, because they are widely used (Kuo & Morgan, 1999), (Bjarnason, 1992). Affine projection (AP) adaptive algorithms and their fast implementations are also known to be efficient when using in ANC systems (Bouchard, 2003 and Bouchard&Albu, 2004). However, AP algorithms possess slower convergence compared to recursive-least-squares (RLS) algorithms (Bouchard, 2003). Also, it is well known that the RLS algorithms have much faster convergence than FX-LMS and MFX-LMS algorithms, but they are too complex and often numerically unstable (e.g., see (Bouchard & Quednau, 2000) and references therein).

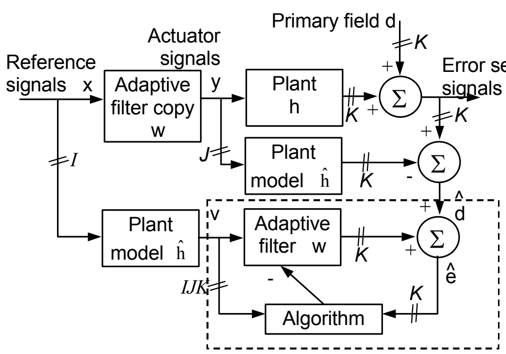

Figure 1. A delay compensated (or modified filtered-x) structure for active noise control.

Extensions of stable realizations of RLS algorithms such as the inverse QR-RLS, the QR decomposition least-squares-lattice (QRD-LSL) algorithms for specific ANC systems were presented in (Bouchard, 2002). However, they are too complex for practical implementations. One of the most useful fast versions of the RLS algorithm is the fast transversal filter (FTF) (Carayannis, Manolakis & Kalouptsidis, 2000). It exploits the time-shift invariance property of the input data. A practical implementation of the MFX-FTF algorithm is presented in (Gonzales, Ferrer, Diego, Pinero & Lopez, 2005). Previously, the numerical complexity of several RLS algorithms for ANC has been reduced by using the Gauss-Seidel algorithm (Albu & Paleologu, 2008a, 2010). However, the Gauss-Seidel algorithm has many multiplications and division operations. They can be avoided by using the Dichotomous Coordinate Descent (DCD) method proposed in (Zakharov & Tozer, 2004). This DCD version was included in an approximated RLS algorithm for ANC in (Albu & Paleologu, 2008b). Therefore, the typical matrix inversion encountered in such algorithms was avoided. However, there was a need to reduce the number of DCD iterations and includes the filter weights update within the DCD algorithm.

In (Zakharov, White & Liu, 2008) a new formulation of the RLS problem in terms of a sequence of auxiliary normal equations with respect to increments of the filter weights has been proposed. This approach was applied to the exponentially weighted RLS algorithm and a new structure of the transversal exponential RLS algorithm was derived (Zakharov, White & Liu, 2008).

In this paper, we adapt the structure of the transversal exponential RLS algorithm for the multichannel filtered-x structure and derive a novel efficient algorithm called the modified filtered-x leading element dichotomous co-ordinate descent exponential recursive least squares (MFX-LEDCD-ERLS) algorithm. It uses the leading element DCD algorithm (Zakharov, White & Liu, 2008). The new algorithm is derived, called the MFX-LEDCD-ERLS algorithm. Its computational complexity is evaluated and compared with that of other algorithms. Finally, simulation results and conclusions are presented.

MFX-LEDCD-ERLS ALGORITHM

In order to describe the algorithms most of notations and definitions from (Albu, 2006) are used. The variable n refers to the discrete time, I is the number of reference sensors, J represents the number of actuators, K is the number of error sensors, L is the length of adaptive FIR filters, M is the length of fixed FIR filters modeling the plant.

The vectors $\mathbf{x}_i(n) = [x_i(n),...,x_i(n-L+1)]^T$ and $\mathbf{x}'_i(n) = [x_i(n),...,x_i(n-M+1)]^T$ consist of the last L and M samples of the reference signal $x_i(n)$, respectively. The vector $\mathbf{y}_j(n) = [y_j(n),...,y_j(n-M+1)]^T$ consists of the last M samples of the actuator signal $y_j(n)$. The samples of the filtered reference signal $v_{i,j,k}(n)$ are collected in $\mathbf{V}_0(n) = \begin{bmatrix} v_{1,1,1}(n) & ... & v_{1,1,K}(n) \\ .... & & ...... \\ v_{I,J,1}(n) & ... & v_{I,J,K}(n) \end{bmatrix}$ and the $IJL \times K$ matrice $\mathbf{V}(n) = [\mathbf{V}_0^T(n)...\mathbf{V}_0^T(n-L+1)]^T = [\mathbf{V}_0^T(n)...\mathbf{V}_r^T(n)]^T$. The vectors $\hat{\mathbf{d}}(n) = [\hat{d}_1(n), \hat{d}_2(n),...,\hat{d}_K(n)]$ and $\hat{\mathbf{e}}(n) = [\hat{e}_1(n), \hat{e}_2(n),...,\hat{e}_K(n)]$ consist of estimates $\hat{d}_k(n)$ of the primary sound field $d_k(n)$ and of alternative error signal samples $\hat{e}_k(n)$, both computed in delay-compensated modified filtered-x structures. The vectors $\mathbf{h}_{j,k} = [h_{j,k,1},...,h_{j,k,M}]^T$ consist of taps $h_{j,k,m}$ of the fixed FIR filter modelling the plant between signals $y_j(n)$ and $e_k(n)$. $\mathbf{R}(n)$ is a $IJL \times IJL$ auto-correlation matrix initialized with an identity matrix multiplied by a regularization factor δ. It is updated as follows: $\mathbf{R}(n) = \begin{bmatrix} \bar{\mathbf{r}}(n) & \mathbf{r}^T(n) \\ \mathbf{r}(n) & \bar{\mathbf{R}}(n-1) \end{bmatrix}$, where $\bar{\mathbf{R}}(n)$ is the top left $IJ(L-1) \times IJ(L-1)$ elements of $\mathbf{R}(n)$, $\mathbf{r}(n)$ is a matrix of size $IJ(L-1) \times IJ$, initialized with zero values and $\bar{\mathbf{r}}(n)$ is a $IJ \times IJ$ matrix, also initialized with zero values. The vector $\mathbf{w}(n) = [[w_{1,1,1}(n)...w_{I,J,1}(n)] \; ... \; [w_{1,1,L}(n)...w_{I,J,L}(n)]]$ consists of the coefficients from all the adaptive FIR filters linking the signals $x_i(n)$ and $y_j(n)$. $e_k(n)$ is the kth error sensor signal, λ is the forgetting factor, and η is a gain scalar. Finally, $\mathbf{a}(n)$, $\mathbf{C}(n)$ and $\bar{\mathbf{w}}(n)$ are initially null $IJL \times 1$ vectors used in solving the auxiliary equations.

In the context of ANC systems, a multichannel feedforward system using an adaptive FIR filter with a modified filtered-x structure and with filter weights adapted with the ERLS-DCD algorithm can be described by the following equations:

$$y_j(n) = \sum_{i=1}^{I} \mathbf{w}_{i,j}^T(n)\mathbf{x}_i(n) \quad (1)$$

$$v_{i,j,k}(n) = \mathbf{h}_{j,k}^T \mathbf{x}'_i(n) \quad (2)$$

$$\hat{d}_k(n) = e_k(n) - \sum_{j=1}^{J} \mathbf{h}_{j,k}^T \mathbf{y}_j(n) \quad (3)$$

$$\hat{\mathbf{e}}^T(n) = \hat{\mathbf{d}}^T(n) + \mathbf{V}^T(n)\mathbf{w}(n) \quad (4)$$

$$\bar{\mathbf{r}}(n) = \lambda\bar{\mathbf{r}}(n-1) + \mathbf{V}_0(n)\mathbf{V}_0^T(n) \quad (5)$$

$$\mathbf{r}(n) = \lambda\mathbf{r}(n-1) + \mathbf{V}_0(n)\mathbf{V}_r^T(n) \quad (6)$$

$$\mathbf{C}(n) = \lambda\mathbf{a}(n-1) - \mathbf{V}(n)\hat{\mathbf{e}}^T(n) \quad (7)$$

$$\mathbf{R}(n) = \begin{bmatrix} \bar{\mathbf{r}}(n) & \mathbf{r}^T(n) \\ \mathbf{r}(n) & \bar{\mathbf{R}}(n-1) \end{bmatrix} \quad (8)$$

$$\mathbf{R}(n)\bar{\mathbf{w}}(n) = \mathbf{C}(n) \quad (9)$$

is solved with the DCD method. The vector $\mathbf{a}(n)$ is also a by-product of the DCD algorithm, being the residual error (Zakharov, White & Liu, 2008). Therefore, an important complexity reduction is obtained if compared with the use of other iterative techniques (e.g. Gauss-Seidel). Finally,

$$\mathbf{w}(n+1) = \mathbf{w}(n) + \eta\bar{\mathbf{w}}(n) \quad (10)$$

The matrices $\mathbf{r}(n)$ and $\bar{\mathbf{r}}(n)$ obtained in equations (5) and (6), respectively, allow the efficient update (8) of the matrix $\mathbf{R}(n)$. This update exploits the time-shift structure of the input data.

The DCD algorithm is a multiplication-free and division-free algorithm based on binary representation of elements of the solution vector with M_b bits within an amplitude range $[-H, H]$ (Zakharov & Tozer, 2004; Zakharov, White & Liu, 2008). An iterative approximation of the solution vector $\bar{\mathbf{w}}(n)$ starts by updating the most significant bit of its elements and proceeds to less significant bits. The algorithm complexity is limited by N_u, the maximum number of "successful" iterations. An efficient DCD version from this point of view was proposed in (Zakharov, White & Liu, 2008). This new version finds a 'leading' (pth) element of the solution to be updated. With N_u updates, the number of additions of this version is upper limited by $2IJLN_u + M_b$ (Zakharov, White & Liu, 2008).

COMPUTATIONAL COMPLEXITY

The numerical complexity of the considered algorithms is measured by the sum of multiplications and additions ($C_{\text{algorithm}}$) per algorithm iteration. Matrix inversions were assumed to be performed with standard LU decomposition that requires $O(X^3/2)$ multiplications, where X is the size of a square matrix (Bouchard, 2002). We investigated the numerical complexity of the MFS-LMS (Bjarnason, 1992), MFX-LEDCD-ERLS, and modified filtered-x fast transversal filter (MFX-FTF) (Bouchard & Quednau, 2000) algorithms:

$$\begin{aligned} C_{MFX-LEDCD-ERLS} &= JKM(I+2) + \\ &+ IJL(2IJ + 2IJK + 4K + 5) + IJK(M-1) - \\ &JK - IJ + K + 2IJLN_u + M_b \end{aligned} \quad (11)$$

$$C_{MFX-LMS} = IJK\left(4L+2M\right)+ IJ\left(2L-K-1\right)+JK\left(2M-1\right)+K \quad (12)$$

$$C_{MFX-FTF} = LIJ\left(14IJK+K^2+3K+2\right)+ K^3-K^2+2K+IJK\left(2M+K-3-9IJ\right)+ IJ\left(IJ+I^2J^2-1\right)+JK\left(M-1\right) \quad (13)$$

It is known than the MFX-FTF algorithm is significantly simpler than the MFX-RLS algorithm and has similar convergence properties if stable (Bouchard & Quednau, 2000). Table 1 evaluates the complexity of the considered algorithms for the multichannel case for the particular case used in simulations ($I=1, J=3, K=2, M=64, L=128$). It can be seen that, in terms of multiplications, the complexity of the MFX-LEDCD-ERLS algorithm in case of $N_u=1$ and $N_u=4$ is significantly lower than that of the MFX-FTF algorithm.

Table 1. Comparison of the computational load of the MFX-LEDCD-ERLS with other multichannel delay-compensated modified filtered-X algorithms for ANC $I=1, J=3, K=2$, $M=64, M_b=24$, $L=128$

Algorithm	The maximum number of multiplies and additions per iteration
MFX-LMS	5363
MFX-LEDCD-ERLS ($N_u=1$)	14219
MFX-LEDCD-ERLS ($N_u=4$)	16523
MFX-FTF	37874

Figure 2 shows the number of multiplications and additions for the MFX-LMS algorithm and the DCD based algorithm when $I=1$, $J=3$, $K=2$, $M=64$, and L is varying; it can be seen that the MFX-LEDCD-ERLS algorithm is less complex than the MFX-FTF algorithm.

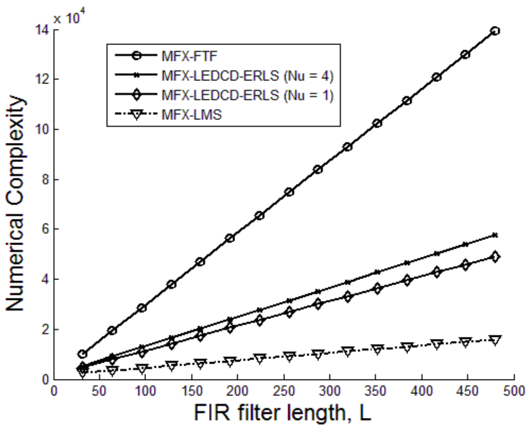

Figure 2. The numerical complexity given by the total number of additions and multiplications of the MFX-LMS, MFX-FTF, MFX-LEDCD-ERLS (N_u has two values, $I=1, J=3, K=2$, $M=64, M_b=24$, and L is varying)

Usually we have $L >> \{I, J, K, N, N_u, M_b\}$ in practical implementations and therefore, the MFX-LEDCD-ERLS algorithm is less complex than the MFX-FTF algorithm in most cases for the considered number of updates (about 60% for one DCD iteration, and 55% in case of 4 DCD iterations).

Typically, the order of increasing the complexity is the following: MFX-LMS, MFX-LEDCD-ERLS, and MFX-FTF algorithms. However, for some particular I, J, K, L, M, M_b values, and very high N_u values this order might change, with MFX-LMS clearly the least complex algorithm.

SIMULATION RESULTS

The MFX-LEDCD-ERLS algorithm was simulated and their performance was compared to the previously published MFX-LMS and MFX-FTF algorithms. In our simulation, we use $I=1, J=3, K=2, \lambda=0.9995$ and the reference signal is a white noise with zero mean and variance one. The simulations are performed with acoustic transfer functions experimentally measured in a duct. The impulse responses used for the multichannel acoustic plant have 64 samples each ($M=64$), while the adaptive filters have 128 coefficients each ($L=128$). The regularization factor is $\delta=2\cdot10^3$ for the ideal plant and $\delta=10^4$ for plant models with SNR of 10 dB. The step size μ for the MFX-LMS algorithm is 2×10^{-5} and the parameter H of the DCD algorithm is set to 2^{-1}. The performance of the algorithms was measured by

$$\text{Attenuation}\left(\text{dB}\right) = 10\cdot\log_{10}\frac{\sum_k E\left[e_k^2\left(n\right)\right]}{\sum_k E\left[d_k^2\left(n\right)\right]} \quad (14)$$

Figure 3 compares the performance of the selected algorithms, with ideal plant models, for a multichannel system ($I=1, J=3, K=2$, $\eta=1$), obtained from simulation. As shown in (Bouchard & Quednau, 2000) a value of $\eta=1$ is an optimal value for the RLS algorithm, and the solution of the least-squares equations is computed at every iteration of the algorithm. Two values of N_u and one for M_b have been used ($N_u=1$, $N_u=4$, $M_b=24$).

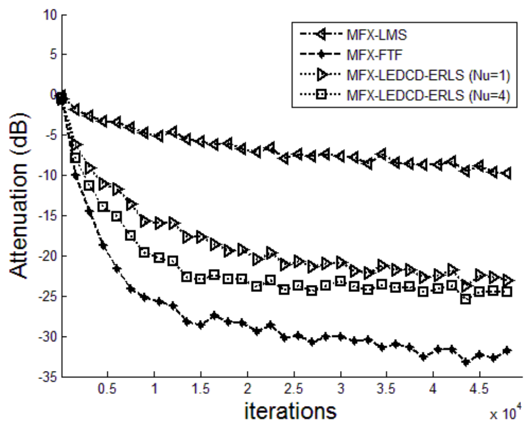

Figure 3. Convergence of multichannel delay-compensated modified filtered-x algorithms for ANC, with ideal plant models ($I=1, J=3, K=2, L=128$, $M=64, M_b=24$).

As expected, increasing the value of N_u, leads to better convergence properties. It can be seen that the MFX-LEDCD-ERLS algorithm using $N_u = 4$ and $M_b = 24$ has a lower performance than the more complex MFX-FTF algorithm. As expected, the convergence speed of the MFX-LEDCD-ERLS algorithm increases if the number of updates is increased (from 1 to 4 in Figure 3). Also, Figure 3 shows that the convergence of the MFX-LEDCD-ERLS algorithm is better than that of the MFX-LMS algorithm.

Figure 4 shows the performance when non-ideal plant models with a 10 dB SNR are used. The noisy plant models with 10 dB SNR accuracy were obtained as in (Bouchard & Quednau, 2000). The value of η has to be reduced to 0.3 for the MFX-FTF and MFX-LEDCD-ERLS algorithm as shown in (Bouchard & Quednau, 2000) for the RLS type algorithms, in order to assure stability, at the price of reduced convergence speed. In this case, the behavior of the MFX-LEDCD-ERLS algorithm is better than that of the MFX-FTF algorithm. Therefore, the MFX-LEDCD-ERLS algorithm is potentially more robust to inaccuracies of the plant model.

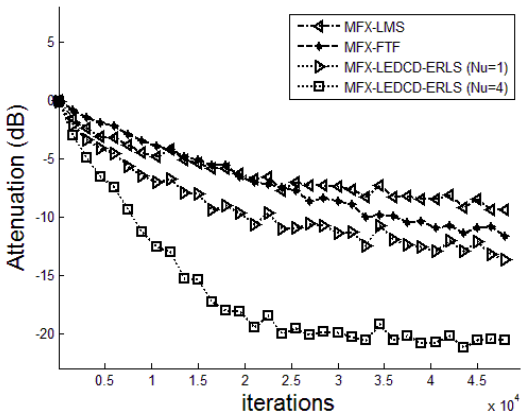

Figure 4. Convergence curves of multichannel delay-compensated modified filtered-x algorithms for ANC, with 10 dB SNR plant models for two values of N_u ($I = 1, J = 3$, $K = 2, L = 128$ $M = 64, M_b = 24$).

CONCLUSIONS

The MFX-LEDCD-ERLS algorithm based on the leading element dichotomous coordinate descent algorithm was introduced for practical active noise control systems using FIR adaptive filters. It was compared with the previously published MFX-FTF and MFX-LMS algorithms. It was shown that the MFX-LEDCD-ERLS algorithm could reduce the complexity in comparison with the MFX-FTF algorithm with acceptable performance for ideal plant models and improved robustness for noisy plant models.

ACKNOWLEDGMENT

This work was supported by a grant of the Romanian National Authority for Scientific Research, CNCS - UEFISCDI, project number PN-II-ID-PCE-2011-3-0097.

REFERENCES

Albu F., 2006, "Efficient multichannel filtered-x affine projection algorithm for active noise control," *Electronics Letters*, vol. 42, no. 7, pp. 421 – 423

Albu, F., Paleologu, C., 2008a, "A Recursive Least Square algorithm for Active Noise Control based on the Gauss-Seidel Method", *in Proc. of ICECS 2008*, St. Juliens, Malta, pp. 830-833

Albu, F., Paleologu, C., 2008b, 'New multichannel modified filtered-x algorithms for active noise control using the dichotomous coordinate descent method', *in Proc. of ACOUSTICS 2008*, Paris, France, pp. 5721-5725

Albu, F., Paleologu, C., 2010, 'New filtered-x recursive least square algorithms for active noise control', *in Proc. of INTERNOISE 2010*, Lisbon, pp. 1-4

Bjarnason, E., 1992, 'Active noise cancellation using a modified form of the filtered-x LMS algorithm,' *in Proc. 6th Eur. Signal Processing Conf.*, Brussels, Belgium, vol. 2, pp.1053–1056.

Bouchard, M. and Quednau, S., 2000, 'Multichannel recursive least-squares algorithms and fast-transversal-filter algorithms for active noise control and sound reproduction systems,' *IEEE Trans. Speech Audio Processing*, vol. 8, no.5, pp.606-618.

Bouchard, M., 2002, 'Numerically stable fast convergence least-squares algorithms for multichannel active sound cancellation systems and sound deconvolution systems,' *Signal Processing* 82(5), pp. 721-736.

Bouchard, M., 2003, 'Multichannel affine and fast affine projection algorithms for active noise control and acoustic equalization systems', *IEEE Trans. on Speech and Audio Processing*, vol. 11, no. 1, pp. 54–60

Bouchard, M., Albu, F., 2004, 'A low-cost and fast convergence Gauss-Seidel pseudo affine projection algorithm for multichanel active noise control', *in Proc. of ICASSP*, vol. 4, pp. 121–124

Carayannis, G., Manolakis, D.G., and Kalouptsidis, N., 1983, 'A fast sequential algorithm for least-squares filtering and prediction,' *IEEE Trans. On Acoustics Speech and Signal Processing*, vol. 31, pp. 1394-1402

Gonzales, A., Ferrer, M., Diego, M., Pinero, G., and Lopez, J., 2005, 'Practical implementation of multichannel adaptive filters based on FTF and AP algorithms for active control,' *International Journal of Adaptive Control and Signal Processing*, vol. 19, pp. 89-105.

Kuo, S.M. and Morgan, D.R., 1999, 'Active noise control: a tutorial review,' *Proc. of the IEEE*, vol. 87, no. 6, pp. 943-973.

Zakharov, Y. and Tozer, T., 2004, 'Multiplication-free iterative algorithm for LS problem, *Electronics Letters*, vol. 40, no. 9, pp. 567-569.

Zakharov, Y., White, G., and Liu, J., 2008, 'Low complexity RLS algorithms using dichotomous coordinate descent iterations,' *IEEE Trans. Signal Processing*, vol.56. No.7, pp. 3150-3161.