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Further Approximation of a Model of Coherent Reflection Loss at the Ocean Surface

Adrian D. Jones (1) and Alex Zinoviev (1)

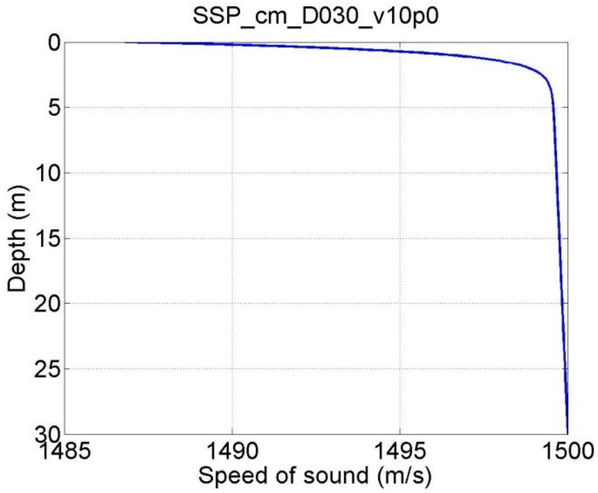
(1) Maritime Division, Defence Science and Technology Group, Edinburgh, Australia

ABSTRACT

The authors previously simplified their analytically complex model “JBZ” of coherent acoustic reflection loss at the ocean surface, and demonstrated that that simplified JBZ model gave loss results similar to those obtained from Monte Carlo parabolic equation (PE) modelling of equivalent scenarios. These models obtained a surface roughness loss inclusive of the refractive effects of a uniformly stratified distribution of near-surface, wind-driven bubbles. Further approximation of the simplified JBZ model has since been carried out. The approximated model, which is the subject of this paper, combines descriptions of roughness loss and bubble-refraction described for the earlier model versions, however, by restricting the ranges of wind speeds, frequencies and grazing angles, achieves an even simpler form. It is now shown that, for a limited, but useful, range of scenarios, for grazing angles as much as about 10 degrees in some cases, the coherent reflection loss at the wind-driven ocean surface, in dB, is well approximated by a function which is linear in grazing angle, where that angle is taken as the angle of acoustic incidence at the bottom of the bubbly region.

1 INTRODUCTION

The description of the coherent loss due to the reflection of sound from the surface of a wind-driven ocean is complicated by the fact that there exists a persistent population of small bubbles near to the surface, and the sound speed in this bubbly layer is significantly reduced due to the compressibility of the bubbles. The resulting sound speed gradient in the bubbly layer is very large, with the effect that a simple application of Snell’s law may not be used to determine angles of surface incidence for sound passing through the layer. A sample sound speed profile, based on the bubble layer model used by Ainslie (2005), is shown in Figure 1. This shows the effect of a bubbly layer of uniform horizontal stratification, for wind speed (referenced to 19.5 m height) $w_{19.5}$ = 10 m/s.



Source (D. W. Bartel, 2011)

Figure 1: Sound speed profile in isothermal ocean including effect of bubbly layer, $w_{19.5}$ = 10 m/s

A model of coherent reflection loss at the wind-driven surface of the ocean was prepared by the authors and the late David Bartel. The model, named “JBZ”, incorporated a description of the refractive effects of an assumed population of near-surface bubbles for which stratification was assumed to be uniform in range. JBZ (Jones et al. 2012) was prepared for use with ray models of sound transmission. The original form of the model involved a determination of the angle of surface incidence that accounted for the refraction in the bubbly layer via a series summation that involved considerable complexity. A simplified version of JBZ was then prepared that avoided the series summation and permitted hand calculations (Jones et al. 2014). Use of this simplified version with a ray model showed excellent agreement with results obtained using Monte Carlo runs of a PE model which described scenarios with equivalent surface roughness and sound speed variation representing that due to wind-driven near-surface bubbles. In this paper, a further approximation is made to this model, for a limited range of wind speeds and frequencies, giving a reasonable fit to the convenient form of $\mathrm{RL} \approx A\beta_0$ dB, where A is a constant with units dB/radian and β_0 is the grazing angle, that is, angle relative to the horizontal, of the incident sound at the depth corresponding with the bottom of the bubbly region. In its use with a ray model which does not explicitly include the refractive effect of the bubble layer, this layer being typically 1.5 to 5 metres deep depending on wind speed, JBZ takes the angle of surface incidence provided by the ray model and assigns it as equivalent to the angle β_0 at the bottom of the bubbly layer. JBZ then determines a grazing angle of sound incidence at the surface β_s that accounts for refraction in the thin bubbly layer, and obtains the desired surface loss value using a model of roughness loss with angle β_s.

2 PREVIOUS SIMPLIFIED JBZ MODEL

2.1 Angle of sound incidence at surface

The approximation used by the simplified JBZ model for the determination of the grazing angle β_s of sound incidence at the ocean surface will not be explained here, but may be shown (Jones et al. 2014) to be given in terms of the acoustic frequency of incident sound and the speed of the wind that gives rise to the bubble layer, by

$$\beta_s \approx \beta_0\left(1+1.95\mu^2-0.4\mu^4+4.9\mu^6\right) \text{ radians} \tag{1}$$

where the non-dimensional parameter μ is given by

$$\mu \approx 7.84\times10^{-6} f\left(w_{19.5}\right)^{3/2} \quad \text{if } w_{19.5} \le 8 \text{ m/s, or} \tag{2}$$

$$\mu \approx 2.47\times10^{-5}\left[0.085\,w_{19.5}-0.35\right] f\left(w_{19.5}\right)^{3/2} \quad \text{if } w_{19.5} > 8 \text{ m/s} \tag{3}$$

where $w_{19.5}$ is wind speed m/s at 19.5 m above the ocean surface, f is frequency, Hz. The constant in Equation (2) has dimensions $\mathrm{m}^{-3/2}\,\mathrm{s}^{5/2}$, and the constant in Equation (3) has dimensions $\mathrm{m}^{-5/2}\,\mathrm{s}^{5/2}$.

Figure 2 shows an example of the determination of surface grazing angle β_s as dependent on the angle below the bubbly region β_0, for wind speed 10 m/s and frequency 1.5 kHz for which the value of $\mu \approx 0.5858$. Here, the red line shows the full solution as used by the full JBZ model, the dashed line shows $\beta_s \approx 1.82\beta_0$ from Equation (1) being the linear approximation for small β_0, the green line represents the zero-refraction case of no bubbles, and the blue line is the Snell's law solution $\beta_s = \arccos\left(\left[c_s \cos\beta_0\right]/c_0\right)$, where c_s and c_0 are sound speed values at the surface and at the base of the bubbly layer, respectively. As shown in the figure, the linear approximation is close to the full solution at small values of β_0, and the full solution approaches the Snell's law result at steeper angles. The simplified JBZ uses the lesser of the linear approximation and Snell's law as a reasonable estimate of the full result.

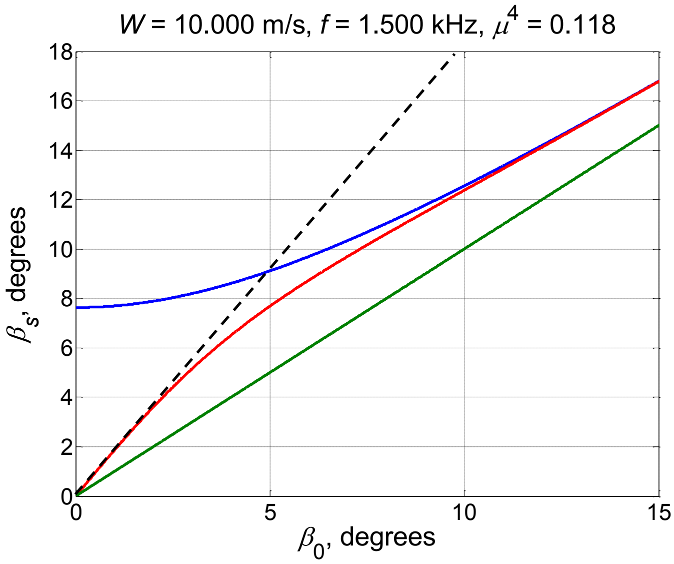


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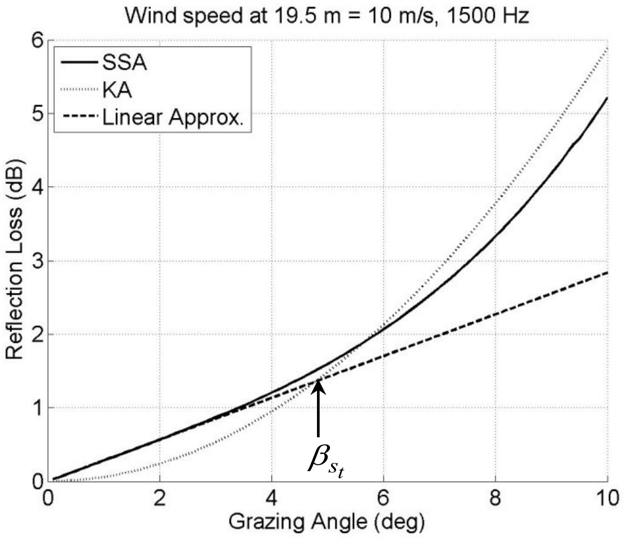
Figure 2: Grazing angle β_s at surface vs grazing angle β_0 below bubbly layer, $w_{19.5}$ = 10 m/s, f = 1.5 kHz. Red line – full solution; dashed line – Equation (1); green line – no bubbles; blue line – Snell's law

2.2 Roughness loss at surface

The JBZ model obtains a value of coherent Reflection Loss (RL) in dB, by applying the derived surface grazing angle β_s to a model of roughness loss applicable to an ocean surface described by a Pierson-Moskowitz (PM) surface wave frequency spectrum. The suitability of some models of roughness loss was considered in some detail by Jones et al. (2016). Figure 3 shows a sample plot of RL versus surface angle β_s similar to those shown by Jones et al. (2016). The solid line is from the small slope approximation (SSA) as described by Williams et al. (2004), the dashed line is the well-known Kirchhoff approximation (KA) and the linear approximation is as described by Jones et al. (2016).



Source (D. W. Bartel, 2011)

Figure 3: Coherent Reflection Loss per bounce vs surface angle β_s; $w_{19.5}$ = 10 m/s, f = 1.5 kHz

Of the three models for which data is shown in Figure 3, the SSA model represents the most accurate determination of coherent RL per reflection. The linear approximation was derived theoretically (Jones et al. 2016) as an approximation of the SSA model for small surface angles β_s. The KA model is used as an approximation of the SSA model for larger angles. The full JBZ model uses the SSA result, whereas the simplified JBZ model approximates the SSA by using the greater of the linear approximation and the KA for each given value of surface angle β_s. The angle of transition, at which the KA loss equals the loss from the linear model, labelled β_{s_t} in the figure, is also a reasonable approximation of the point of divergence of the SSA model from the linear approximation.

3 FURTHER APPROXIMATION OF SIMPLIFIED JBZ MODEL

3.1 Angle below bubbly layer under 5 degrees

The linear approximation to the SSA model has the following form (Jones et al. 2016)

$$\mathrm{RL} \approx 2.79\times10^{-7} f^{3/2}\left(w_{19.5}\right)^{3} \beta_s \ \ \mathrm{dB}. \tag{4}$$

It then follows from Equations (1) and (4) that for small angles of incidence at the bottom of the bubbly layer β_0, and for all wind speed-frequency combinations for which these equations are valid, the RL is a linear function of the angle β_0 below the bubbly layer, and is given by

$$\mathrm{RL} \approx 2.79\times10^{-7} f^{3/2}\left(w_{19.5}\right)^{3} \beta_0\left(1+1.95\mu^2-0.4\mu^4+4.9\mu^6\right) \ \ \mathrm{dB}. \tag{5}$$

From, for example, Figure 2, the dashed line showing the linear approximation of the full function for the surface angle is valid for small values only of angle β_0, in this case to $\beta_0 \approx 3°$. From Figure 3, the linear approximation to the SSA model is likewise valid for small angles only of surface angle β_s, in this case to $\beta_s \approx 4°$. However, the trend in Figure 2 is for the linear function in β_0 to exceed the full solution at steeper angles β_0, and the trend in Figure 3 is for the linear approximation in β_s to under-estimate the SSA result. Thus, the trend of the error in each of the two linear functions is to approximately compensate for the other, with the result that Equation (5) may be expected to be a reasonable approximation to the overall loss function for some angles $\beta_0 > 4°$. This is illustrated in Figure 4 in which the coherent RL derived by the full JBZ model is shown as a function of the angle of incidence at the bottom of the bubbly layer β_0, for the example of wind speed 10 m/s and acoustic frequency 1.5 kHz.

As shown in Figure 4, the values of RL obtained by the full JBZ model diverge from the linear function $\mathrm{RL} \approx 29.5\beta_0$ dB/radian (same as $\mathrm{RL} \approx 0.515\beta_0$ dB/degree) from Equation (5) relatively slowly with angle β_0. Figure 5 shows corresponding data for wind speed 8 m/s and frequency 4 kHz, and illustrates a similar slow divergence from Equation (5) for a high loss scenario. By consideration of these, and many other examples, of the approximation of Equation (5) to the full JBZ solution, it is reasonable to claim that Equation (5) provides an adequate approximation of coherent RL for angles β_0 between zero and 5° for all combinations of wind speed and acoustic frequency of practical interest, for frequency $\le$ 4 kHz. In work to date, this has been established for the wind speed-frequency combinations indicated in Table 1. The limitation to frequencies below about 4 kHz relates to the fact that the JBZ model does not include the loss effects of the scattering from the bubbles in the bubbly layer. This loss mechanism becomes relevant at higher frequencies, as discussed by Ainslie (2005), for example.

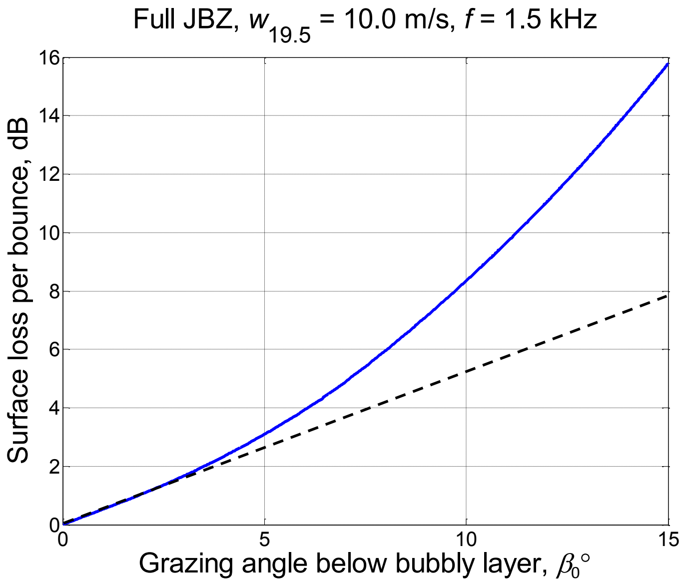


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Figure 4: Coherent RL per bounce vs incidence angle β_0, $w_{19.5}$ = 10 m/s, f = 1.5 kHz. Blue line - full JBZ, dashed line – $\mathrm{RL} \approx 29.5\beta_0$ dB/radian from Equation (5).

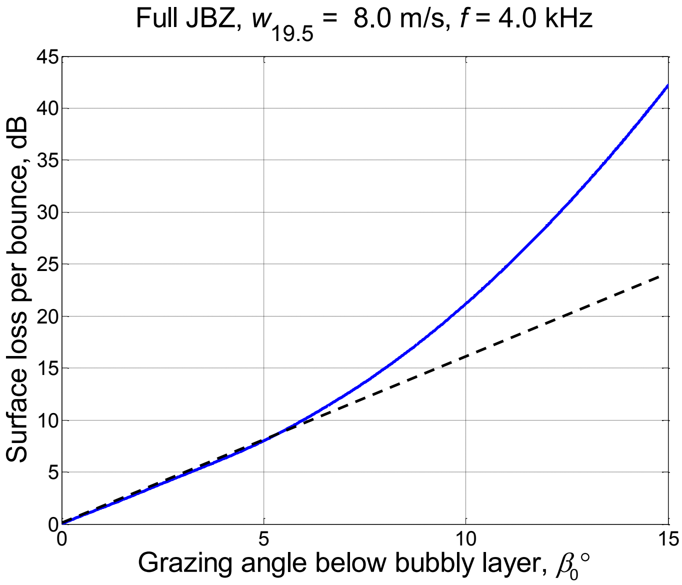

Figure 5: Coherent RL per bounce vs incidence angle β_0, $w_{19.5}$ = 8 m/s, f = 4.0 kHz. Blue line - full JBZ, dashed line – $\mathrm{RL} \approx 90.56\beta_0$ dB/radian from Equation (5).

Table 1: Wind speed – frequency pairs for which RL well approximated by Equation (5), for angles β_0 < 5°

Wind speed $w_{19.5}$	Frequency Values f
5 m/s	4 kHz, 3 kHz, 2 kHz, 1 kHz and less
6 m/s	4 kHz, 3 kHz, 2 kHz, 1 kHz and less
7.5 m/s	4 kHz, 3 kHz, 1 kHz and less
8 m/s	4 kHz, 3 kHz, 2 kHz, 1.0 kHz, 0.5 kHz and less
10 m/s	2 kHz, 1.5 kHz, 1 kHz, 0.5 kHz and less

3.2 Angle below bubbly layer under 10 degrees

Now, the divergence of the RL values from the linear approximation of Equation (5), away from the full JBZ solution, may be partially associated with the divergence of the SSA model away from the linear approximation of Equation (4). The angle of the latter divergence, approximated as angle β_{s_t} in Figure 3, was shown by Jones et al. (2016) for a sea surface with a PM surface wave frequency spectrum to be

$$\beta_{s_t} \approx \frac{0.833}{w_{19.5}} \sqrt{\frac{c_s}{f}} \text{ radians.} \tag{6}$$

The larger the value of β_{s_t}, the larger will be the angle at which the linear approximation of Equation (4) is a good substitute for the full SSA model. For such angles, the approximation of Equation (5) will then be expected to be a good substitute for the full JBZ loss function. Clearly, this will apply for lower values of wind speed $w_{19.5}$ and to a lesser degree, lower frequencies f. The frequency associated with each wind speed value and transition angle β_{s_t} may be found by inverting Equation (6), to give the following:

$$f \approx \frac{0.694}{\left(w_{19.5}\right)^2} \frac{c_s}{{\beta_{s_t}}^2} \text{ Hz.} \tag{7}$$

The value of the non-dimensional parameter μ associated with each value of transition angle β_{s_t} and wind speed $w_{19.5} \le 8$ m/s may now be found by substituting for frequency in Equation (2) using Equation (7) to give

$$\mu \approx \frac{5.44 \times 10^{-6}}{\sqrt{w_{19.5}}} \frac{c_s}{{\beta_{s_t}}^2}. \tag{8}$$

Clearly, for values of transition angle $\beta_{s_t} = 10°$, the relevant values of μ are small. For wind speed 5 m/s, μ = 0.12 and Equation (1) may be approximated as $\beta_s \approx \beta_0\left(1+1.95\mu^2\right)$, or even as $\beta_s \approx \beta_0$ to an error of merely 3%. There is thus little refraction within the bubbly zone and from Equation (5) the Reflection Loss may be approximated as

$$\mathrm{RL} \approx 2.79 \times 10^{-7} f^{3/2} \left(w_{19.5}\right)^3 \beta_0 \left(1+1.95\mu^2\right) \text{ dB.} \tag{9}$$

Figure 6 shows an example of the values of RL obtained by the full JBZ model and by the linear function $\mathrm{RL} \approx 1.96\beta_0$ dB/radian obtained from Equation (9), for wind speed 6 m/s and frequency 1.0 kHz. This frequency is close to the value 0.950 kHz obtained from Equation (7) for a transition angle β_{s_t} set to 10°. Clearly, the full JBZ data do not diverge greatly from the linear function for grazing angles below the bubbly layer $\beta_0 \le 10°$. By consideration of the loss function in Figure 6, and those for other wind speed-frequency combinations for which transition angle β_{s_t} given by Equation (6) is 10°, it is reasonable to claim that Equation (9) provides an adequate approximation of coherent RL for angles β_0 between zero and 10° for frequencies up to those given by Equation (7). In work to date, this has been established for the wind speed-frequency combinations indicated in the first two columns of Table 2. The table also shows the value of frequency obtained from Equation (7) for each wind speed, and it is obvious that these values are close to those in the 2nd column for the frequencies for which the linear function of Equation (9) was judged to be close to the full JBZ loss function.

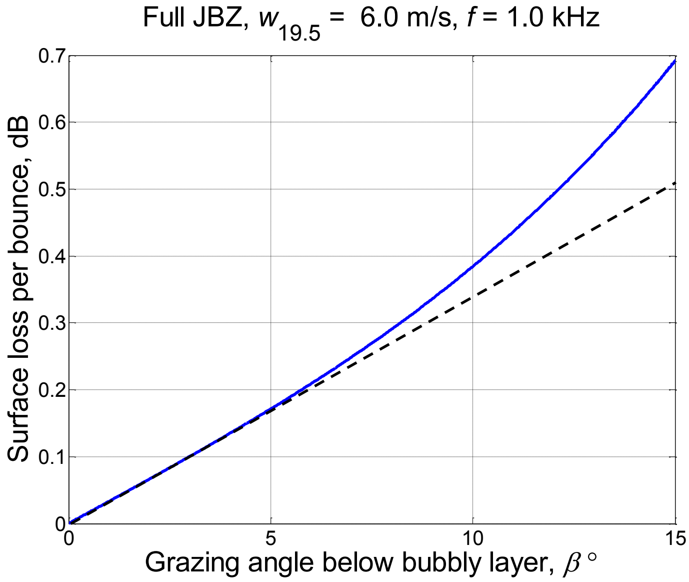

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Figure 6: Coherent RL per bounce vs incidence angle β_0, $w_{19.5}$ = 6 m/s, f = 1.0 kHz. Blue line - full JBZ, dashed line – RL $\approx 1.96\beta_0$ dB/radian from Equation (9).

Table 2: Wind speed – frequency pairs for which RL well approximated by Equation (9), for angles β_0 < 10°

Wind speed $w_{19.5}$	Frequency Values f	Frequency for β_{s_t} = 10° (Equ. (7))
5 m/s	1.5 kHz	1367 Hz
6 m/s	1.0 kHz	950 Hz
7.5 m/s	1.0 kHz	608 Hz
8 m/s	0.6 kHz	534 Hz
10 m/s	0.4 kHz, 0.3 kHz	341 Hz

Inspection of simulations for other wind speed-frequency combinations, not shown here, showed that the frequency limit indicated by Equation (7) with $\beta_{s_t} = 10°$ was conservative in setting a limit for Equation (9) to be useful in approximating the full JBZ loss function. It appears that Equation (9) may be used to approximate the loss function for angles $\beta_0 \leq 10°$ for slightly higher frequencies. An example is shown in Figure 7 for wind speed 8.0 m/s for frequency 0.8 kHz. This frequency is beyond the value 534 Hz obtained by the limit discussed above, yet the divergence of the full JBZ solution from the linear function of Equation (9) is not overly large, confirming that the use of Equation (7) to set a frequency limit is conservative.

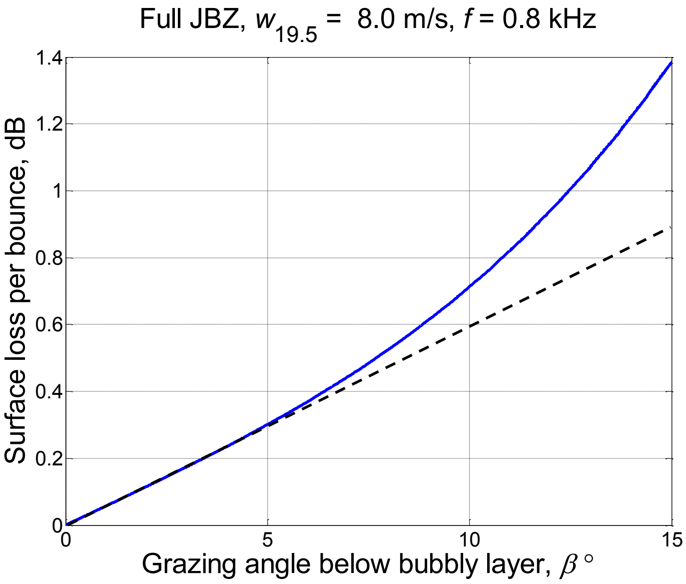

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Figure 7: Coherent RL per bounce vs incidence angle β_0, $w_{19.5}$ = 8 m/s, f = 0.8 kHz. Blue line - full JBZ, dashed line – RL $\approx 3.36\beta_0$ dB/radian from Equation (9).

3.3 Discussion

The approximation of Equation (5), which appears adequate for all wind speed – frequency combinations of practical interest, for $\beta_0 < 5°$, and $f \le 4$ kHz, is particularly relevant to transmission within a mixed layer surface duct, as such transmission is confined to incidence angles β_0 less than about 3.5° as is well-known. The authors believe that Equation (5) may then be used with confidence as a very good substitute for the full JBZ model for obtaining coherent RL values, inclusive of the refractive effects of a near-surface bubble layer, for all surface ducted situations for $f \le 4$ kHz. The JBZ model, in turn, was prepared for an ocean with a surface according to the PM surface wave frequency spectrum, and a near-surface sound speed variation which results from uniform horizontal stratification of bubbles after the model described by Ainslie (2005).

For angles of incidence β_0 as large as 10°, and $f \le 4$ kHz, an approximated model of loss, Equation (9), may be applied, to provide a model in which the coherent RL in dB has a linear variation in grazing angle, however, restrictions must be applied to the frequency values over which the approximation may be used. The approximate upper frequency limit for the use of Equation (9) is given by Equation (7), when the angle β_{s_t} is set to 10°. This approximated model then lends itself to application to transmission in shallow water, for which relevant angles of incidence exceed those of a surface duct.

4 CONCLUSIONS

An approximation has been made to the authors' previously simplified model "JBZ" of coherent acoustic reflection loss at the ocean surface. The approximated model provides values of coherent RL in dB as a linear function of the grazing angle of sound incident at the bottom of the near-surface bubbly region of an ocean with a wind-driven surface according to the Pierson-Moskowitz (PM) surface wave frequency spectrum. The approximated model is subject to limitations on this incidence angle, and on the acoustic frequency for which it may be used. For application to incidence angles less than 5°, the approximated model may be used for all wind speed-frequency combinations of interest. For application to incidence angle less than 10°, the frequency value must be less than a value determined by an algorithm. As with the full JBZ model, the model is limited in application to acoustic frequencies below 4 kHz.

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