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SOUND, SCIENCE AND SOCIETY

Simple experiments in vibration and acoustics

Hugh Hunt(1)

(1) Department of Engineering, University of Cambridge, UK

ABSTRACT

Sound and vibration are mostly invisible, so no wonder solving noise problems is tricky. Mechanical vibration is taught to undergraduates as if it is a simple science. The mass-on-a-spring, uni-axial vibration of a rod, viscous damping, modal analysis - all these are the bread and butter of vibration science. As for rigid-body dynamics (which this talk will dip into) undergraduate courses remain fixed in 2-D planar motion. But real dynamic and vibrating systems just don't behave simply. There are pitfalls in even the most ordinary cases and some of these will be demonstrated: a tuning fork; a bottle of coke; a bending beam; a turbocharger wheel, a bouncing ball ... and boomerangs. All of these things behave counter-intuitively.

The paper describes many practical demonstrations - seeing is believing. All are demonstrations that can be repeated at home and they will be shown live at the conference!

1 INTRODUCTION

This paper describes counter-intuitive phenomena in dynamics, vibration and acoustics. Something is counter-intuitive if:

- it requires advanced/specialist knowledge;
- it is obscure or difficult to observe;
- it doesn't fit with our experience;
- we've never noticed it before;
- we believed what our teachers said.

Specialist knowledge, for instance, is required to understand quantum physics and relativity and these effects are also difficult to observe. But some things are easy to observe, and in acoustics is full of them. For example we hear strange sounds or we see musical instruments doing remarkable things. We are used to these being complicated to analyse and impossible to understand simply. Seismic waves move through the ground and are refracted within the layered crust of the Earth – well outside of our normal knowledge and experience. But what do we learn when we tap on the side of a cup, or blow a note on a bottle, or listen to a tuning fork, or even when we throw a boomerang? We just don't always think about the simples of problems and we don't notice just how strange they are. We believe that we understand all the simple problems, because we were taught to understand them, and we look no further. As Professor Charles Inglis said in his 1944 James Forrest Lecture [1]

> *Just as in this age of mechanisation we welcome the advent of any mechanical process which makes a demand on craftsmanship and manual skill, so some of us at any rate may feel grateful that, in problems relating to vibrations, nature has provided us with a range of mysteries which for their elucidation require the exercise of a certain amount of mathematical dexterity. In many directions of engineering practice, that vague commodity known as common sense will carry one a long way, but no ordinary mortal is endowed with an inborn instinct for vibrations; mechanical vibrations in general are too rapid for the utilization of our sense of sight, and common sense applied to these phenomena is too common to be other than a source of danger.*

There are some important concepts that will get us a long way. These relate to the simple mass-spring-damper "*mkc*" system shown in Figure 1. The key elements of the model are:

- Mass *m* – but note that it is usually distributed;
- Stiffness *k* – but it rarely looks like a coil spring;
- Damping *c* – and it hardly ever looks like a viscous dashpot;
- Applied force *f* – hardly ever sinusoidal.

The natural frequency $\sqrt{\frac{k}{m}}$ is a simple concept, but may systems have multiple modes and multiple natural frequencies. The simple "*mkc"* model gives us no means of understanding nodal points, mode shapes and coupling between modes. And it tells us nothing of non-linearity We are led through the standard undergraduate vibration courses through free vibration, forced vibration, resonance, damping, impulse and step responses, acceleration through resonance, random vibration, spectral analysis, base isolation … there is so much that this model can do!
What it cannot do is more important to understand. This paper, and in particular the talk, will open your eyes.

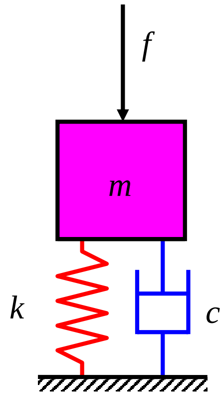

Figure 1

2 THE EXPERIMENTS

The particular examples that will be demonstrated are:

- The Helmholtz resonator and the plastic drinks bottle;
- The tuning fork and the importance of non-linearity;
- The coffee cup and the subtleties of axisymmetric systems;
- The bending beam, and its application to solving vibration problems in buildings;
- The turbocharger and yet more subtleties of axisymmetry;
- Constrained-layer damping, and how easy it is to see;
- Tuned vibration absorbers and how a glass of wine helps.

2.1 The Helmholtz resonator and the plastic drinks bottle

This is a remarkable experiment, and so simple to perform. Take a plastic drinks bottle, it has to be circular in cross section. If you blow across the top of the bottle you can produce a note. This is the situation depicted in Figure 2(a). Now pour some water into the bottle, as shown in Figure 2(b) and of course the frequency rises – you get a higher note – no doubt because the volume has reduced. Now tip that water out, and instead squeeze the bottle so that the volume is reduced by about the same amount, as shown in Figure 2(c). Astonishingly, the note produced by blowing over the top is *lower*. How can this be? Surely the volume is reduced in both of these cases and the note should rise? The conundrum is only further confounded when the bottle is lowered into a bucket of water, as shown in Figure 1(d) and the note produced goes up, when the volume inside the scrunched-up bottle hasn't changed.

The explanation? Well, think of a Helmholtz resonator. The mass *m* in Figure 1 is the mass of air in the narrow opening of the bottle, called the "neck plug" and the stiffness *k* is the elasticity of the air contained in the bottle. Everything is explained if you consider that a cylindrical plastic bottle is very rigid so the stiffness is governed by the elasticity of the contained air. Squeeze the bottle and you lower the stiffness, so the note drops. Put the bottle under water and the walls are stiffened up again and the note rises.

To demonstrate this in a lecture requires a bit of planning – a bucket of water and a jug to pour water around from one bottle to the other. It's actually much easier to do in a swimming pool. Usually there'll be an empty bottle of soda around somewhere (look in the bin!) and you can impress your friends with an in-pool experiment.

This experiment is entirely governed by linear theory, can be explained using the mass-on-spring model and is very easy to perform. It ought to be a standard case study for all undergraduates in acoustics and vibration because it illustrates the counter-intuitivity criterion that "we've never noticed it before". The next experiment fits exactly into the same category, but this time there is a tiny touch of non-linearity.

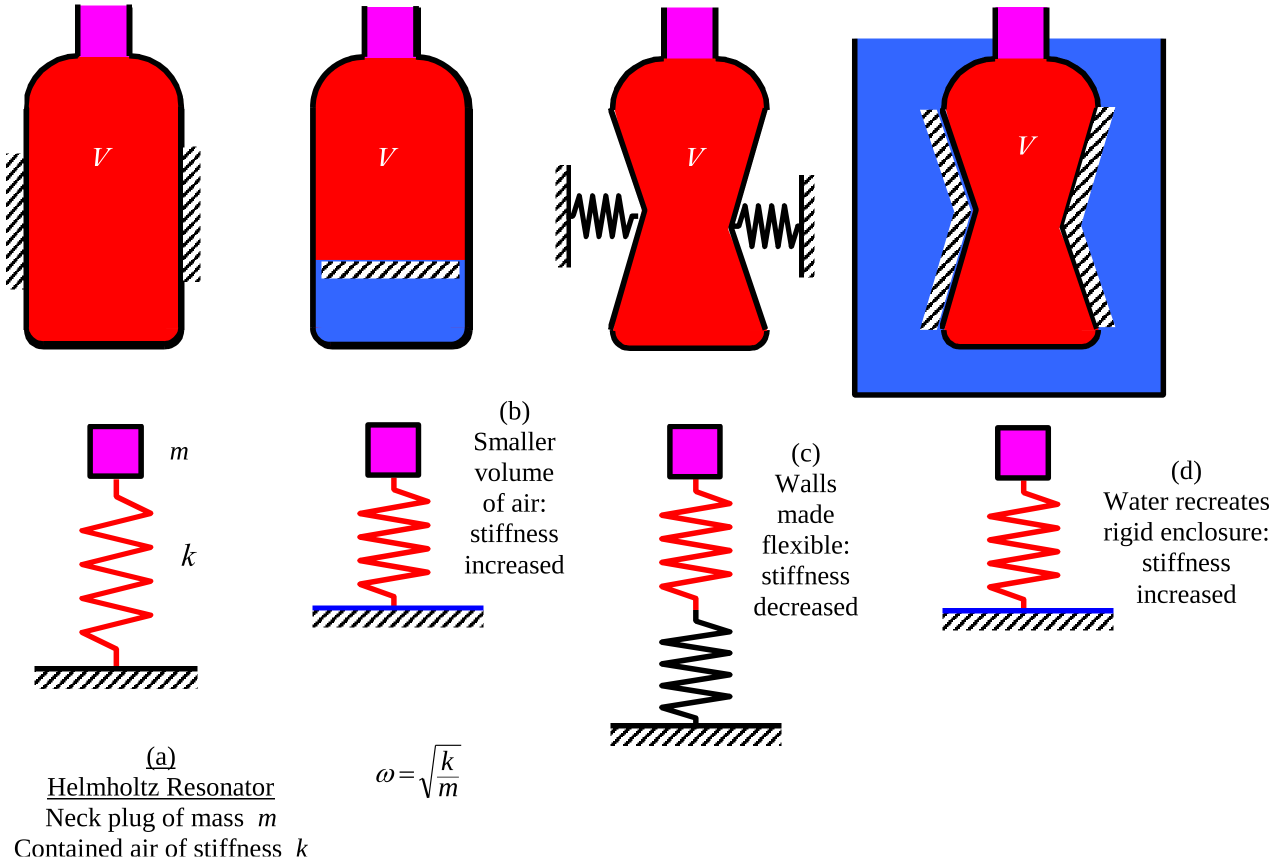

Figure 2

2.2 The tuning fork and the importance of non-linearity

What could be more common an example of vibration than a tuning fork? It produces a pure tone, and with a simple spectrum analyser app on your smartphone you see that there are no harmonics to speak of (well, there are some high tones of you whack the tuning fork on a hard surface, but these tones die out quickly). Let's suppose the tuning fork is the standard orchestral "A" at 440Hz. The app shows us that it's 440Hz, perhaps 339Hz or 441Hz – that's probably the inaccuracy of the app than the tuning fork in my experience.

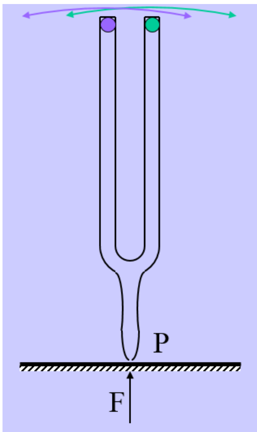

Figure 3

You'll have noticed that a tuning fork is very quiet, and to produce a loud note it's easy just to put the bottom end of the tuning fork, labelled P in Figure 3, onto a table. The sound is much louder because, we're told, the table acts as a sounding board and the large surface area of the table radiates sound very efficiently. But surely point P is a nodal point for vibration in the simplest [1,-1] mode, and if so then putting it onto a table should make no difference. Take a look at the spectrum analyser app and you'll see that the peak frequency is 880Hz. The force F that is transmitted to the table is at 880Hz. Why is that? Well, consider that the tips of the tuning fork are moving backwards and forwards on arcs of a circle. Centrifugal inertia forces are generated and these a maximum twice per cycle when the speed is maximum. Suppose the tip amplitude is 0.2mm, the oscillating frequency is 440Hz, the moving mass is, say, 20% of the fork mass, then the 880Hz component of tip force F is about 10% of the weight of the fork.

Again, this is so simple to observe, and linear analysis explains the pure sinusoidal motion really well. But sometimes small non-linear effects are responsible for the biggest surprises.

2.3 The coffee cup and the subtleties of axisymmetric systems

Morning coffee, tap the mug with a teaspoon (why not a coffee spoon, I don't know). The note you get depends on where you hit it. In fact, the note changes by as much as a semitone (that's 6%) if you move the tapping point by just 45° around the circumference of the rim. Just a little thought and you can understand the importance of the mass of the handle. In Figure 4 you see that the mode shape responsible for vibration of the cup is an elliptical form with four nodal points. If you tap at any of the points labelled A then you excite a mode that involves the mass of the handle. The *mkc* model tells us that higher mass should give a lower frequency, exactly as observed because tapping at the points B give a higher note.

Incidentally, the note you get for a cup full of coffee is lower that for an empty cup – try it. It's all about added mass. Interesting things happen when there are bubbles around. Try doing the same experiment with your next glass of bubbly – you'll wonder why people spend so much money on champagne flutes that ring like a clear bell when the bubbles just deaden the sound completely.

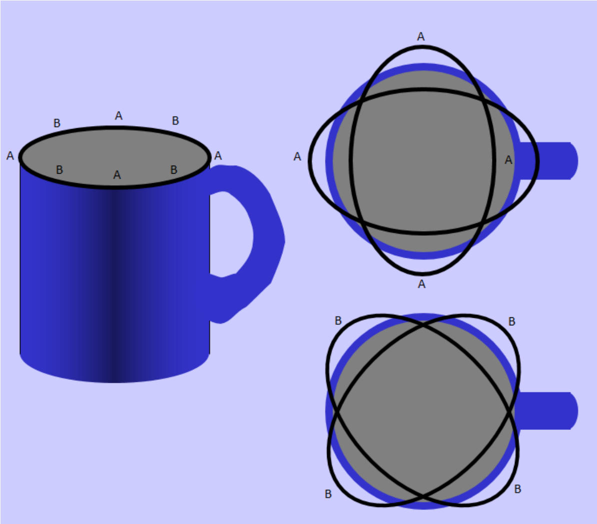

Figure 4

2.4 The bending beam, and its application to solving vibration problems in buildings

A simple beam will vibrate, and there are lovely things to demonstrate if you know a little bit about nodal points and modes. The axial modes of vibration of a free-free bar are shown in Figure 5, up to the 4th mode. The frequency of each mode is in a linear "harmonic" sequence. The position of the nodal points is straightforward because the mode shapes are simple cosines. The odd modes all have a nodal point at the centre and the nodal points are al evenly spaced.

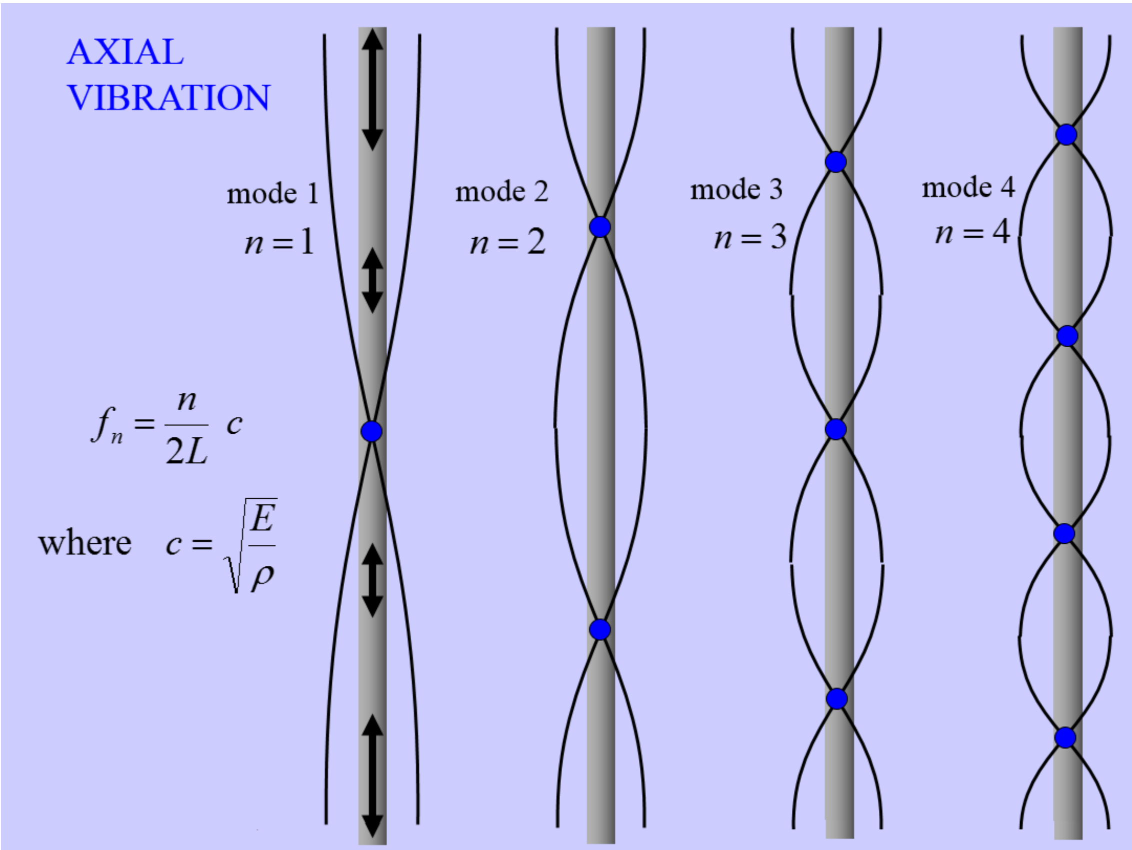

Figure 5

Bending modes are a bit more complicated and are shown in Figure 6. The complexity is only in the calculation of the modal frequencies and the position of the nodal points. The frequencies do not form a harmonic sequence at all, and their spacing is quadratic, for high frequencies. The position of the nodal points is also tricky to compute, but they are given in Figure 7 .for the very important reason that you really must make one of these bars for yourself as a demonstration of nodal points and modes.

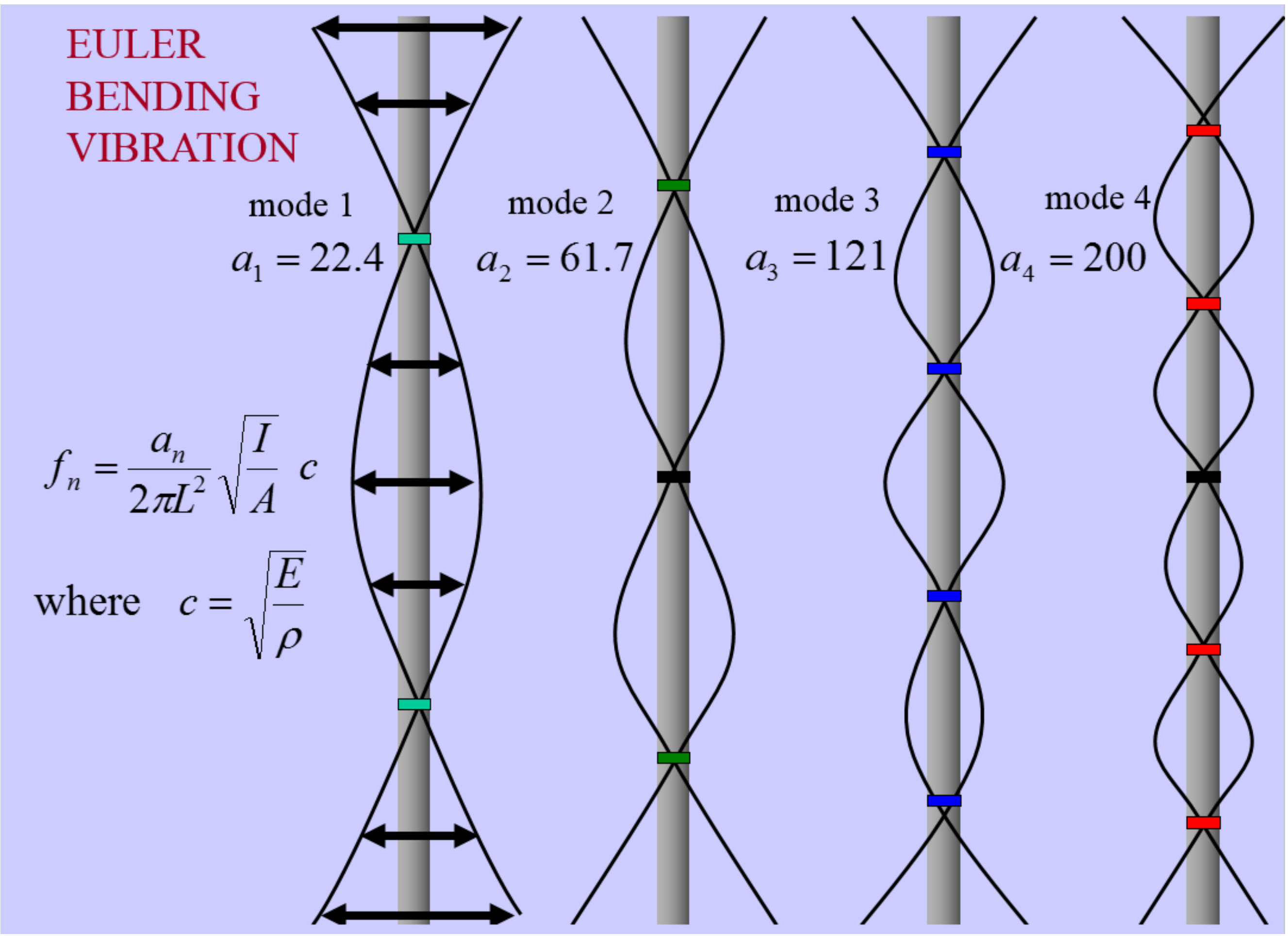

Figure 6

Go to a hardware store and buy a length of 25mm diameter thin-walled aluminium tube, about 2 metres long works really well, but shorter is fine too. Use the information in Figure 7 to mark with coloured tape the positions of the nodal points on your beam. You need to scale the distances unless your beam happens to be exactly 1000mm long.

Position of nodal points for a beam of L=1000mm
(measured in mm from one end)

mode 1:	224	776						
mode 2:	132	500	868					
mode 3:	94	356	644	906				
mode 4:	73	277	500	723	927			
mode 5:	60	226	409	591	774	940		
mode 6:	51	192	346	500	654	808	949	
mode 7:	44	166	300	433	567	700	834	956
mode 8:	39	147	265	382	500	618	735	853	961

Figure 7

Now find some suitable donger to hit the beam with. For the low frequencies I find that a tightly rolled up newspaper or the rubber sole of a running shoe work fine – call this donger A. For the higher frequencies avoid using anything too hard like a ruler or a pen. The ideal is something heavy and soft, like a short length of three-phase cable – it has heavy copper inside and a soft exterior – this is donger B. Use your knuckles if you have a high pain threshold! For very high frequencies use a hard object, like a steel ruler or a spoon, that's donger C.

Now the experiments. First hold the bar between your fingers at the middle. This is a nodal point for all the odd axial modes and the even bending modes. Hit the end of the bar axially with a hard donger C and you'll excite the odd high-frequency axial modes. Hit the end transversely with the medium donger B and you'll get a rich sound of all the even bending modes. Now, while this sound is ongoing pinch a nodal point of mode 4 between your fingers (it's coloured red in Figure 6) and magically only mode 4 will carry on vibrating. That's because only mode 4 has a nodal point both in the centre and at the other red point. Do the same for mode 6, mode 8, mode 2 (you might need to use donger A). Try all the nodal points that you have now demonstrated that the coloured marks for the even modes are all in the right place. Try pinching one of the odd modes and you'll find that there is no sound because none of the odd modes are nodal at the centre.

Now check the odd modes. Start by holding the beam at a nodal point of mode 3, say. Hit the beam with donger B, and it'll sound at the frequency of mode 3. Try pinching another node of mode 3, it keeps ringing. But pinch it anywhere else it will stop. What about mode 1? You'll note that the nodal point for modes 1 and 5 are very close together so to get mode one (and it's quite now, so you see it rather than hear it) use the soft donger A.

The grand finale of all this is to demonstrate that you can't excite a mode at one of its nodal points, so hold the beam at a nodal point for mode 5, say, and use donger B to hit another nodal point for 5. It won't sound. It's like trying to move a seesaw by pushing at the pivot. In just the same way, you can't stop a mode at a nodal point. The corollary is useful, that you can stop a mode at the same point that you started it.

The message is powerful. Suppose you have a vibration problem in a building caused by the spin cycle of a washing machine (this is exactly what happened in a building in Sydney where a coin laundry was causing problems for residents above). What do you do? Stiffen the building? Add damping? Change the spin speed? No, all you do is to find where the nodal points are in the laundry for the frequency that is causing the problem and mode the washing machines to these points. It's remarkably effective and a very cheap solution. It helps to have an understanding of nodal points in your mind when solving vibration problems.

2.5 The turbocharger and yet more subtleties of axisymmetry

Just when you thought that you understood nodal points and modes, we get onto a system with axial symmetry, very common in industry and transport – the turbine. A car turbocharger, a steam turbine, an aircraft turbofan engine… The common thread is that they have blades that are all identical and radially disposed around a hub. A model for this is shown in Figure 8, which I call the paddle-wheel – for obvious reasons. I've made one machined out of aluminium allow, about 150mm diameter with 12 blades each about 5mm thick.

If you hold the hub and tap one of the blades and it rings like a bell because the hub is a nodal point for many of the modes. A key feature of bladed systems is that there are many modes all very-closely spaced in frequency. The strange thing is that for the vibrating beam we found that you can always stop a mode at the same point that you used to excite it. Well, it doesn't work for axisymmetric systems like this. It's a case of a system that looks simple but that is full of complexity.

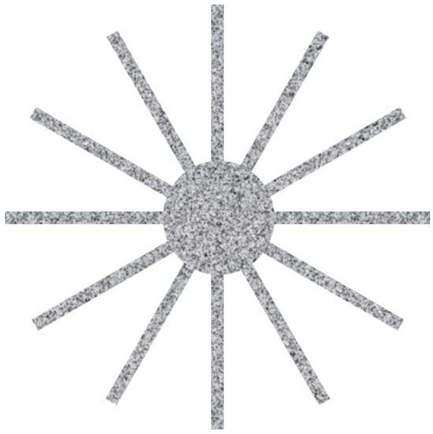

Figure 8

If you add a tiny amount of mass to one of the blades, to mess up the symmetry, then all of a sudden the normal rules apply. It's really a good case for Inglis' law - *common sense applied to these phenomena is too common to be other than a source of danger.*

2.6 Constrained-layer damping, and how easy it is to see

If you take an aluminium bar or plate and you hold it at a nodal point – experiment until you find one – then it will ring like a bell when struck with a donger, as we have seen above. But take two of them and stick them together with double-sided adhesive tape then the vibration is dead. Such a simple way of controlling noise, but you have to think of this in advance. Surface application of adhesive tape doesn't work at all well. Why not? Think of bending beam theory, where the maximum shearing stress in a beam is along the neutral axis. If you allow the beam to shear a little along this plane, as illustrated in Figure 9, and if the material you are using there is capable of absorbing energy then you have a result. You can try gluing the two beams together with a hard epoxy adhesive – and it doesn't work.

The demonstration is simple and very dramatic. It's important to understand the mechanism though because many efforts to control noise with surface treatments just don't turn out to be effective

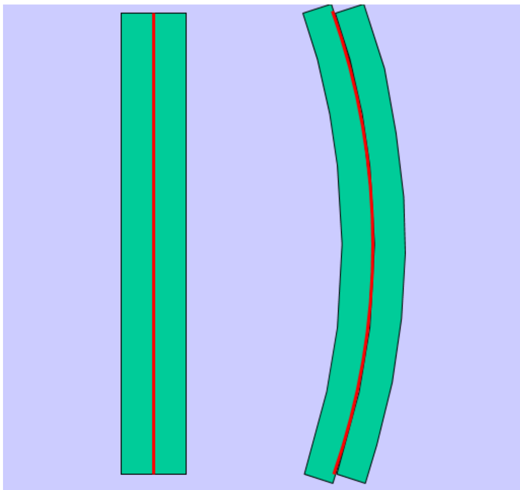

Figure 9

2.7 Tuned vibration absorbers and how a glass of wine helps

Some of the best experiments involve a bit of wine. This one illustrates the principle of a tuned vibration absorber, again a technique for adding damping to a vibrating system but this time it can be done as a retrofit. Suppose you have a problem vibration, like the ringing of a wine glass. Now attach a small mass and spring system to it, in this case I've superglued a straightened-out paperclip to the rim. There are now two vibration modes, instead of just the original one. If you choose the frequency of the paperclip so that it exactly matches that of the glass then there is an effective interaction between the two modes and a little bit of damping at the joint (ie in the superglue) is enough to kill the vibration. So you have created and extra mode but you've added damping.

In this experiment you keep the paperclip wire fixed in length and you add wine (or remove it, which is more enjoyable – but be careful not to poke yourself with the wire). As the frequency changes with a lowering level of wine in the glass there will come a point where the frequencies match and there is heavy damping of the wine-glass ringing mode. As further wine is consumed the ringing reappears.

This illustrates the importance of accurate tuning of a tuned-mass damper and there is a great deal of literature in this area. Many examples of tuned absorbers are to be found in easily observed places, for instance the Stockbridge dampers that are fixed to power lines to control wind-induced vibration.

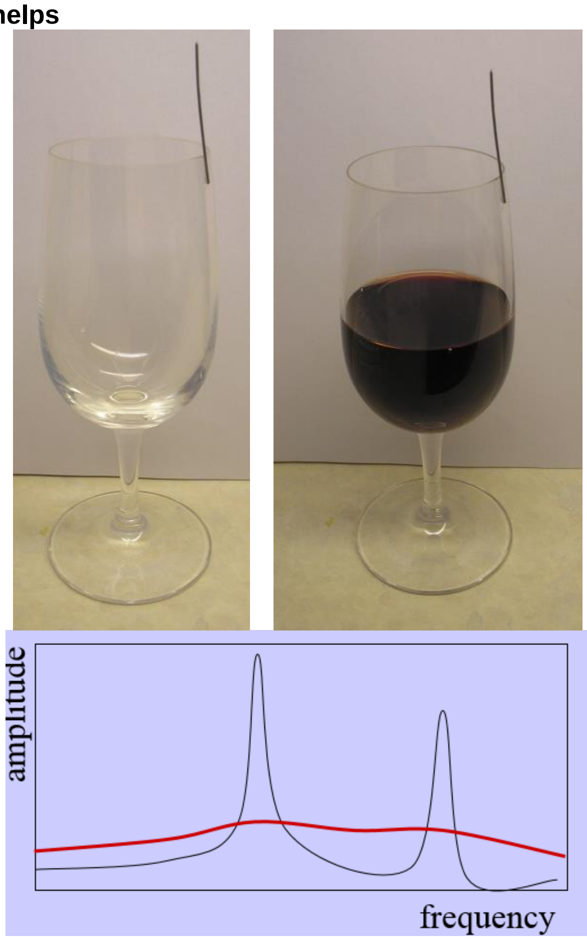

Figure 10

3 CONCLUSIONS

This paper has illustrated some simple and intriguing demonstrations which can help development of an understanding of the simple underlying principles of vibration and acoustics, as well as pointing out a few of the complicated bits. The lecture in the conference will demonstrate all of these experiments, and a few more as they come to mind.