ACOUSTICS 2017 PERTH
SOUND, SCIENCE AND SOCIETY

Coherent Leakage of Sound from Ocean Surface Ducts of Non-linear Sound Speed Profile

Adrian D. Jones (1), Alec J. Duncan (2) and Zhi Yong Zhang (1)

(1) Defence Science and Technology Group, Edinburgh, Australia
(2) Centre for Marine Science & Technology, Curtin University, Perth, Australia

ABSTRACT

The sound speed variation with depth that defines a surface acoustic duct does not necessarily adhere to the uniform gradient of perfectly isothermal water. Prior consideration of the coherent leakage of sound from a surface duct has almost always been made for an isothermal layer, and not for a realistic duct with a nonlinear sound speed profile. This paper presents the results from a brief study in which the rate of leakage has been obtained by using a normal mode model, for an ocean with various surface duct types for which the sound speed varies nonlinearly with depth. Initial results indicate that the rate of variation of leakage with acoustic frequency is strongly linked to the duct trapping frequency, but that the absolute level of leakage is related to the depth of the duct. The formation of these conclusions is illustrated by results from numerical modelling. Reference is also made to theoretical expectations for ducts of uniform gradients weaker than for isothermal water.

1 INTRODUCTION

For a sound source located within a mixed-layer surface duct, the signal transmitted within the duct beyond the first surface skip of a limiting ray is constrained by refraction at the lower duct boundary, and by reflection at the ocean surface. Beyond this range, the acoustic intensity received within the duct will be subject to a near-cylindrical spreading loss, and the loss of energy scattered from the rough sea surface to angles too great to be constrained within the duct. There may also be a loss of intensity if the process of refraction at the lower duct boundary does not return all sound energy to the duct. The last mentioned phenomenon is the leakage of sound which is considered in this paper. Of these loss mechanisms, the spreading effect is not associated with any loss of acoustic energy, whereas the leakage effect results in coherent acoustic energy leaving the duct. The acoustic energy lost from the duct due to rough surface scattering is regarded as incoherent, as it is appropriate to average effects over many reflections from sea surface shapes of different random form.

In earlier work, the authors considered the issue of leakage from a perfectly isothermal surface duct (Jones et al, 2015, 2016). In more recent work, the authors' analysis was extended to the development of a generalised expression for leakage for an isothermal duct of any depth, and most recently, consideration has been given to the issue of leakage from a surface duct of nonlinear sound speed profile (SSP). The present paper includes a brief description of earlier relevant work, but the reader is referred to the earlier papers (Jones et al., 2015, 2016) for more complete details, including descriptions of prior work in the literature.

2 DETERMINATION OF DUCT LEAKAGE

As is well-known, the leakage of sound from a surface duct represents incompleteness in the refraction of sound at the lower duct boundary. With reference to Figure 1, not all the energy represented by the limiting ray depicted in red is refracted as the ray proceeds past the turning point at the bottom of the surface duct, the latter being at depth D in the figure. As may be shown, the leakage effect is significant at, and below, frequencies close to the duct trapping frequency, but the leakage becomes insignificant at much greater frequencies.

Strictly, duct leakage is a modal effect, and as is well-known, duct trapping frequencies $f_{c,m}$ Hz correspond with cut-on of modes of index numbers m. At a given frequency, the rate of leakage for each mode, in units of dB/km, is uniform with range, but is larger for each successive mode of higher index m. Conversely, the acoustic signal transmitted in the surface duct is associated with the least leakage of energy, and so for practical applications it is necessary to consider the leakage rate of the first mode, only.

2.1 Duct Trapping

As mentioned above, duct trapping corresponds with cut-on of modal transmission within the surface duct. The wave-based mathematical derivation of the corresponding cut-on frequencies $f_{c,m}$ Hz is complex and will not be considered here, however the concept may be readily explained with reference to the ray-based depiction in Figure 1. Here, duct trapping occurs when the phase change along a single skip of a limiting ray, after accounting for the phase change at the caustic near the turning point, and for the phase change on reflection from the pressure release surface, is $2(m-1)\pi$ for $m = 1, 2, 3, ...$ With this phase change of a multiple of 2π radians, the refracted sound exactly reinforces the sound at the indicated arrow on the wavefront corresponding with the previous surface reflection. This onset of duct trapping may be considered as the cut-on of the relevant mode. Further, the mode may be expressed in terms of a vertical wave and a horizontal wave. The vertical wave travels to the bottom of the duct and returns to the surface, and so the phase interference may be seen to result from the vertical component alone (e.g., Freehafer in section 1.5 of the text edited by Kerr (1951)). Considering the phase change associated with the vertical component, the cut-on frequencies are

$$f_{c,m} = \frac{\left(m-\frac{1}{4}\right)c_w}{2\int_0^D \sqrt{N_z^2 - N_D^2}\,dz} \approx \frac{\left(m-\frac{1}{4}\right)c_w}{2\sqrt{2}\int_0^D \sqrt{N_z - N_D}\,dz} \tag{1}$$

where c_w m/s is speed of sound at the ocean surface, D m is the depth of the duct, $N_z = c_w/c_z$ is index of refraction at depth z at which sound speed is c_z.

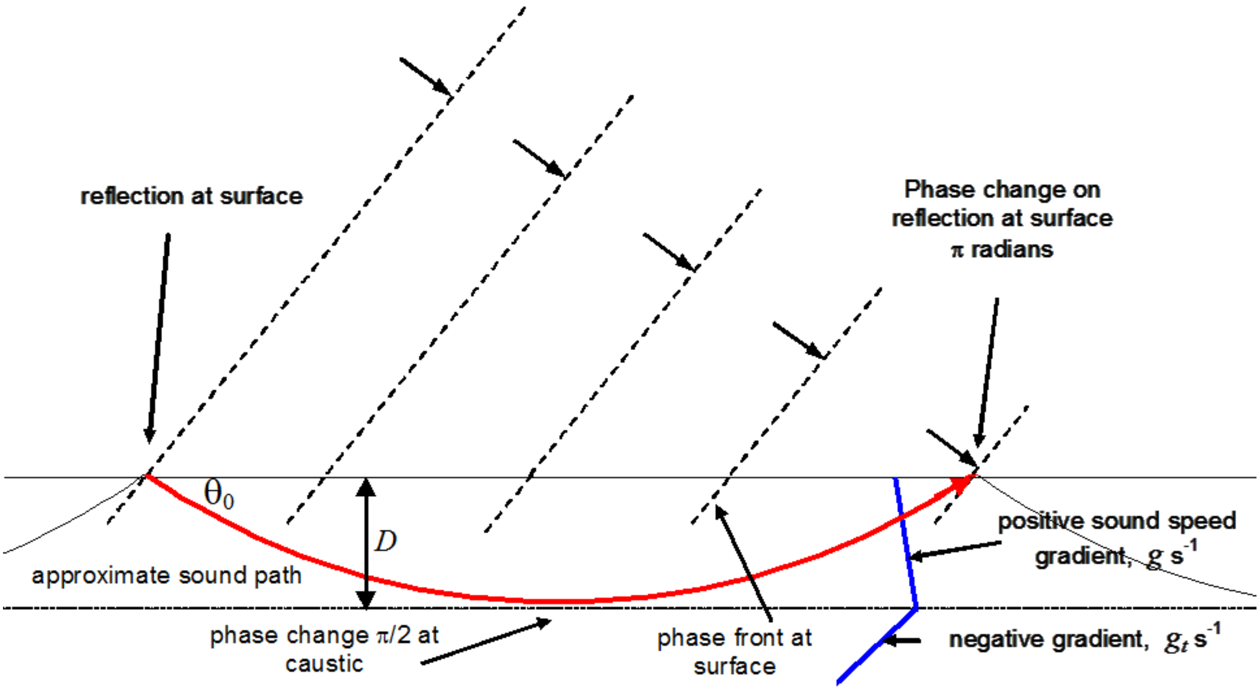

Figure 1: Ray-based considerations of duct trapping – example of linear SSP

For a uniform sound speed gradient g s^{-1} within the surface duct, $c_z = c_w + gz$, and from Equation (1) it follows that the trapping frequencies may be expressed as

$$f_{c,m} \approx \frac{3\left(m-\frac{1}{4}\right)}{4\sqrt{2g}}\left(\frac{c_w}{D}\right)^{3/2}. \tag{2}$$

The commonly used wave-based analysis makes the assumption that $N_z^2 = 1 - 2gz/c_w$, as (e.g. Jensen et al., 2011 page 141), this permits a solution of the homogeneous depth-separated wave equation using Airy functions. This is the so-called "N^2-linear" assumption, and is valid for values of gz/c_w <<1, which is virtually always the case. The resulting wave-based expression for trapping frequencies is

$$f_{c,m} \approx \frac{1}{2\pi\sqrt{2g}}\left(\frac{-a_m c_w}{D}\right)^{3/2} \tag{3}$$

where a_m is the m^{th} zero of the Airy function, that is $\text{Ai}(a_m)=0$. Evaluation for the first mode ($m=1$), gives $f_{c,1} \approx \frac{0.3977}{\sqrt{g}}\left(\frac{c_w}{D}\right)^{3/2}$ from Equation 2 and, using $a_1=-2.3381$, $f_{c,1} \approx \frac{0.4023}{\sqrt{g}}\left(\frac{c_w}{D}\right)^{3/2}$ from Equation 3, indicating that the ray-based expression is a very good approximation.

2.2 Leakage from Surface Duct of Linear Profile

As is true of any transmission scenario involving modal leakage, the attenuation rate for mode m, in nepers/m, is equal to the imaginary part of the horizontal wave number of the mode, $\text{Im}(k_{r,m})$, and the intensity loss is

$$A_m = -1000 \times 20[\log_{10}(e)]\,\text{Im}(k_{r,m}) \approx -8686\,\text{Im}(k_{r,m}) \text{ dB/km}. \tag{4}$$

For a surface duct of uniform gradient, $\text{Im}(k_{r,m})$ may be approximated (e.g. Jones et al., 2016) as

$$\text{Im}(k_{r,m}) \approx -\left(\left[\pi f g^2\right]^{1/3}/c_w\right)\text{Im}(Mx_m) \tag{5}$$

where the complex quantity Mx_m (e.g. Pedersen and Gordon (1965)) is usually obtained by an iterative process involving Hankel functions. Power series solutions for $\text{Im}(Mx_m)$ for frequencies either well below the trapping frequency $f_{c,m}$ Hz, or well above the trapping frequency, may be obtained from the analysis of Furry, as included in the text by Kerr (1951), with reference to Furry's notation provided by Pedersen and Gordon (1970). These power series are described in the authors' earlier paper (Jones et al., 2016).

For $f > f_{c,1}$ Hz, if frequency is sufficiently large, the first term in the relevant power series provides an adequate approximation for $\text{Im}(Mx_m)$. Taking this expression for $\text{Im}(Mx_m)$, substituting into Equations (5) and (4), further manipulation gives the following expression for duct leakage for the first mode (Jones et al., 2016)

$$A_1(f) \approx \frac{8686 \times \frac{1}{4}\alpha_1}{c_w}\left(\left[\pi f g^2\right]^{1/3}\right)\exp\left[-\tfrac{4}{3}(1-g/g_t)|\varsigma_1|^{3/2}\left(\left(\frac{f}{f_{c,1}}\right)^{2/3}-1\right)^{3/2}\right] \text{dB/km} \tag{6}$$

where g_t is the sound speed gradient below the duct, and is assumed to be uniform; $\alpha_m = -\text{Bi}(-|\varsigma_m|)/\text{Ai}'(-|\varsigma_m|)$ where $\text{Ai}(x)$ and $\text{Bi}(x)$ are Airy functions, and (e.g. Furry (Kerr, 1951) page 151) $\alpha_1 \approx 0.6474$; ς_m are solutions of the modified Hankel function of order one-third $h_2(\varsigma_m)=0$ (e.g. Freehafer page 94 in text edited by Kerr (1951)) and it may be shown that $|\varsigma_1| \approx 2.3381$. As the first mode has the least leakage, Equation (6) approximates the leakage rate for the total signal for frequencies for which it is appropriate.

Likewise, for $f < f_{c,1}$ Hz and frequency sufficiently small, the first term in the relevant power series provides an adequate approximation for $\text{Im}(Mx_m)$ and the leakage rate for the first mode becomes (Jones et al., 2016)

$$A_1(f) \approx \frac{8686\,\text{Im}(\varsigma_1)}{c_w}\left[\pi f g_t^2\right]^{1/3} \text{ dB/km} \tag{7}$$

where $\varsigma_m = |\varsigma_m|e^{2\pi i/3}$, and as $e^{i\theta} = \cos\theta + i\sin\theta$, $\text{Im}(e^{2\pi i/3}) = 0.8660$ hence $\text{Im}(\varsigma_m) = 0.8660|\varsigma_m|$, giving $\text{Im}(\varsigma_1) = 2.0249$.

3 SIMULATIONS OF LEAKAGE FROM SURFACE DUCT OF LINEAR SOUND SPEED PROFILE

In prior work by the authors (Jones et al., 2016, 2015), a number of simulations of sound transmission within a surface duct were made using the normal mode model ORCA (Westwood et al., 1996). These simulations were interrogated to extract the rate of leakage of sound from the surface duct, for the first mode, for a range of frequency values from below that for trapping of the first mode, to that for which at least two modes were expected to be trapped. The leakage rate was found directly from the model's complex mode finder. These leakage values were compared to expressions which included Equation (6) and (7) (Jones et al., 2016, 2015).

The scenario was for a sound source at 7 m depth in a surface duct of 50 m over a thermocline typical of a deep ocean. The sound speed gradient in the surface duct was $g = 0.016\,\mathrm{s}^{-1}$, the gradient in the 25 m layer immediately below the duct was $g_t = -0.1512\,\mathrm{s}^{-1}$ and sound speed at the surface $c_w = 1539.7483\,\mathrm{m/s}$. The thermocline was not uniform, but the latter value of gradient below the duct was presumed. ORCA runs included Thorp absorption (e.g. Urick (1983) page 108), with the ORCA code incorporating the absorption coefficient a_V as

$$a_V = 0.1F^2/(1+F^2) + 40F^2/(4100+F^2) + 2.75\times10^{-4}F^2 \ \ \mathrm{dB/kyd} \tag{8}$$

where F is frequency in kHz. This is converted to $\mathrm{dB/km}$ by multiplying by 1.09361.

Values of leakage rate cited in this paper have had the Thorp attenuation described by Equation (8) removed unless indicated. From Equation (3), the duct trapping frequency for the first mode $f_{c,1}$ follows as 543.6 Hz. Likewise the trapping frequency for the second mode is $f_{c,2} = 1{,}256.7$ Hz. From the approximate expression, Equation (2), the duct trapping frequency for the first mode is 537.36 Hz, and the trapping frequency for the second mode is 1,253.8 Hz.

3.1 Comparison with Theory

Leakage rates for the 1st mode in the surface duct, as determined using Equations (6) and (7) are shown in Figure 2 by the red and green continuous lines, respectively, for frequency ranges both within and beyond those for which the expressions are relevant. Now, Equations (6) and (7) are for the first term only in the full power series expansions of $A_1(f)$ for high and low frequencies. The dashed red and green lines in Figure 2 were obtained using the first two terms of the power series expansions as given in Jones et.al. (2016). The vertical lines drawn at 125 Hz and 891 Hz are the frequencies at which the second terms in the expansions are equal to half of the first terms. These are shown, as the first terms in the power series may be expected to be reasonably valid in regions for which the continuous and dashed red and green lines have not diverged greatly. These leakage rates are compared with those obtained numerically for the 1st mode using ORCA, inclusive of Thorp absorption in magenta, and with Thorp absorption subtracted in yellow. The Thorp absorption rate is shown separately as the light blue line. Clearly, for the surface duct of depth 50 m, the leakage rate of the 1st mode is less than the in-water absorption at frequencies greater than about 1100 Hz, and at progressively higher frequencies the issue of modal leakage is irrelevant.

The leakage rates determined by Equation (6), for $f \geq 891\,\mathrm{Hz}$, describe the zero-absorption data from ORCA very well. There are no ORCA data at a frequency sufficiently low to enable a comparison with rates determined by Equation (7) for $f \leq 125\,\mathrm{Hz}$, however, the trend of the ORCA data is to "join the gap" between the regions of the predicted curves for which data may be expected to be valid.

Values of leakage determined using the formulation of Packman (1990) are shown in the figure as the dark blue line, and leakage data obtained using the algorithm of Duan et al. (2016) are shown by the black dashed line. The curve derived using the algorithm of Packman under-estimates ORCA data. As is described in more detail by Jones et al. (2015), the Packman algorithm assumes a ratio of sound speeds g/g_t equal to -1, and is not suitable for all situations. Similarly, the algorithm of Duan et al. also under-estimates the ORCA data, but not as greatly. The algorithm of Duan et al. was prepared for application to surface ducts for which the below-layer gradient was strong, however, it was derived empirically from simulated data.

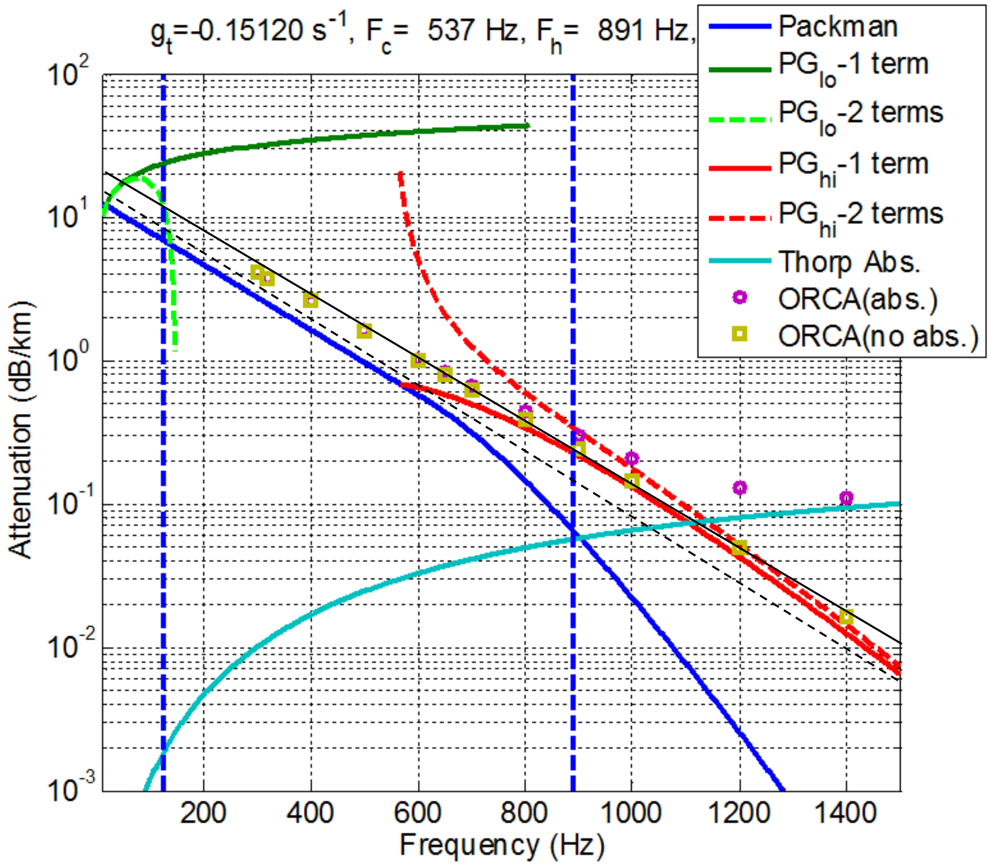

Figure 2: Leakage rate from 10 Hz to 1500 Hz for 1st mode for 50 m surface duct, $g/g_t = -0.1058$, vertical lines at frequencies 125 Hz and 891 Hz at which 2nd terms in power series forms of Equations (7) and (6) respectively are equal to half of 1st terms, black line: tangent to Equation (6) data at 891 Hz, black dashed line: algorithm of Duan et al. (2016), blue line: algorithm of Packman (1990)

4 ALGORITHM FOR LEAKAGE FOR SURFACE DUCT OF UNIFORM GRADIENT

In Figure 2, the solid black line is drawn as a tangent, at the frequency 891 Hz, to the solid red curve according to Equation (6), as described by Jones et al. (2016). This was done as it was simply a coincidence that for $f \le 891\text{Hz}$, the line was a very close fit to the ORCA data for the 1st mode. Further, it was observed (Jones et al., 2016) that this same line was a very good fit to all ORCA 1st mode data obtained for frequencies less than about 891 Hz, for all simulations for the 50 m isothermal surface duct, for which the below layer gradient was as strong or stronger than $g_t = -0.05\,\text{s}^{-1}$. The strongest below layer gradient for these simulations was $g_t = -0.40\,\text{s}^{-1}$. Jones et al. showed that the solid black line was described by the following function

$$A_1(f) \approx 22.0 \exp(-0.00514 f) \text{ dB/km} \tag{9}$$

which may then be regarded as an algorithm for leakage vs frequency for an isothermal duct of depth 50 m.

In order to obtain a more generalized form of algorithm, it is useful to re-consider Equation (6). As is well-known, the magnitude of the below layer gradient $|g_t|$ is commonly much greater than that for the duct itself, so that $1 - g/g_t \approx 1$, and Equation (6) becomes

$$A_1(f) \approx \frac{8686 \times \frac{1}{4}\alpha_1}{c_w}\left(\left[\pi f g^2\right]^{1/3}\right) \exp\left[-\tfrac{4}{3}|\varsigma_1|^{3/2}\left(\left(\frac{f}{f_{c,1}}\right)^{2/3} - 1\right)^{3/2}\right] \text{dB/km} . \tag{10}$$

Now, for surface duct of uniform gradient g, the skip distance of the limiting ray may be expressed in terms of the 1st mode trapping frequency $f_{c,1}$, as

$$r_s \approx c_w \left[\frac{3^2}{g^2 f_{c,1}} \right]^{\frac{1}{3}} \text{m}. \tag{11}$$

Using this expression, the gradient g may be expressed in terms of skip distance and trapping frequency, and substituting this form of g into Equation (10) gives

$$A_1(f) \approx 8.686 \times \tfrac{1}{4} \times 3^{2/3} \pi^{1/3} \alpha_1 \left[\frac{f}{f_{c,1}} \right]^{1/3} \exp\left[-\tfrac{4}{3} |\varsigma_1|^{3/2} \left(\left(\frac{f}{f_{c,1}} \right)^{2/3} - 1 \right)^{3/2} \right] \text{dB/skip} \tag{12}$$

which may be seen to be entirely in terms of $f/f_{c,1}$, and is otherwise unrelated to either the depth of the surface layer, or to the sound speed gradient in the surface layer. Bearing in mind the use of the tangent line mentioned earlier, it may be speculated that the entire function of leakage per skip is similarly unchanged for particular values of $f/f_{c,1}$, and that, with suitable re-scaling, one plot of leakage values will provide data for all situations with $|g/g_t|$ small. With this assumption, the leakage data for case of the 50 m duct of gradient 0.016s^{-1} may be re-scaled as dB/skip as a function of $f/f_{c,1}$, using the value of r_s obtained from Equation (11) with $c_w = 1539.7483 \text{m/s}$, $g = 0.016 \text{s}^{-1}$, $D = 50 \text{m}$, and $f_{c,1} \approx 543.6 \text{Hz}$ from Equation (3). Also, using these values, Equation (9) may be generalised as loss per skip for a duct of linear profile, as a function of $f/f_{c,1}$, as

$$A_1(f) \approx 136.0 \exp(-2.79\, f/f_{c,1}) \text{ dB/skip}. \tag{13}$$

Using the expression for the skip distance in Equation (11), this generalized form of leakage may be written as

$$A_1(f) \approx \frac{65.4 \times 10^3}{c_w} \left(g^2 f_{c,1} \right)^{1/3} e^{-2.79 f/f_{c,1}} \text{ dB/km}. \tag{14}$$

In this paper, there is insufficient space to detail the comparisons which have been made between Equation (14) and with 1st mode leakage data obtained using the ORCA model, however, it may be stated that Equation (14) was quite accurate in describing leakage data for isothermal ducts of depths 20 m and 34.46 m, for all frequencies of interest. Further, Equation (14) was also found to be adequate for describing the leakage from surface ducts of greater depth, but with uniform gradients weaker, than that for isothermal conditions. In that test, surface ducts for which ORCA simulations were obtained were designed to all have the same trapping frequency $f_{c,1}$ as the 50 m isothermal duct, but were of depth 76.3 m, 92.3 m and 120.2 m. The sound speed gradients for these surface ducts were 0.00450s^{-1}, 0.00254s^{-1} and 0.00115s^{-1}, respectively. These values of gradient were selected by inversion of Equation (2). It is intended to describe these comparisons in a later document.

5 SURFACE DUCTS OF NONLINEAR SOUND SPEED PROFILE

The surface ducts considered in the preceding sections are all of a linear profile, that is, they have a uniform gradient $g\,\text{s}^{-1}$. As is well known, this is an ideal form, and is based on an assumption that good mixing occurs at all depths to the base of the duct, and that no mixing occurs at greater depths so that below the duct there is a thermocline of, usually, strong negative gradient. For a typical ocean, it is more likely that there will be a zone of transition from an isothermal region to the thermocline, or that the SSP in the surface layer will be nonlinear for other reasons. In order to study the leakage applicable to surface ducts with such SSPs, a number of profiles of nonlinear form were considered. For convenience, each of these profiles was designed so that the function of sound speed with depth, c_z, followed a mathematical form for which the trapping frequency $f_{c,1}$ might be readily determined by application of Equation (1). The selected functions of sound speed are as follows:

exponential: $c_z = c_w + \Delta_{Ex}\left(1 - e^{-z/L}\right)$ (15)

where L metres is the depth decay constant, and c_z approaches $c_w + \Delta_{Ex}$ at $z \to \infty$;

elliptical: $c_z = c_w - \Delta_{El} + 2\Delta_{El}\sqrt{1 - \frac{3}{4}\left[\left(z/D_{El}\right) - 1\right]^2}$ (16)

where $c_z = c_w + \Delta_{El}$ at the base of the duct at depth D_{El} at which $dc_z/dz = 0$; and

quadratic: $c_z = c_w + g_Q z - g_Q z^2 / \left(2D_Q\right)$ (17)

where g_Q is sound speed gradient at the surface, and the duct depth D_Q is that at which $dc_z/dz = 0$.

The design of each of these elliptical and quadratic SSPs is such that the sound speed gradient is monotonically decreasing with depth, and is zero at the base of each duct. Of course the exponential profile has no depth limit, but for present purposes the depth of the duct D_{Ex} was selected as equal to 5 times the depth constant L. At this depth, the term $1 - e^{-z/L}$ in Equation (15) is about 0.993, and the intended approximation, that is, c_z approaches $c_w + \Delta_{Ex}$ at depth D_{Ex}, is sufficiently valid. Using Equations (15), (16) and (17), finding relevant expressions for the index of refraction $N_z = c_w/c_z$, for each profile type a closed form solution was found for the duct trapping frequencies $f_{c,m}$, through application of Equation (1). In each instance, some small level of approximation was made, that is, none of the resulting expressions was exact.

The set of SSPs selected for study was designed so that the sound speed gradient at depth $z = 0$ was 0.016s^{-1} for each profile, and that the trapping frequency $f_{c,1}$ obtained using the respective formula determined from Equation (1) was the same as for the 50 m isothermal duct as determined using Equation (2). The resulting SSPs are shown in Figure 3, with the 50 m isothermal profile. Each is designed to have the same below layer gradient $g_t = -0.1512\text{s}^{-1}$, and the same surface sound speed $c_w = 1539.7483$ m/s as used for the simulations of Section 3. Duct depths are 120.2 m (exponential), 92.3 m (elliptical) and 76.3 m (quadratic).

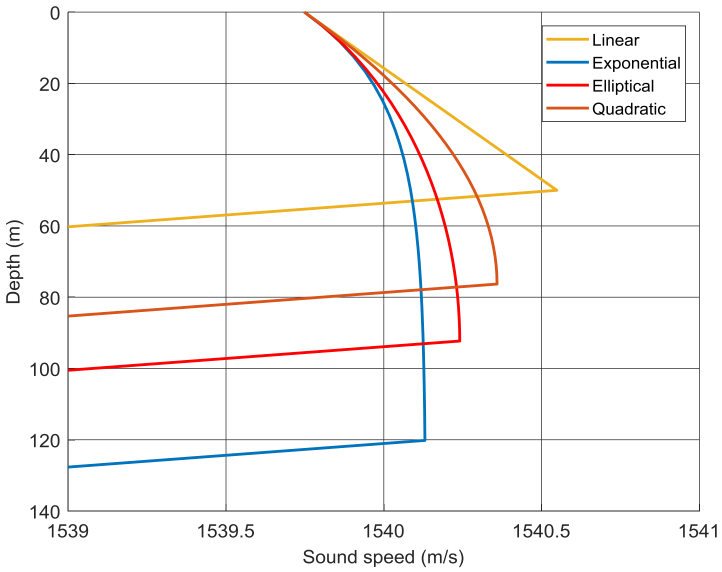

Figure 3: Sound speed profiles for surface ducts of nonlinear profile, ray-based trapping frequencies all 537 Hz

5.1 Simulations of Leakage from Surface Ducts of Nonlinear Sound Speed Profile

As for the work described in Section 3, a number of simulations of sound transmission within a surface duct were made using the normal mode model ORCA, for ocean scenarios with each of the exponential, elliptical and quadratic SSPs as shown in Figure 3. These simulations were interrogated to extract the rate of leakage of sound from the surface duct, for the first mode, for a range of frequency values from below that for trapping of the first mode, to that for which at least two modes were expected to be trapped. The leakage rate was found directly from the model's complex mode finder. For each scenario, the sound source was at 7 m depth. As for the simulations of Section 3, the ORCA runs included Thorp absorption, but leakage data presented below have had the Thorp attenuation removed.

As mentioned earlier, each scenario has a ray-based trapping frequency $f_{c,1} = 537.36\,\mathrm{Hz}$. Although not shown here, the duct trapping frequency expression derived for each of the nonlinear profiles has the same term $(m-1/4)$ to express the relativity between trapping frequencies for modes of index number m as included in Equations (1) and (2) for the duct of linear gradient. Thus, the ray-based duct trapping frequency for mode $f_{c,2}$ follows as $537.36\times\left(2-\frac{1}{4}\right)/\left(1-\frac{1}{4}\right)=1253.84\,\mathrm{Hz}$ for every duct type shown in Figure 3.

The derived 1st mode leakage data are shown in Figure 4 together with the leakage data for the duct of linear gradient (shown in Figure 2), the line according to the algorithm for leakage vs frequency for an isothermal duct of depth 50 m, from Equation (9), and the solid and dashed red and green lines discussed in Section 3.1.

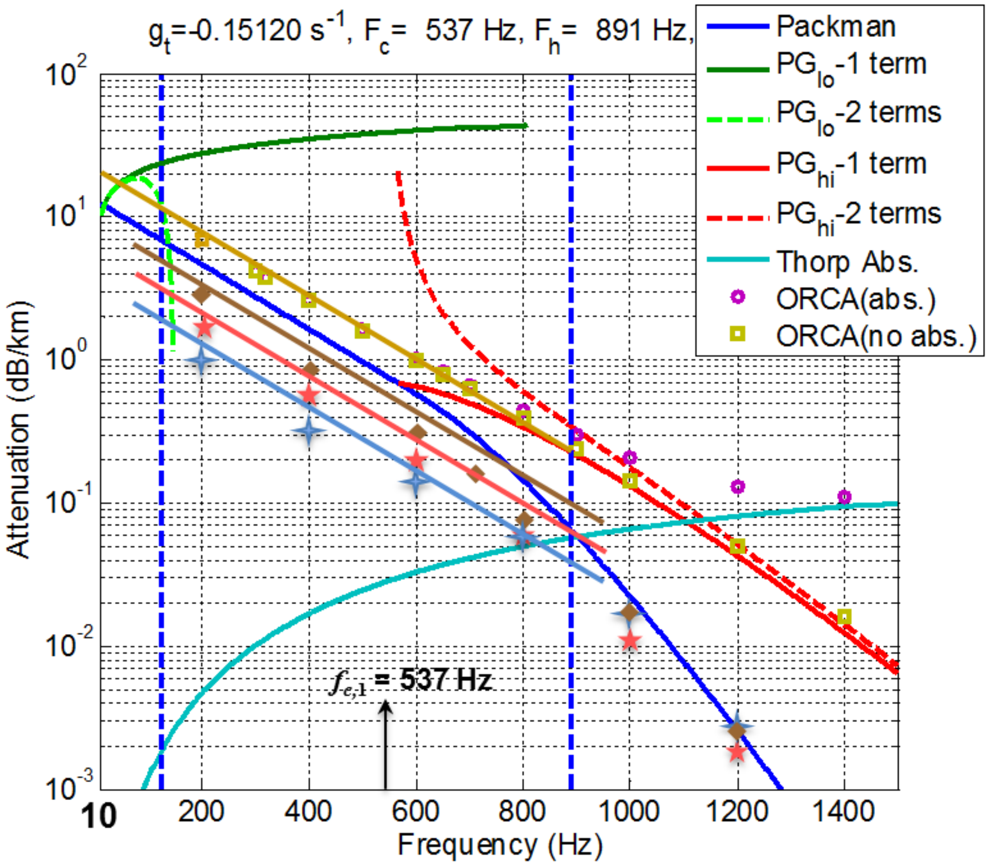

Figure 4: Leakage rate estimates for surface ducts of nonlinear gradient, trapping frequencies all 537 Hz, vs 1st mode ORCA data (no abs.): ✦ exponential SSP, ★ elliptical SSP, ♦ quadratic SSP, □ linear SSP

5.2 Proposal of Equivalent Uniform Gradient

A noteworthy feature of the ORCA data for each of the exponential, elliptical and quadratic SSPs is that, for the larger leakage values (at frequencies below about 900 Hz) each set of data has a very similar rate of variation with frequency as for the data for the linear duct, and is a good fit to a line of the form $A_1(f)\approx K\exp(-0.00514\,f)$ dB/km where K is a constant of different value for each set. More importantly, by reference to the comparison mentioned in Section 4, of leakage from linear ducts of deep depth and weak gradient versus predictions from Equation (14), each set of ORCA data in Figure 4 is very close to the leakage data for the linear duct of the same depth and trapping frequency. For example, the ORCA leakage data obtained for the 120.2 m exponential duct of trapping frequency $f_{c,1} = 537.36\,\text{Hz}$ is very close to the ORCA leakage data at corresponding frequencies for the 120.2 m duct of linear SSP and gradient 0.00115s^{-1} for which the ray-based trapping frequency $f_{c,1}$ is also $537.36\,\text{Hz}$. From this correspondence of datum values, it seems reasonable to propose that each surface duct of nonlinear profile has an "equivalent uniform gradient", with the definition being: that gradient of a linear duct of equal depth which enables the trapping frequencies to be identical. Mathematically it can be expressed as the gradient g obtained by inverting Equation (2) for mode $m = 1$, as

$$\text{equivalent uniform gradient } g \approx \left(\frac{9}{16}\right)^2 \frac{1}{2\left(f_{c,1}\right)^2}\left(\frac{c_w}{D}\right)^3 \text{ s}^{-1} \tag{18}$$

where the depth D, the trapping frequency $f_{c,1}$ and sound speed at the surface c_w are all obtained from the nonlinear SSP in question.

With this definition, the leakage data for each of the exponential, elliptical and quadratic ducts may be estimated using Equation (14), with the values of c_w and $f_{c,1}$ being those for the nonlinear SSP in question, and the value g being obtained from Equation (18). The predicted values of leakage are shown in Figure 4 by the straight lines for which the colour corresponds with the respective set of ORCA datum points. Whilst the agreement between the ORCA data and the lines obtained using Equation (14) is not perfect, it is nonetheless, reasonable, and quite adequate for practical use in obtaining leakage estimates.

5.3 Estimate of Leakage Rates for Exponential Duct of Trapping Frequency 939 Hz

As a further indication of the applicability of the concept of equivalent uniform gradient for a nonlinear duct, and of Equation (14) as a means of making estimates of leakage, the technique was applied to the case of an exponential duct of the form described earlier, with trapping frequency 939 Hz and depth 82.8 m.

From Equation (18), the equivalent uniform gradient follows as 0.00115s^{-1}, and from Equation (14) the expected function of leakage is

$$A_1(f) \approx 4.57\,e^{-0.00297\,f}\ \text{dB/km}\ . \tag{19}$$

Figure 5 shows this line, together with 1st mode ORCA datum values obtained from relevant simulations, as well as the ORCA data for the 50 m isothermal duct and a line according to Equation (9). Clearly, the leakage function derived for the 82.8 m exponential duct, using this process, is a very reasonable fit to the ORCA data.

6 DISCUSSION

Whilst the estimate of leakage rate determined by Equation (19) is reasonably close to the ORCA data in Figure 5, it nonetheless over-estimates the dB/km values of the simulated ORCA data, approximately by 30%. Likewise, the estimates of leakage obtained using Equation (14) with the relevant values of equivalent uniform gradient, shown in Figure 4, also over-estimate the corresponding ORCA data. Although not shown, ORCA leakage data obtained for linear ducts of weaker gradient than isothermal water were also over-estimated by Equation (14), to a similar degree to that shown in Figure 4. However, the estimates of leakage rate made using Equation (14) for isothermal ducts of depths 20 m, 34.46 m and 50 m were extremely close to the corresponding ORCA data. This is not an unreasonable outcome, as the algorithm shown in Equation (14) is merely an approximation based on the work of Furry (Kerr, 1951) and observations of data from ORCA simulations. Equation (12), which is derived from Furry's analysis, is relevant only to frequencies above duct trapping for a duct of uniform gradient, and gives an expectation, but not proof, that there is but one set of leakage data as a function of $f/f_{c,1}$. Further, the concept of a skip distance for a limiting ray has little meaning for ducts with zones of sound speed gradient near zero, such as the nonlinear ones studied, so the success shown in the paper by the use of the concept of an equivalent uniform gradient is, with present knowledge, fortuitous, but very convenient.

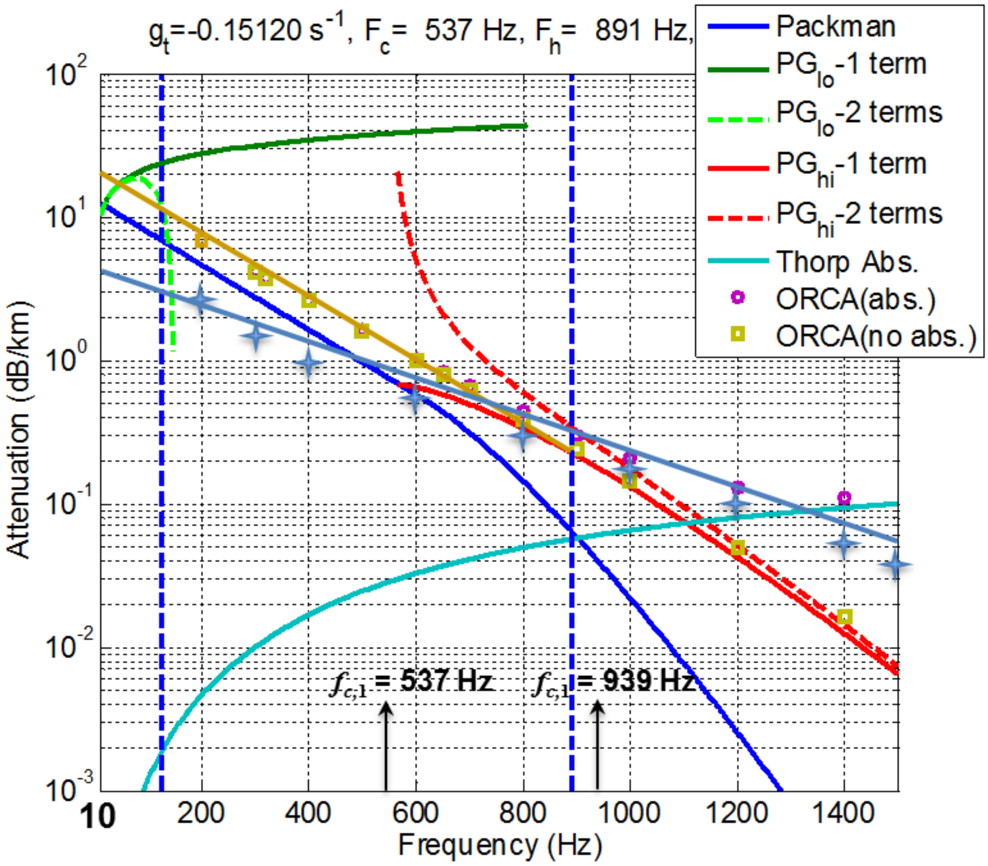

Figure 5: Leakage rate estimate for exponential duct — vs 1st mode ORCA data (no abs.): ✦ exponential duct of ray-based trapping frequency 939 Hz, ◻ linear SSP of ray-based trapping frequency 537 Hz

7 CONCLUSIONS

A part-empirical algorithm, for the estimation of the coherent leakage rate for sound travelling underwater in a surface duct scenario, has been extended to describe the leakage for surface ducts with linear sound speed profile for which the sound speed gradient is less than for isothermal conditions. By then using a modal model to simulate the leakage rate for surface ducts of a number of forms of nonlinear profile, it was observed that the leakage rate versus frequency was very similar to that for corresponding ducts of uniform gradient and identical trapping frequencies and depths. The concept of an equivalent uniform gradient has been proposed for surface ducts of nonlinear profile. Estimations of leakage rate made for nonlinear surface ducts using the algorithm intended for linear ducts, but applying the relevant equivalent uniform gradient, have been found to agree well with data from simulations, and give promise for application to nonlinear sound speed profiles measured at sea.

REFERENCES

Duan, R.; Yang, K.; Ma, Y. and Chapman, N.R. 2016, ‘A simple expression for sound attenuation due to surface duct energy leakage in low-latitude oceans’, *J. Acoust. Soc. Am.* 139, EL118

Jones, A.D., Duncan, A.J. and Zhang, Z.Y. 2015, Coherent Leakage of Sound from a Mixed Layer Surface Duct - Revisited, *Proceedings of Acoustics Hunter Valley*, 15-18 November, Hunter Valley, Australia

Jones, A.D., Duncan, AJ. and Zhang, Z.Y. 2016, Estimates of the coherent leakage of sound from an ocean surface duct, *Proceedings of Acoustics 2016 Brisbane*, 9-11 November 2016, Brisbane, Australia

Kerr, D. E., ed. 1951, *Propagation of Short Radio Waves*, McGraw-Hill

Packman, M.N. 1990, ‘A Review of Surface Duct Decay Constants’, *Proc. I.O.A.*, Vol. 12, Part 1, 139-146

Pedersen, M.A. and Gordon, D.F. 1965, ‘Normal-Mode Theory Applied to Short-Range Propagation in an Underwater Acoustic Surface Duct’, *J. Acoust. Soc. Am.*, 37, 105-118

Pedersen, M.A. and Gordon, D.F. 1970, ‘Theoretical Investigation of a Double Family of Normal Modes in an Underwater Acoustic Surface Duct’, *J. Acoust. Soc. Am.*, 47, 304-326

Urick, R.J. 1983, *Principles of Underwater Sound*, 3rd edition, Peninsula Publishing

Westwood, E.K., Tindle, C.T., and Chapman, N.R. 1996, ‘A normal mode model for acousto-elastic ocean environments’, *J. Acoust. Soc. Am.*, vol. 100, 3631-3645