Effective properties of metamaterials using inverse methods

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ABSTRACT

Metamaterials can be engineered for noise and vibration control due to their ability to manipulate waves in band gaps and due to local resonance effects. Metamaterials are often characterised in terms of effective parameters derived from an equivalent homogeneous medium. This work compares two inverse approaches to derive the frequency-dependent effective properties of symmetric metamaterial designs, including a multilayered medium and a periodic array of resonant inclusions in a host rubber medium. The effective properties corresponding to the effective complex wavenumber and the effective impedance obtained using the two inverse methods are identical for the 1D multilayered medium but exhibit slight variations for the 2D locally resonant medium.

1 INTRODUCTION

Acoustic metamaterials have been receiving growing interest in the last decade due to their capacity to exhibit unusual properties such as sub-wavelength band gaps, negative values and/or strong variations in the equivalent material properties. These unusual properties arise from the collective manifestation of the internal constituent units in the structure, such as resonant inclusions arranged in a host matrix (Ma and Sheng, 2016). A common metamaterial design comprises a periodic distribution of inclusions embedded in a host medium, for example, as a viscoelastic coating applied to the outer hull of a marine vessel (Sharma et al., 2019). Mechanisms governing acoustic performance are associated with local resonances of the inclusions which facilitate the conversion of longitudinal waves into shear waves, thereby increasing sound dissipation in the viscoelastic medium. A method to derive the effective wavenumber using two finite media which differ only by their length was proposed by Bianco and Parodi (1976). Wave propagation in the length difference is used to represent wave propagation in a medium of theoretically infinite length. The technique has been applied to obtain the dispersion curves of acoustic metamaterials comprising periodic structures and resonant inclusions (Meresse et al., 2015). Fokin et al. (2007) proposed an alternative inverse approach to estimate effective material properties for a locally resonant acoustic metamaterial from measured reflected and transmitted pressures of the metamaterial immersed in a fluid. This paper compares the effective properties of acoustic metamaterials for two symmetric designs corresponding to a multilayered medium and resonant inclusions in rubber. Each design is characterized by the repetition of a periodic unit cell, as respectively shown in Figure 1(a) and 1(b) for the multilayered medium and resonant inclusions in a host matrix. Each periodic unit is described in terms of a transfer matrix to relate the acoustic pressure and normal particle velocity on either side of the unit cell. The transfer matrix is obtained using a differential inverse approach corresponding to an extension of the Bianco and Parodi (1976) method, as well as by the direct inversion method by Fokin (2007).

2 MODELS

Two segmented media comprising periodic repetitions of a constituent unit cell are shown in Figure 2. The segmented media differ in length by one unit cell. The shorter length comprises n number of unit cells and hence the number of unit cells in the longer medium is $n' = n + 1$. R and T respectively correspond to the reflection and transmission coefficients calculated at the interfaces between the medium and the surrounding water. In the direct inversion method proposed by Fokin (2007), the reflection and transmission coefficients of the shorter segmented medium are used. Using a differential inverse method, corresponding to an extension of the Bianco and Parodi (1976) method and described in detail by Roux et al. (2018), the reflection and transmission coefficients of both the shorter and longer segmented media are required.

3 RESULTS

Figure 3 presents the dispersion curves kL_u and effective impedance Z associated with the two symmetric designs. The 1D multilayered medium comprises a 15mm layer of silicone, a 10mm layer of aluminium and another 15mm layer of silicone. The 2D design comprises a cylindrical steel inclusion of 4mm diameter in a square polyurethane matrix of 1cm^2. Results for the multilayered medium (green curves) obtained using the two inverse methods are identical. Stop bands occur when the real part of the effective reduced wavenumber is equal to 0 or π. The imaginary part of the effective wavenumber shows that a local maximum within these stop bands occurs, which corresponds to highest attenuation of acoustic waves. Slight variations in the results obtained between the two inverse methods for the locally resonant design (blue lines) can be observed. The peak around 14 kHz corresponds to dipole resonance of the scatterers, resulting in conversion of longitudinal to shear waves that are subsequently dissipated in the host rubber.

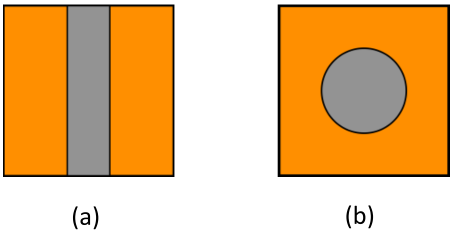

Figure 1: (a) Unit cell of a multilayered medium. (b) Unit cell of a resonant inclusion in rubber.

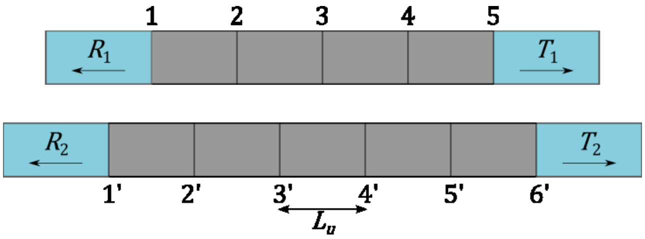

Figure 2: Schematic diagram of two segmented media of identical unit cells of length L_u which differ by a single cell.

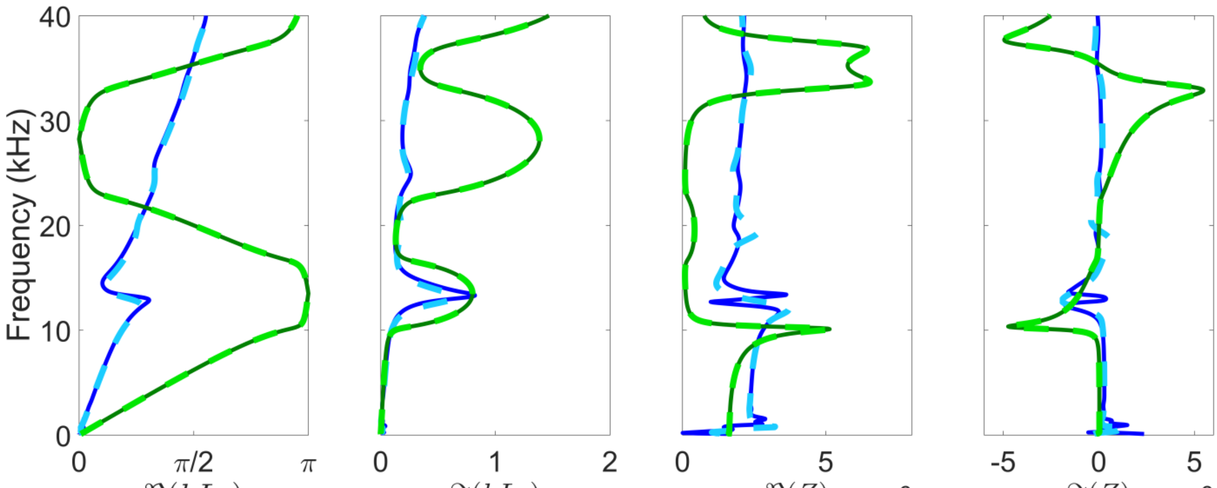

Figure 3: Real and imaginary parts of the effective reduced wavenumber (kL_u) and effective impedance (Z) for the multilayered medium (green lines) and resonant inclusion in rubber (blue lines), obtained using the differential inverse method (solid line) and the direct inverse method (dashed line).

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