ACOUSTIC PERFORMANCE OF PERIODIC COMPOSITE MATERIALS

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Abstract

Sonic crystals are finite arrays of periodically arranged scatterers, for which very low noise transmission occurs in certain frequency bands known as stop bands or band gaps. The location of these band gaps can be tuned by changing the spacing between adjacent scatterers in the periodic array. Sonic crystals are receiving recent interest for practical applications ranging from vibro-absorbing coatings for attenuation of structure-borne noise to acoustic barriers for environmental noise reduction. In this work, analytical and numerical models of a simple sonic crystal comprising periodically arranged inclusions in a host medium are developed. The analytical model is based on the effective medium approximation method, whereby the composite material is modelled as a homogeneous viscoelastic material, determined by the volume fraction of the inclusions in the composite. A finite element model of the sonic crystal using the commercial software COMSOL Multiphysics is also developed. The acoustic performance of the sonic crystal array obtained analytically and numerically is compared.

1. Introduction

Sonic crystals comprise arrays of scatterers periodically arranged in a homogenous fluid medium. Minimal transmission of sound through sonic crystals occurs at certain frequencies, attributed to destructive interference of scattered waves within the lattice structure due to the periodic arrangement of the scatterers. The range of frequencies at which sound waves are attenuated is known as the band gap and it is repeated after a certain period. The centre frequency of these band gaps can be approximately predicted and is given by Bragg's law. These frequencies can be altered by changing the distance between the scatterers, known as the lattice constant.

A simple application of the sonic crystal principle for noise control is as an outdoor barrier to attenuate road traffic noise [1-6]. A sonic crystal barrier has an advantage over a conventional solid noise barrier to allow air flow, whereby free flow of air reduces wind loading on the barrier. Martinez-Sala et al. [1] experimentally demonstrated high noise attenuation at selected frequencies by a sculpture in Madrid, Spain, which consists of hollow cylinders made of steel and arranged in a square lattice pattern. Martinez-Sala et al. [2] also performed experimental studies of the noise attenuation by trees arranged periodically. Kessissoglou et al. [5] studied the acoustic performance of 2D and 3D sonic crystals as outdoor noise barrier applications using a quasi-periodic boundary element technique

[7]. Numerical simulations using a 2D boundary element method were performed by Koussa et al. [6] with the aim to reduce road and railway transportation noise using low height sonic crystal based noise barriers. Local acoustic resonance and absorption was used with the cylindrical scatterers and was shown to result in significant reduction in noise over a broad range of frequencies.

There is a vast amount of literature on analytical and numerical methods for calculating the acoustic properties of composite materials, for example, see [8-12] and the references therein. The aim of this study is to compare analytical and numerical models of a composite material comprising a host medium embedded with cylindrical scatterers in a 2D periodic configuration. The analytical model is based on the effective medium approximation method, whereby the composite material is modelled as a homogeneous viscoelastic material, determined by the volume fraction of the inclusions in the composite. A finite element model of the sonic crystal array using commercial software COMSOL Multiphysics is also developed. The acoustic performance of the sonic crystal obtained analytically and numerically is compared.

2. Model Description

The 2D model examined in this work is shown in Fig. 1. It consists of scatterers in the form of solid cylinders in a square lattice arrangement. The cylinders are made of steel and are assumed to be infinitely rigid. The fluid medium between the cylinders is air. The diameter of the cylinders is *d*. Due to the square lattice arrangement, the spacing between adjacent cylinders known as the lattice constant *a* is the same in both directions. A periodic boundary condition is applied to consider an array of infinite length in the *y*-direction. A non-reflective boundary condition is applied on all other surfaces to model anechoic termination of waves. The sonic crystal is subjected to plane acoustic wave excitation in the *x*-direction, as shown in Fig. 1.

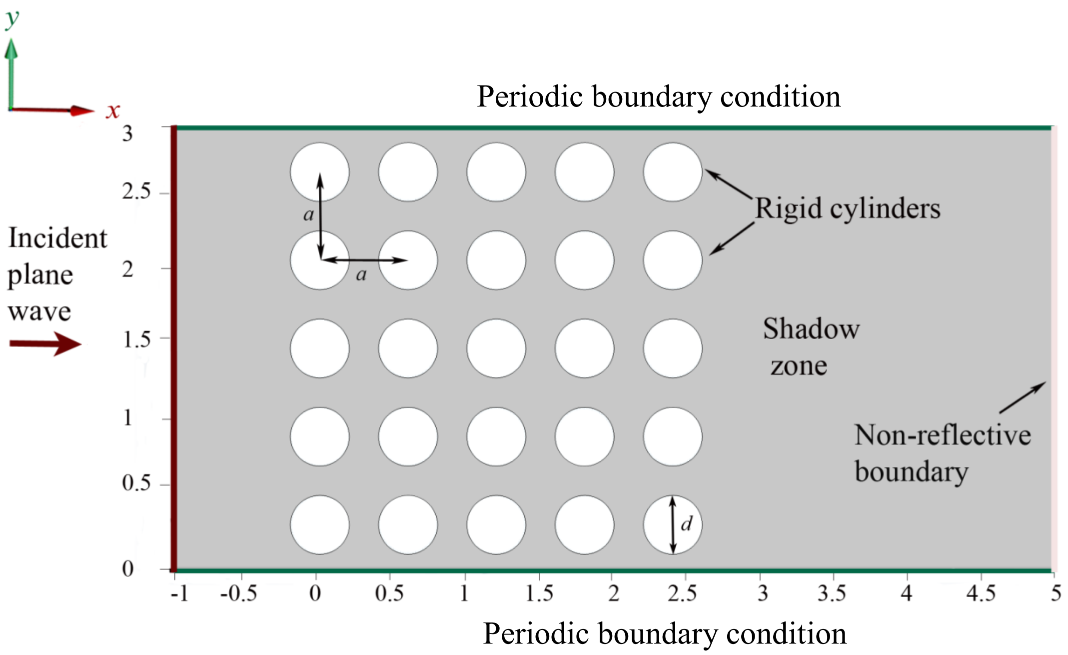


Figure 1. Two-dimensional finite element model of a sonic crystal array using solid circular cylindrical scatterers under incident plane wave excitation

The creation of band gaps in the sonic crystal depends on the shape of the scatterers, the lattice constant (a) and the filling fraction (f_f). Filling fraction is the ratio of volume occupied by the scatterers to the total volume of the sonic crystal. For cylinders in a square lattice, the filling fraction is given by

$$f_f = \frac{\pi d^2}{4a^2} \tag{1}$$

The centre frequencies of the band gaps created by periodic scatterers can be approximately predicted by Bragg's law as follows

$$\omega_c = \frac{nc}{2a}; \;\; n = 1,2,3 \ldots \tag{2}$$

where c is the speed of sound in the medium.

3. Effective Medium Approximation

Effective Medium Approximation (EMA) is a technique to approximately model inhomogeneous composite media as continuous homogenous media. At sufficiently low frequencies, the composite material can be modelled as a homogeneous visco-elastic media with some effective elastic moduli that are mostly determined by the volume fraction of inclusions in the material. Using the row homogenization (RH) EMA approach, sound propagation only in a horizontal period of the structure is considered, as shown in Fig. 2. This process can be conceptually thought of as sound propagation in a 2D duct with rigid boundaries and varying cross-section (rigid boundaries are imposed due to periodicity of the system). An analytical treatment of this model can be implemented by employing the standard model of duct acoustics [13, 14], in which the cross section of the duct is appropriately modified due to the presence of scatterers, as shown in Fig. 2.

At a relatively low frequency, the scattering caused by the changing cross section of the duct is not sensitive to the shape of this change, but instead is only determined by the change in the size of the opening from its initial size. For the 2D duct considered here, the initial size prior to the change in cross section corresponds to the lattice constant a. The effective minimum of the duct denoted by b in Fig. 3 is found by

$$b = a - d_{eff} \tag{3}$$

where d_{eff} is an appropriately chosen effective or lumped value for the duct constriction, which in reality varies continuously across the hemi-cylindrical blockage of diameter d (see Fig. 3). The effective duct constriction for the periodic scatterers becomes

$$d_{eff} = \frac{\sqrt{\pi} d}{2} \tag{4}$$

with a downstream width of the same value, which keeps the reduction in the cross-sectional area in the plane of Fig. 3 caused by the restriction, d_{eff}^2, equal to the area of the two semi-circular constrictions.

For a single step change in the duct, the simplest estimation of the energy transmission coefficient is given by [13, 14]

$$T = 1 - \left(\frac{a - b}{a + b}\right)^2 \tag{5}$$

For multiple periods of the duct constriction as shown in the left figure of Fig. 2, the transmission coefficient will be different and is obtained in what follows. The duct acoustics model for this work uses the acoustic impedance $Y = p/Su$, where p is the acoustic pressure, u is the particle velocity and S is the duct cross sectional area. For a duct element of length l, the matrix equation that connects the element output quantities for acoustic pressure and particle velocity, p_o and v_o, to the element input quantities p_i and v_i is

$$\begin{bmatrix} p_o \\ v_o \end{bmatrix} = \boldsymbol{M_e} \begin{bmatrix} p_i \\ v_i \end{bmatrix} \tag{6}$$

where $\boldsymbol{M_e}$ is the transfer matrix and is given by

$$\boldsymbol{M_e} = \begin{bmatrix} \cos kl & jY \sin kl \\ \dfrac{j \sin kl}{Y} & \cos kl \end{bmatrix} \tag{7}$$

$k = \omega/c$ is the acoustic wavenumber. The full transfer matrix $\boldsymbol{M}$ for the system with the four elements M_{11}, M_{12}, M_{21} and M_{22} is obtained as the product of all the periodically repeated element matrices $\boldsymbol{M_e}$ of the system. The final transmission loss for the system with the assumption of an infinite inlet and a non-reflective or infinite outlet is given as [14]

$$T = 10 \log_{10}\left(\frac{Y_o}{4Y_i}\left|M_{11} + \frac{M_{12}}{Y_o} + Y_i M_{21} + \frac{Y_i}{Y_o} M_{22}\right|^2\right) \tag{8}$$

where Y_i and Y_o are the acoustic impedances at the inlet and outlet of the duct.

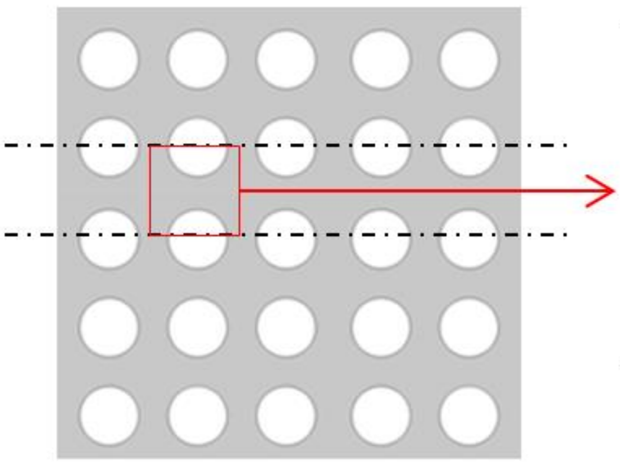


Figure 2. Row homogenization applied to the periodic array of cylindrical scatterers

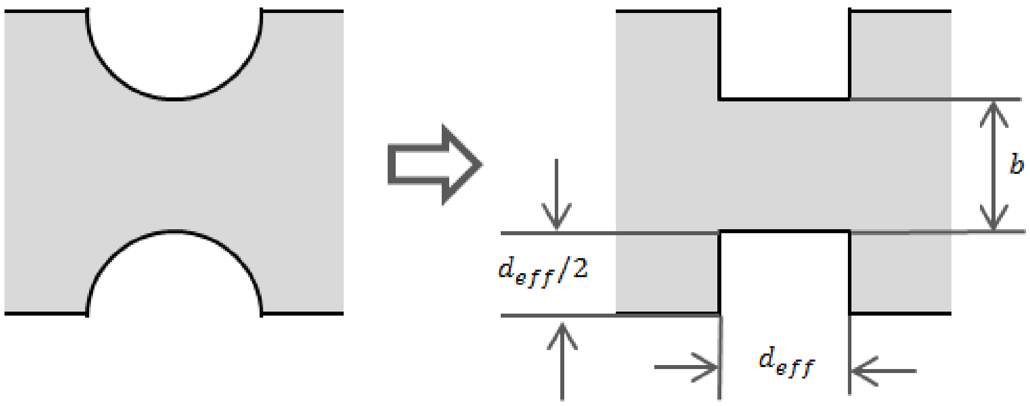


Figure 3. Row homogenization mapping for conversion to lumped parameters of duct acoustics

4. Numerical Model

A finite element (FE) model of a straight 2D sonic crystal array using COMSOL Multiphysics (v4.3b) [15] was developed as follows. The Helmholtz wave equation that governs the acoustic pressure p in the frequency domain at a point $\mathbf{x}$ in the acoustic domain is given by [16]

$$\Delta p(\mathbf{x}) + k^2 p(\mathbf{x}) = -q \tag{9}$$

where Δ is the Laplacian operator, k is the acoustic wavenumber and q is the acoustic source term. Notice the above equation is obtained with the assumption that the acoustic forcing is harmonic in nature.

Discretisation of the acoustic domain leads to a piece-wise representation of acoustic pressure where the acoustic pressures at the nodal locations in the acoustic domain are in the acoustic pressure vector $\mathbf{p}$. Following the discretisation process presented in detail by Marburg and Nolte [16], the discretised form of Eq. (9) is given by

$$(\mathbf{K} - ik\mathbf{C} - k^2\mathbf{M})\mathbf{p} = \mathbf{p}_i \tag{10}$$

where $\mathbf{K}$, $\mathbf{C}$ and $\mathbf{M}$ are the stiffness, damping and mass matrices, respectively. The vector $\mathbf{p}_i$ represents the incident acoustic pressure at the nodal locations in the acoustic domain. To reduce the model size, only a slice of the overall problem is modelled and periodic boundary conditions are applied to selected boundaries of the computational domain as indicated in Fig.1. Given the geometric periodicity of the sonic crystal array and for a normal incident pressure field, a simple continuity of the acoustic pressure on two sides of a periodic boundary is

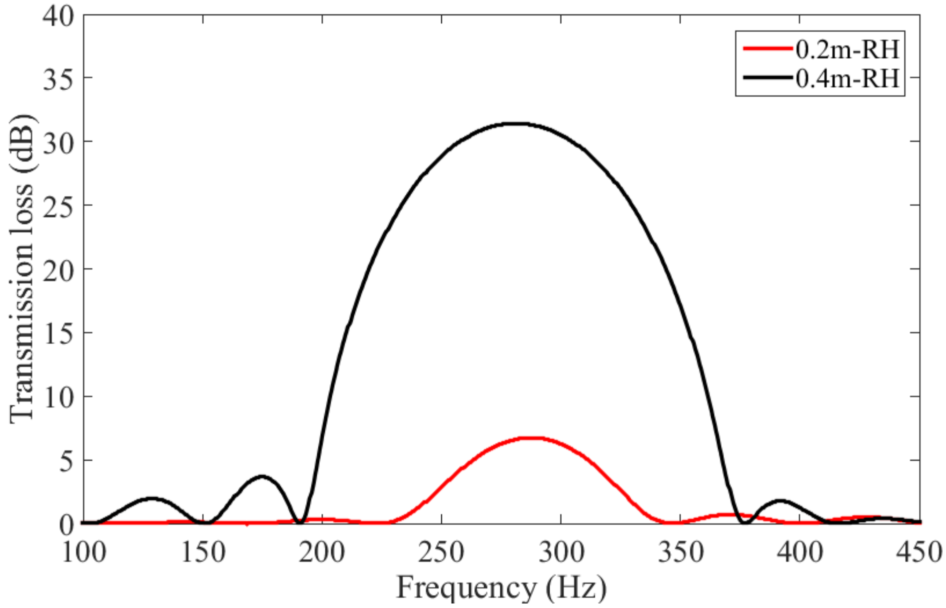
$$\mathbf{p}_1 = \mathbf{p}_2 \tag{11}$$

where the subscripts 1 and 2 refer to the two sides that the periodic boundary condition is applied to.

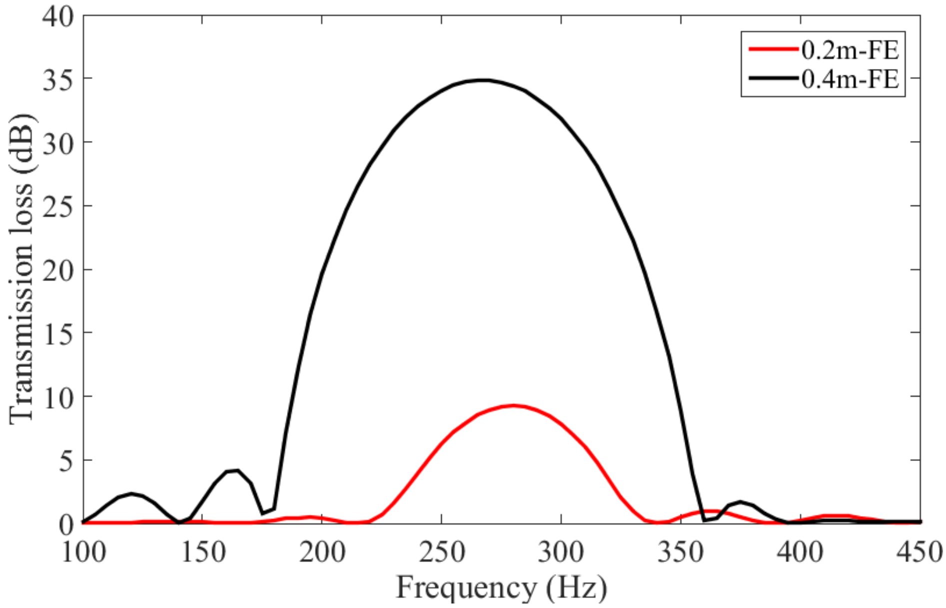
5. Results and Discussion

Noise reduction by the sonic crystal array is quantified by transmission loss corresponding to the ratio of the acoustic power of the transmitted waves to the power of the incident waves. The host medium is air with density ρ = 1.21 kg/m^3 and speed of sound c = 343 m/s. The cylindrical scatterers are assumed to be infinitely rigid.

Figure 4 shows the effect of altering the filling fraction on the transmission loss of the sonic crystal array. Results obtained analytically using the row homogenization based EMA (Fig. 4(a)) and numerically using the finite element (FE) method (Fig. 4(b)) are presented. The lattice constant was kept constant at $a = 0.6$ m. The filling fraction was altered by changing the diameter of the cylinders from d = 0.2 m to d = 0.4 m. The corresponding filling fractions are 0.087 (d = 0.2 m) and 0.35 (d = 0.4 m). A significant increase in the transmission loss and width of the band gap of attenuation occurs for an increased size of scatterers, due to an increase in the blocking of the sound waves. The centre frequency of the band gap does not depend on filling fraction and is not significantly affected by the increase in cylinder diameter. According to Eq. (2), the centre frequency of the first band gap occurs at approximately 265 Hz, which corresponds to the frequency of peak transmission loss in Fig. 4. The results obtained by both the techniques are in good agreement. However, it is noted that the RH method slightly under predicts the transmission loss compared to the FE method. Also, a slight shift in the band gap is observed between the two methods.



(a)



(b)

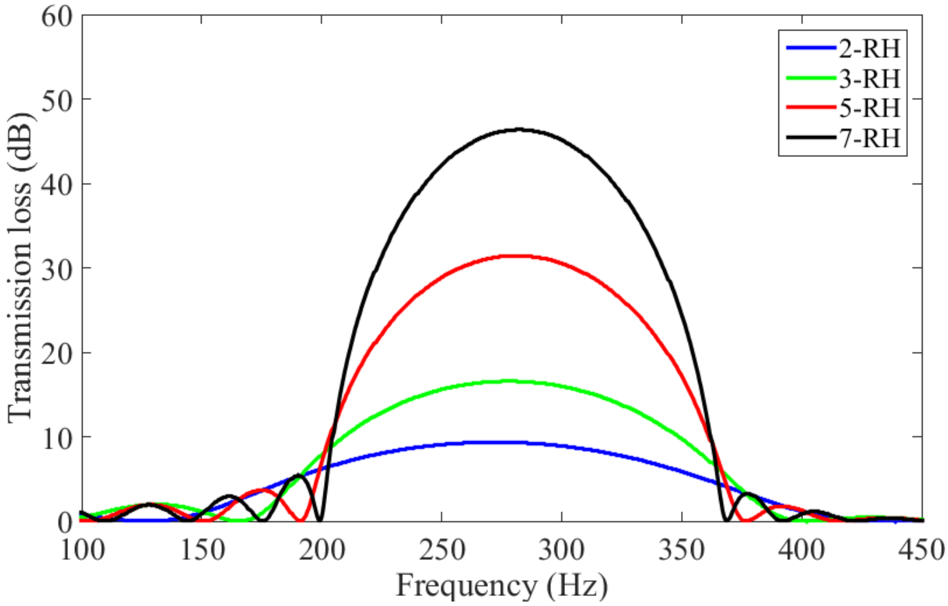
Figure 4. Transmission loss for a square lattice sonic crystal array of fixed lattice constant ($a = 0.6$) with cylinder diameters of 0.4m ($f_f = 0.35$) and 0.2m ($f_f = 0.087$) obtained (a) analytically using the RH method and (b) numerically using the FE method

The effect of increasing the number of cylinders on the transmission loss using both the row homogenization based EMA approach and the FE method is shown in Fig. 5. Only the width of the domain was altered as the number of cylinders in the *x*-direction was varied between 2 to 7. The same periodic boundary condition in the *y*-direction was maintained. The diameter of the scatterers is d = 0.4m, with a lattice constant of a = 0.6 m and filling fraction f_f = 0.35. Increasing the number of cylinders increases the transmission loss. Again, the results from both the methods are in good agreement.

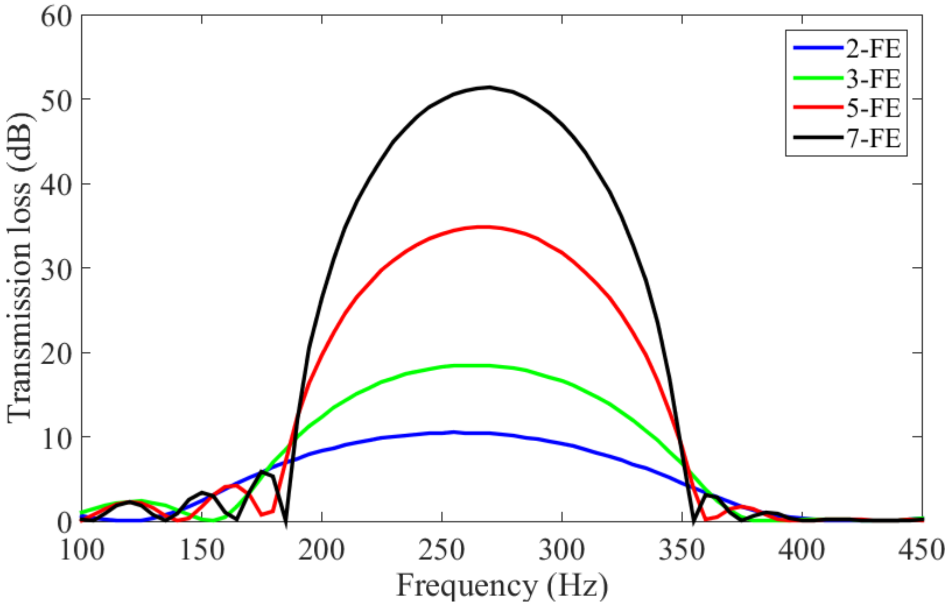
Dispersion curves obtained from both RH and FE methods are shown in Fig. 6. These plots correspond to the square lattice sonic crystal array with periodic boundary conditions in both the *x*- and *y*-directions. The shaded region represents the band gap. In this region there is no solution of the frequency for the given wavenumber, thus representing the frequencies at which waves cannot propagate in the crystal structure. The dispersion relation obtained from both the methods is in good agreement.

6. Summary

The acoustic performance of a sonic crystal array consisting of a periodic arrangement of solid cylinders in air is studied analytically and numerically. The simple analytical model is based on duct acoustics theory using an effective medium approximation method. Numerical simulations are performed using commercial finite element software COMSOL Multiphysics. In the low frequency range, good agreement in the results for transmission loss and the dispersion relation from both these methods for a square lattice arrangement of solid cylinders in air is obtained.

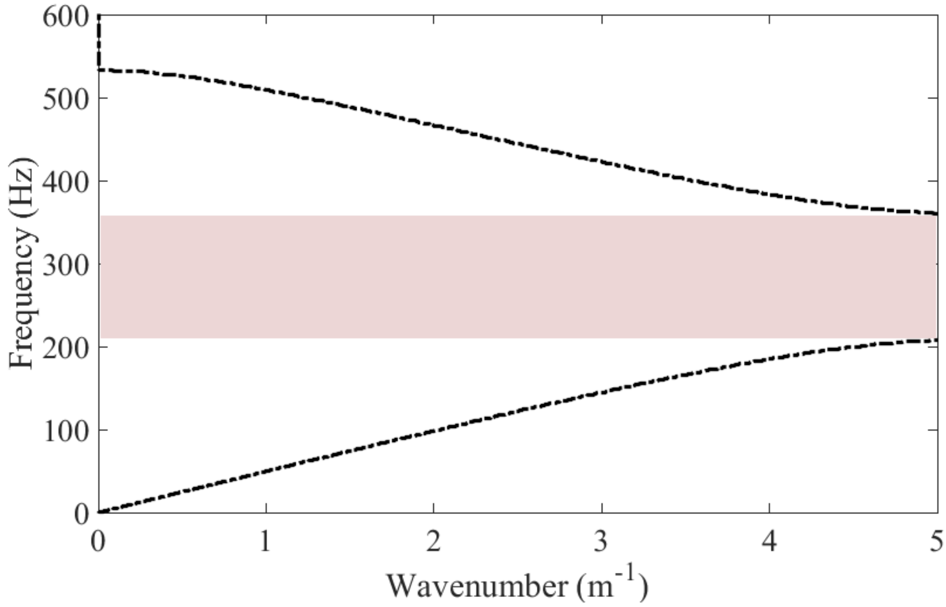


(a)

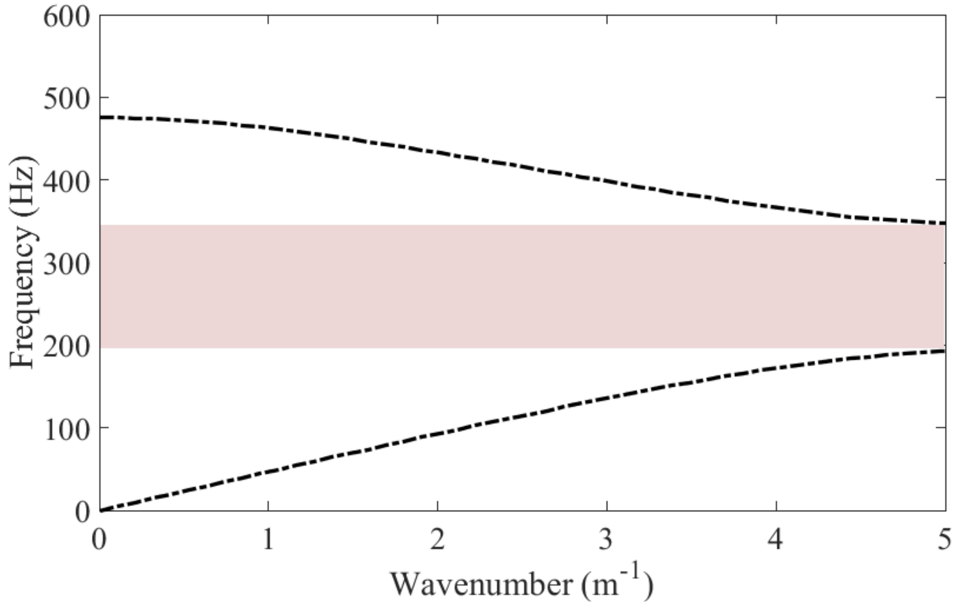


(b)

Figure 5. Transmission loss for a sonic crystal array of cylinder diameter $d = 0.4$m, lattice constant $a = 0.6$ and filling fraction$f_f = 0.35$, for different number of cylinders obtained using (a) the RH method and (b) the FE method



(a)



(b)

Figure 6. Dispersion relation for a sonic crystal array of cylinder diameter $d = 0.4$m, lattice constant $a = 0.6$ obtained from the (a) RH method; (b) FE method

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