COHERENT LEAKAGE OF SOUND FROM A MIXED LAYER SURFACE DUCT - REVISITED

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Abstract

The coherent leakage of a signal from a mixed-layer surface duct with no rough surface scattering is a subject which received considerable attention decades ago. As is well known, coherent leakage of sound from a surface duct reduces progressively as frequency rises from below a nominal trapping frequency to higher values. Considered as modal leakage, or attenuation, the rate of coherent leakage with range for a surface ducted scenario is related to the imaginary part of the horizontal wave number, and may be determined through the use of a modal model of transmission. Such a calculation is performed through an iterative technique, and so when speed is desirable in the calculation, use of a direct analytic expression for leakage would be preferred. To that end, a brief study was made of the suitability of some of the expressions derived originally by Furry and described by Pederson and Gordon (JASA, 47, 304-326, 1970), as the basis for such a determination for the first acoustical mode, in particular.

This work includes comparison of leakage rates obtained from expressions based on early work by Furry against results from both the ORCA modal model, and from simulations based on the wave number integration model SCOOTER, for a mixed-layer surface ducted scenario with frequencies relevant to the onset of duct trapping. The work also includes a brief review of some of the early literature relating to leakage of sound from the surface duct.

1. Introduction

In a deep ocean, sound may travel within the mixed layer surface duct with less Transmission Loss (*TL*) to long ranges than at other depths. Sound travelling in the duct within a small span of angles about the horizontal is constrained, by refraction at the lower duct boundary, and by reflection at the ocean surface, to remain in the mixed layer duct. Of course, the duct is formed as the motion of the sea surface mixes the water to an approximate depth boundary, and the resulting uniform temperature causes a rise in sound speed of about 0.016 s^{-1}, due to the relative effects by pressure on the water bulk modulus and density. A surface duct is shown in Figure 1, above a region of declining temperature.

Sound travelling within the surface duct will be subject to a spreading loss, but beyond a short range from the source, the spreading will be approximately cylindrical. Other losses to transmission will occur if sound impinging on the rough sea surface is scattered to angles too great to be constrained within the duct, and if the process of refraction at the lower duct boundary does not return all sound energy to the duct. The latter phenomenon is the leakage of sound which is the subject of this paper.

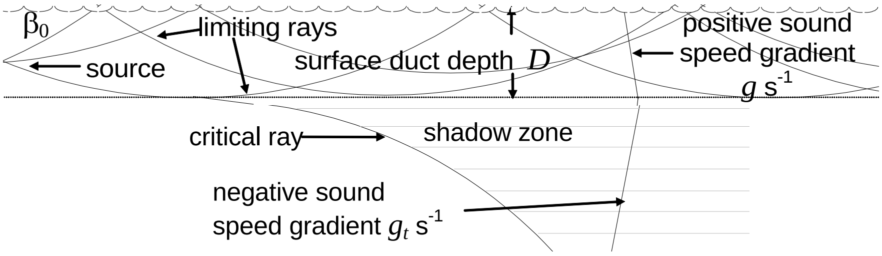

Figure 1. Sea surface interaction in mixed layer, isothermal surface duct above thermocline

The study of leakage of sound from the surface duct has a long history. Early work includes that of Marsh [1] in consideration of modal transmission. Using analytical results obtained by Furry, Marsh obtained an expression for the imaginary component of the horizontal wave number of modes in a surface duct, this being directly related to modal leakage, for a duct with a linear increase in sound speed with depth. Furry's work had related to the surface radar duct, was carried out during the Second World War, and was published most fully [2] after Marsh's work was completed. Subsequently, Pederson and Gordon [3], [4] used Marsh's results, making corrections, modifications and extensions, with reference to Furry [2]. Then, using results such as these, duct leakage expressions were incorporated in some models of transmission which did not otherwise describe the relevant physics. For example, the analysis of Pederson and Gordon was used [5] so that the NISSM model contained pre-computed loss values for the combination of sound speed gradients g in the duct and g_t below the duct, such that $\rho = (g/g_t)^{1/3} = -0.48$. (Note that with g_t being negative, the value of ρ is real, and is the same as $-(g/|g_t|)^{1/3}$.)

In recent times the study of sound transmission losses within the surface duct has been concerned primarily with effects due to scattering at the wind driven ocean surface. Consideration of the coherent leakage phenomena has been somewhat neglected, probably due to the ready availability of models of *TL* that describe the relevant physics and model the coherent leakage along with all other effects. The available work of Pederson and Gordon [3], [4] does not, however, include an explicitly stated solution for the leakage coefficient, and includes an incomplete transcription of a key expression derived by Furry [2], so it is appropriate to re-visit the subject. The key parts of the analysis of Pederson and Gordon, as based on the work of Furry, are outlined in Section 2, and the expression for leakage determined in the present work is provided. Comparisons of this expression of leakage with both data obtained by the use of wave-type models of transmission, and with other expressions in the literature are given in Section 3. A discussion follows in Section 4.

2. Leakage from Surface Duct

As is well known, for sound travelling near to horizontal, the phenomenon of duct trapping may be considered to result from the in-phase reinforcement of sound reflected downward from the ocean surface with sound refracted upward from the region of the lower duct boundary. An approximate expression for the lowest frequency, $f_{c,m}$ Hz, for which this reinforcement may occur for mode number $m = 1, 2, 3...$, may be shown to be

$$f_{c,m} \approx \frac{1}{2\pi\sqrt{2g}} \left(\frac{-a_m\, c_w}{D} \right)^{3/2} \tag{1}$$

where c_w m/s is sound speed at the surface, D in metres is depth of the duct, and the values a_m are the

m^{th} zeros of $\text{Ai}(a)$, the Airy function. Using the well-known approximation $a_m \approx -\left[3\pi\left(m-\frac{1}{4}\right)/2\right]^{2/3}$, the more familiar expression is obtained as

$$f_{c,m} \approx \frac{3(m-1/4)}{4\sqrt{2g}}\left(\frac{c_w}{D}\right)^{3/2}. \tag{2}$$

This expression may be shown to correspond with a ray-based description of the sound path, so is not exact, however it is within a few percent of the result in Equation (1). Strictly, a precise determination of $f_{c,m}$ Hz requires consideration of the sound speed gradient below the duct, however, the exactness of the determination is immaterial, as the range-rate of leakage varies continuously as frequency changes, with no step change at frequency $f_{c,m}$.

Pederson and Gordon [3] described the surface duct "strength" by the dimensionless parameter *M*:

$$M = \left(2k^2 g/c_w\right)^{1/3} D. \tag{3}$$

where $k = 2\pi f/c_w$ is acoustic wave number, m^{-1}. Substituting using Equation (2) gives the value of *M* for cut-on of mode *m* as $M_m \approx -a_m$, and for example $M_1 \approx 2.3381$ and $M_2 \approx 4.0879$.

As is well known, the attenuation rate for sound pressure amplitude for mode *m*, in nepers/m, is equal to the imaginary part of the horizontal wave number of the mode, so the intensity loss becomes

$$A_m = -1000 \times 20\left[\log_{10}(e)\right]\text{Im}(\lambda_m) \approx -8686\,\text{Im}(\lambda_m)\,\text{dB/km}, \tag{4}$$

where λ_m is the horizontal wave number, and is given by (e.g. equ (10) of ref. [3])

$$\lambda_m = \left[k^2 - Mx_m (M/D)^2\right]^{1/2} \tag{5}$$

where Mx_m is a root of a characteristic equation involving Hankel functions for which Pederson and Gordon obtained a solution through an iterative technique. It is readily shown that the wave number term *k* dominates Equation (5) and a suitable approximation for $\text{Im}(\lambda_m)$ becomes

$$\text{Im}(\lambda_m) \approx -\left(\left[\pi f g^2\right]^{1/3}/c_w\right)\text{Im}(Mx_m). \tag{6}$$

A solution for the leakage rate in dB/km then follows from this expression for the imaginary part of the horizontal wave number using the imaginary component of Mx_m. In his section 2.18, Furry [2] derived several asymptotic approximations by which each of $\text{Re}(Mx_m)$ and $\text{Im}(Mx_m)$ might be determined for combinations of ρ and *M* for which convergence occurs.

2.1 Frequencies above mode cut-on

From Furry's equ. (548), which is a suitable approximation when ρ is negative and *M* large:

$$\text{Im}(Mx_m) \approx \tfrac{1}{4}\alpha_m\left(\exp\left[-\tfrac{4}{3}\left(1-\rho^3\right)\left|M-|\varsigma_m|\right|^{3/2}\right]\right)\left[1+\tfrac{1}{24}\left(1-\rho^{-3}\right)\left|M-|\varsigma_m|\right|^{-3/2}+\ldots\right] \tag{7}$$

where $m = 1, 2, 3...$ is mode number, ς_m are solutions of the hankel function $h_2(\varsigma_m) = 0$ and it may be shown that $|\varsigma_1| \approx 2.3381$, $|\varsigma_2| \approx 4.0879$, etc. (e.g. Kerr [2] page 95), and also $|\varsigma_m| = -a_m$ used above. Also $\alpha_m = -\mathrm{Bi}(-|\varsigma_m|)/\mathrm{Ai}'(-|\varsigma_m|)$ where $\mathrm{Ai}(x)$ and $\mathrm{Bi}(x)$ are Airy functions, and (e.g. Furry [2] page 151) $\alpha_1 \approx 0.6474$, $\alpha_2 \approx 0.4935$, $\alpha_1 \approx 0.4252$, with $\alpha_m \approx \left[\frac{3}{2}\left(m - \frac{1}{4}\right)\pi\right]^{-1/3}$ an approximation. It needs to be noted that, apart from the inclusion of the higher order term in the square bracket, Equation (7) differs from equ. (8) of Pederson and Gordon [4] in that the latter does not include the factor $\frac{1}{4}$, hence that expression could not be used without error. By calculating leakage rates for the first mode based on Equation (7) it is clear that, at frequencies above but not near to that for mode trapping, the leakage values agree with those shown by Pederson and Gordon in their fig. 11, hence it has been presumed that the factor $\frac{1}{4}$ was omitted from their text by oversight.

If the term in the square brackets is assumed equal to 1, for the first mode Equation (7) becomes $\mathrm{Im}(Mx_1) \approx \frac{1}{4}\alpha_1 \exp\left(-\frac{4}{3}\left(1-\rho^3\right)\left|M - |\varsigma_1|\right|^{3/2}\right)$ and the attenuation rate A_1 is

$$A_1 \approx \frac{1406}{c_w}\left(\left[\pi f g^2\right]^{1/3}\right)\exp\left[-\tfrac{4}{3}\left(1+|g/g_t|\right)|M - 2.3381|^{3/2}\right]\mathrm{dB/km} \qquad (8)$$

where the term $|\varsigma_1|$ has been replaced by its value, the term $-g/g_t$ has replaced $-\rho^3$, and as the sound speed gradient in the thermocline g_t is negative, the positive term $|g/g_t|$ is used for convenience. From Equations (8) and (3) it follows that the attenuation rapidly reduces as frequency rises, due to the term in M in the exponent. The attenuation also decreases as the magnitude of the gradient in the thermocline, $|g_t|$, decreases. At a first level approximation, it may be assumed that Equation (8) may be used so long as the higher order terms indicated in Figure (7) may be neglected. For example, a criterion based on the higher order term in the square brackets having a value ≤ 0.25, with an implied underestimation of attenuation by Equation (8) of less than about 20%, requires $M \geq 2.3381 + \left(\left[1 - g_t/g\right]/6\right)^{2/3}$. For example, if the gradients in the surface duct and thermocline are equal in amplitude, the requirement is for $M \geq 2.82$. From Equation (3) it follows that this corresponds with a frequency 33% greater than $f_{c,1}$ for trapping of the first mode. Also, the value $M - 2.3381$ is zero at the duct trapping frequency $f_{c,1}$, with the result that the first higher order term in Equation (7) becomes infinite. Use of the expression is then limited to values of M at least greater than 2.3381. An equivalent form of Equation (8) in terms of frequency f is

$$A_1 \approx \frac{8686 \times \frac{1}{4}\alpha_1}{c_w}\left(\left[\pi f g^2\right]^{1/3}\right)\exp\left[-\tfrac{4}{3}\left(1+|g/g_t|\right)|\varsigma_1|^{3/2}\left|\left(\frac{f}{f_{c,1}}\right)^{2/3} - 1\right|^{3/2}\right]\mathrm{dB/km}\,. \qquad (9)$$

2.2 Frequencies below mode cut-on

Furry's equ. (541) is a suitable approximation when ρ is negative and $-M/\rho$ is small, and is used by Pederson and Gordon [4]. By considering imaginary parts, it may be shown that:

$$\mathrm{Im}(Mx_m) \approx \frac{\left[\mathrm{Im}(\varsigma_m)\right]}{\rho^2}\left[1 - \frac{\left(1-\rho^3\right)}{90}\left(\frac{M}{\rho}\right)^6 + \ldots\right] \qquad (10)$$

where $\varsigma_m = |\varsigma_m| e^{2\pi i/3}$ and hence $\mathrm{Im}(\varsigma_m) = 0.8660|\varsigma_m|$, giving e.g. $\mathrm{Im}(\varsigma_1) = 2.0249$. If $-M/\rho$ is small, the first higher order term (and all other higher order terms) shown in the square brackets is negligible, and $\mathrm{Im}(Mx_m) \approx \mathrm{Im}(\varsigma_m)/\rho^2$. Pederson and Gordon [4] show an effectively identical result in their equ. (17). For mode m = 1, $\mathrm{Im}(Mx_1) \approx 2.0249/\rho^2$ and the attenuation rate A_1 is

$$A_1 \approx \frac{8686\,\mathrm{Im}(\varsigma_1)}{\rho^2\, c_w}\left(\left[\pi\; f\; g^2\right]^{1/3}\right) \approx \frac{17.6\times10^3}{\rho^2\, c_w}\left(\left[\pi\; f\; g^2\right]^{1/3}\right)\ \text{dB/km}\,. \tag{11}$$

Now in Equation (10), the term $(1-\rho^3) > 1$ for a surface duct over a thermocline, so that the first higher order term tends to reduce the value of $\mathrm{Im}(Mx_m)$ as frequency (and the value M) rises from a value much less than that for mode cut-on. Of course, if $-M/\rho$ is no longer small, the higher order terms will not converge and Equation (11) may not be used, but it may be presumed that the expression is valid so long as the first higher order term in Equation (10) is less than a nominal value. Taking this value as 0.2 (Equation (11) over-estimating by 20%), the requirement is $M \le 1.619|\rho|/(1-\rho^3)^{1/6}$, that is $M \le 1.619|g/g_t|^{1/3}/(1-g/g_t)^{1/6}$. For example, if the gradients in the surface duct and thermocline are equal in amplitude, the requirement is for $M \le 1.44$.

2.3 Leakage of total signal

Now, at a particular frequency and value of M, there is a unique leakage rate for each particular mode m, however for present practical considerations, in order to estimate the leakage of the total transmitting signal it is necessary to consider the leakage rate of the first mode, only. Firstly, at frequencies near to cut-on for the first mode, the leakage rates for higher order modes are very large and so transmission via these modes need not be considered. Secondly, near to cut-on of each subsequent mode m = 2, 3, etc. the leakage rate of each mode of order m – 1 and less is very small and a very approximate but reasonable approach is to estimate the leakage of total energy as that described by the first mode. This is illustrated in the example in Section 3.

The attenuation rates for the total signal may then be estimated using Equations (8) and (11), so long as the value of M is appropriate for an acceptable error. For values of M too great for the latter expression and too small for the former, a number of data-fitting schemes might be considered, although such work has not been followed through within this initial study.

3. Simulations of Transmission in Surface Duct

A number of simulations of sound transmission within a surface duct were made using models known to describe the relevant physics responsible for the occurrence of duct leakage. These simulations were interrogated so that the rate of leakage of sound from the surface duct might be determined for a range of frequency values from below that for trapping of the first mode, to that for which at least two modes were expected to be trapped. The models used include SCOOTER [6], based on wave number integration, and ORCA [7], based on normal modes. The scenario was for a sound source at 7 m depth in a surface duct of 50 m over a thermocline of typical sound speed variation. The sound speed at various depths is as shown in Table 1, with the assumption of linear variation in speed being made at intermediate points, and the seafloor commencing at depth 200 m. Simulations made using ORCA included Thorp absorption (e.g. Urick [8] page 108) whereas there was no absorption incorporated in the simulations made using SCOOTER.

With SCOOTER, the seafloor properties were matched to those of the water at depth 200 m, so that the influence of seafloor reflections might be removed. In the case of ORCA, seafloor reflections were minimised by using a seabed consisting of layers of increasing attenuation overlying a half-space. For frequencies to 1500 Hz, each layer was 80 m thick, had the same sound speed as the water at 200 m depth, and had attenuation increasing linearly with depth. Over the five layers, the attenuation varied from zero to 10 dB per wavelength. The half-space was given a sound speed of 1550 m/s and attenuation of 20 dB per wavelength. For frequencies over 1500 Hz, the thickness of each layer was reduced to 10 m, so that the maximum number of modes was not exceeded.

Table 1. Water column sound speed profile

Depth (m)	Sound Speed (m/s)
0	1539.7483
50	1540.5483
75	1536.7688
100	1531.9767
125	1528.2929
150	1525.5472
200	1520.2753

3.1 Rates of leakage from duct determined from simulations

In the case of ORCA, the leakage rate for the first surface duct mode was determined directly based on data from the model's complex mode finder. These values are given in the 3rd column of Table 2 for particular frequencies. In the case of SCOOTER, for which individual modes may not be separated, an overall leakage rate was obtained from the variation with range of the mean-square pressure averaged over the duct depth, as shown in Figure 2. Here, the cylindrical spreading was removed, so that the remaining variation was a function of leakage of all modes combined. To obtain a result free from close-range effects, the leakage values were based on data between 10 km and 20 km.

The leakage values from SCOOTER, which are shown in the 4th column in Table 2, are virtually the same as those obtained by ORCA for the 1st mode. This may be attributed to the fact that at about the frequency at which the second mode is trapped, which follows from Equation (1) to be 1257 Hz, the leakage of the second mode is at a much greater rate than for the 1st mode and so the second mode contributes little to the total received signal at the ranges involved. Leakage rates were also determined from the mean-square pressure averaged over duct depth obtained from the ORCA runs, with all modal contributions summed. These results are shown in the 2nd column in Table 2, and confirm the SCOOTER data.

Table 2. Duct Leakage Rate Obtained from Transmission Modelling

Frequency (Hz)	Duct leakage rate over 10 km to 20 km from ORCA mean-square pressure (dB/km)	Attenuation rate of 1st mode from ORCA (dB/km)	Duct leakage rate over 10 km to 20 km from SCOOTER mean-square pressure (dB/km)
300	4.2	4.2	4.3
400	2.6	2.6	2.6
500	1.6	1.6	1.6
600	1.0	1.0	1.0
700	0.67	0.67	0.67
800	0.44	0.44	0.44
1000	0.22	0.21	0.21
1200	0.16	0.13	0.14
1500	0.18	0.11	0.20

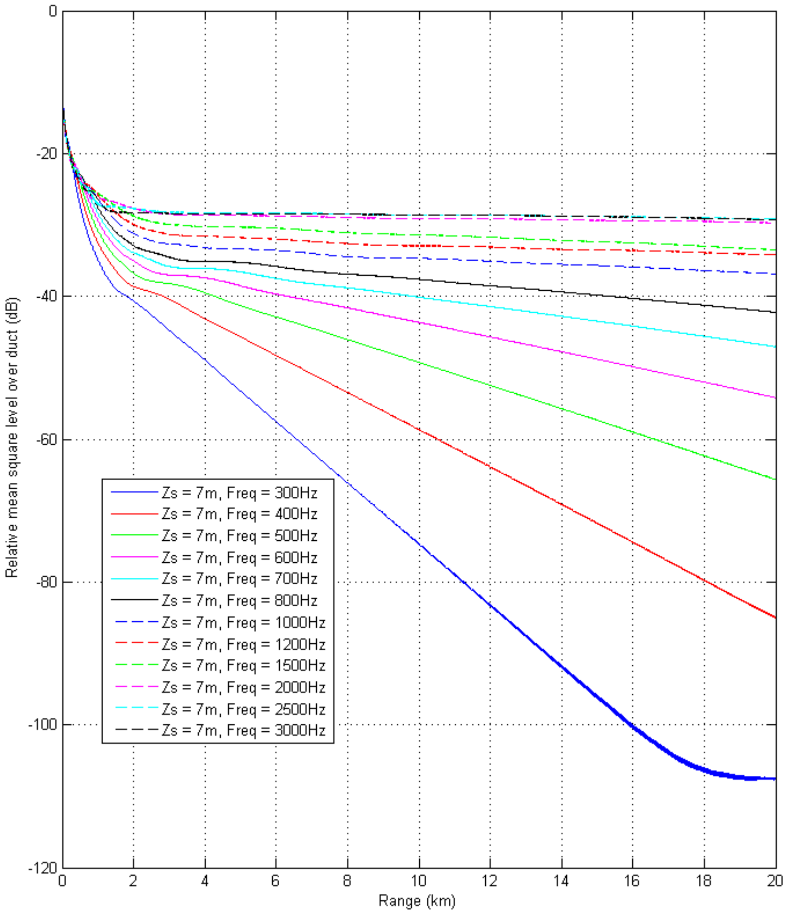

Figure 2. Averaged mean-square pressure level at each range in duct, source at 7m depth, cylindrical spreading removed, SCOOTER

3.2 Comparison with theoretical rates of leakage from duct

Based on the data in Table 1, the sound speed gradient in the 50 m thick surface duct is $g = 0.016\,\mathrm{s}^{-1}$, and the gradient in the 25 m layer immediately below the duct is $g_t = -0.1512\,\mathrm{s}^{-1}$. The entire thermocline is not uniform, but for present purposes the latter value of gradient below the duct will be presumed, with the value $\rho = (g/g_t)^{1/3} = -0.473$. From the argument of Section 2.1, the attenuation rate at frequencies above duct trapping may be estimated using Equation (8) so long as $M \geq 2.3381 + ([1 - g_t/g]/6)^{2/3}$, for an under-estimation of 20% or less, and for frequencies below duct trapping may be estimated using Equation (11) so long as $M \leq 1.619|g/g_t|^{1/3} / (1 - g/g_t)^{1/6}$, with an over-estimation by less than 20%. These values of M are 3.79 and 0.75, respectively, corresponding with frequencies 1,122 Hz and 99 Hz respectively. From Equations (8) and (11), the corresponding leakage rates are 0.067 dB/km and 22 dB/km.

Of course, leakage rates may be determined whilst incorporating the higher order term shown in each of Equations (7) and (10), for an improved result, so long as the value of the higher order term is not too great. If a maximum value of 0.5 is permitted for each of these terms, the allowable values of M become $M \geq 2.3381 + ([1 - g_t/g]/12)^{2/3}$ in the case of the Equation (7) and $M \leq 1.886|g/g_t|^{1/3} / (1 - g/g_t)^{1/6}$ in the case of Equation (10). For the water column of Table 1 it follows that versions of Equations (8) and (11) incorporating the higher order terms may then be used for values of $M \geq 3.25$ and $M \leq 0.88$, that is for frequencies ≥ 891 Hz and ≤ 125 Hz, respectively.

Leakage rates for the 1st mode in the surface duct, as determined using the versions of Equations (8) and (11) which incorporate the respective first higher order terms, are shown in Figure 3 in red, for frequency ranges both within and beyond those for which they are relevant. The leakage rates determined using Equations (8) and (11) "as is", without any higher order terms, are shown in the figure in green. These leakage rates are compared with those obtained numerically for the 1st mode (as shown in the 3rd column of Table 2) using ORCA inclusive of Thorp absorption, in magenta, and with Thorp absorption (as specified by Urick [8] on his page 108) subtracted, in yellow. Further, values of leakage determined using the formulation of Packman [9] are shown in the figure as the dark blue line. These calculations used Packman's algorithm as it is shown in the text by Ainslie [10], with one exception – the duct trapping frequency was determined using the actual sound speed and sound speed gradient values for the duct rather than using pre-determined values. The Thorp absorption rate is shown separately as the light blue line. Clearly, for the surface duct of depth 50 m, the leakage rate of the 1st mode is less than the in-water absorption at frequencies greater than about 1100 Hz, and at progressively higher frequencies the issue of modal leakage is irrelevant.

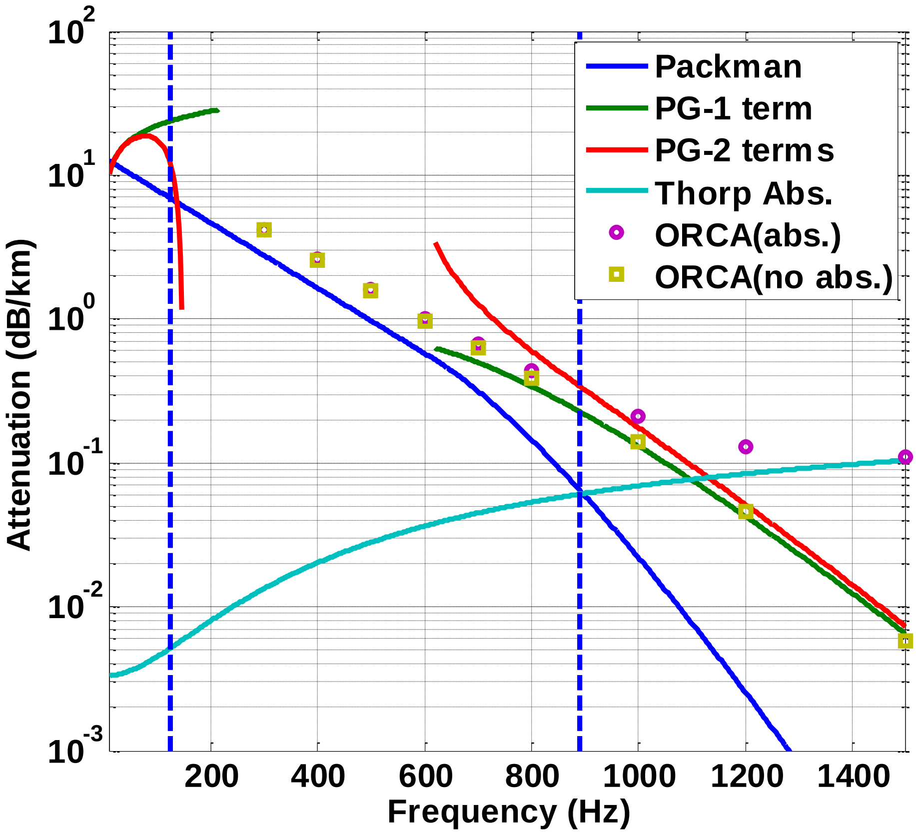

Figure 3. Leakage rate for 1st mode for surface duct of Table 1 from 10 Hz to 1500 Hz, vertical lines at frequencies 125 Hz and 890 Hz

From the figure, for frequencies greater than about 600 Hz, it is clear that the leakage rates determined by the expression of Packman under-estimate both those obtained by use of Equation (8) and those determined numerically by the ORCA model. At lower frequencies, for which the use of Equation (8) is no longer expected to be valid, the data from Packman's algorithm also under-estimate the ORCA data. At the region of lowest frequencies from 10 Hz to about 100 Hz, for which Equation (11) may be expected to be accurate, the results from the Packman algorithm under-estimate those from the former expression, with the exception of the at the very lowest frequencies for which data are shown.

3.3 Comments on leakage rate expression of Packman

Packman's expression for duct leakage [9], his equation (1), may be considered with reference to Equation (9). Packman assumed that the amplitude of the sound speed gradient below the duct was the same as in the duct, and substituted the value $1+|g/g_t|=2$ in his work. Now using this substitution, and knowing that $|\varsigma_m|$ may be approximated as $\left[3\pi\left(m-\frac{1}{4}\right)/2\right]^{2/3}$, 3π may be substituted for

$\frac{4}{3}\left(1+|g/g_t|\right)|\varsigma_1|^{3/2}$, and Equation (9) becomes

$$A_1 \approx 8686\,\mathrm{Im}(\lambda_1) \approx 8686 \times \tfrac{1}{4}\,\alpha_1\left(\left[\pi\, f\, g^2\right]^{1/3}\big/c_w\right)\exp\left[-3\pi\left|\left(\frac{f}{f_{c,1}}\right)^{2/3}-1\right|^{3/2}\right]\mathrm{dB/km}\,. \tag{12}$$

Packman's expression includes a term $\left[g/(2\,c_w\,D)\right]^{1/2}$ in place of the term $\alpha_1\left(\left[\pi\, f\, g^2\right]^{1/3}\big/c_w\right)$ in Equation (12), but is otherwise identical to $\mathrm{Im}(\lambda_1)$ in Equation (12). If α_1 is replaced by the approximation $\left[\frac{3}{2}\left(m-\frac{1}{4}\right)\pi\right]^{-1/3}$ with $m=1$, the terms are identical at the duct trapping frequency $f_{c,1}$ defined by Equation (2). It follows that, at frequency $> f_{c,1}$, Packman's [9] expression (1) gives leakage values $\left(f_{c,1}/f\right)^{1/3}$ times those from Equations (8) and (9) where $|g/g_t|$ is assigned the value 1.

For all frequencies below 1.16 $f_{c,1}$ Packman makes an approximation, which he indicated that he used to ensure a good fit to data published by Kerr [2]. This approximation has the effect that when his resultant leakage data in dB/km are plotted on the logarithmic axes used in Figure 3, the slope of the curve on this figure is the same as the slope computed by his algorithm at the frequency 1.16 $f_{c,1}$. As his determination of $f_{c,1}$ is made using Equation (2), the frequency 1.16 $f_{c,1}$ for the purposes of the present scenario is 623 Hz. It is apparent from Figure 3 that the Packman data do fit a straight line for frequencies less than about 623 Hz. An issue with this approach is that the derived values of leakage for the lowest frequencies will not follow the shape of the curve determined using Equation (11), which in turn is based on Furry's equ. (541) and may be regarded as accurate for small frequencies. However, leakage rates are not needed at very low frequencies for practical purposes, so this is of little consequence.

4. Discussion

From the data shown in Figure 3 for frequencies from about 700 Hz to 1200 Hz, the values of leakage for the 1st mode in the surface duct, as obtained using the ORCA model without absorption, lie between the results from the two versions of Equation (8), but are generally closer to the values obtained without the addition of the first higher order term. At 1500 Hz, the predictions of leakage by the two forms of Equation (8) slightly over-estimate the ORCA result, however above 1200 Hz in-water absorption dominates as shown in Figure 3 and modal leakage is irrelevant for the surface duct of depth 50 m. Of course, for surface ducts of lesser depth, the duct trapping frequency for the first mode is higher in accordance with Equation (1), and it follows that the frequency above which in-water absorption is greater than the leakage loss is higher. For example, if the duct trapping frequency is 5000 Hz, it is easy to show that the leakage loss greatly exceeds absorption losses at this frequency.

As expected, at frequencies below 890 Hz, the leakage rate determined using Equation (8) "as is", with no higher order term included, under-estimates the correct leakage rates, which may be presumed to align with the ORCA data. There are no data at very low frequencies to compare with the version of Equation (11) which incorporates the first higher order term, however as an approximation, the ORCA data do appear to trend toward the leakage value at 125 Hz which corresponds with that obtained by the modified form of Equation (11). It is preferable that the suitability of the modified and un-modified forms of Equations (8) and (11) is tested by further comparison against modelled leakage data for a range of different below-layer sound speed gradient values, and different duct depths.

As stated in Section 3.2, Packman's algorithm under-estimated the leakage rates determined by the ORCA model. However, Packman's algorithm is defined for a below-layer gradient that has the same amplitude as the duct gradient, and such under-prediction might be expected. There is some potential that the Packman algorithm might be modified for a generalised below-layer gradient, however, this may not necessarily be the best way to proceed to obtain an algorithm for a useful range of frequencies and practical circumstances. Such work is beyond the scope of the present paper.

5. Conclusions

For practical application to studies of underwater sound transmission in a mixed-layer surface duct, reasonable estimates of modal leakage of the total signal may be made through consideration of leakage of the first mode only. For any particular surface duct scenario, modal leakage will be an issue that will need consideration only for those frequencies below some value at which absorption effects dominate. However, for surface ducts with cut-on frequencies for the first mode of at least 5000 Hz, leakage effects greatly dominate over absorption at the cut-on frequency.

Simple and rapid calculation of the leakage rate for the first mode at all frequencies for all surface duct scenarios appears potentially feasible, although a commonly used algorithm has been shown to be deficient. It appears likely that an improved algorithm might be prepared. This may involve incorporation of the first higher order term in each of two expressions derived by Furry for the low frequency and high frequency limit, respectively. In any event, further comparison with simulations of sound transmission in a surface duct is necessary before any proposed algorithm might be considered suitable.

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