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Estimates of Coherent Leakage of Sound from Ocean Surface Ducts for First and Higher Order Modes

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ABSTRACT

In earlier work, the authors established that the coherent leakage of sound from a mixed layer surface duct could be estimated for the first acoustic mode with reference to the analysis of Furry (in D. E. Kerr, ed., Propagation of Short Radio Waves, McGraw-Hill, 1951). By further consideration of Furry's analysis, is it straightforward to arrive at the conclusion that leakage rates for any mode, when expressed as a dB loss over the distance of a surface skip for a limiting ray in the duct, will be expected to be a function of two ratios: frequency as a proportion of duct trapping frequency, and the ratio of sound speed gradients in the duct and below. Based on this expectation, algorithms have been prepared to estimate the rate of leakage of the coherent signal from a surface duct, for the first mode and for higher order modes. The effectiveness of the algorithms is demonstrated by comparison of leakage estimates against values obtained by a nominally exact iterative technique.

1 INTRODUCTION

As is well known, sound travelling within a mixed layer surface duct suffers from leakage to the thermocline below the duct at a rate that is uniform with range for each acoustic mode. The leakage rate is highly dependent on frequency, in particular it is highly dependent on the ratio of the signal frequency f to the nominal trapping frequency $f_{c,m}$ of each mode m. As shown by Jones et al. (Jones et al. 2017), for example, the leak rates may be estimated using expressions within the analysis of ducted radio waves prepared by Furry (in Kerr, 1951). These were based on the assumption of "N^2-linear" sound speed profiles and the use of Airy functions, as stated by Freehafer, section 2.10 (in Kerr, 1951).

2 DUCT LEAKAGE PER SURFACE SKIP

As shown by Jones et al. for mode 1, for a surface duct with a uniform positive sound speed gradient g over a thermocline with a uniform negative gradient g_t, the leakage rate for frequencies at or above duct trapping frequency $f_{c,m}$ for mode m may be expressed in units of the dB loss over the distance of a surface skip. In new work, this leakage rate $A_m(f)$ dB/skip for each mode m may be approximated as

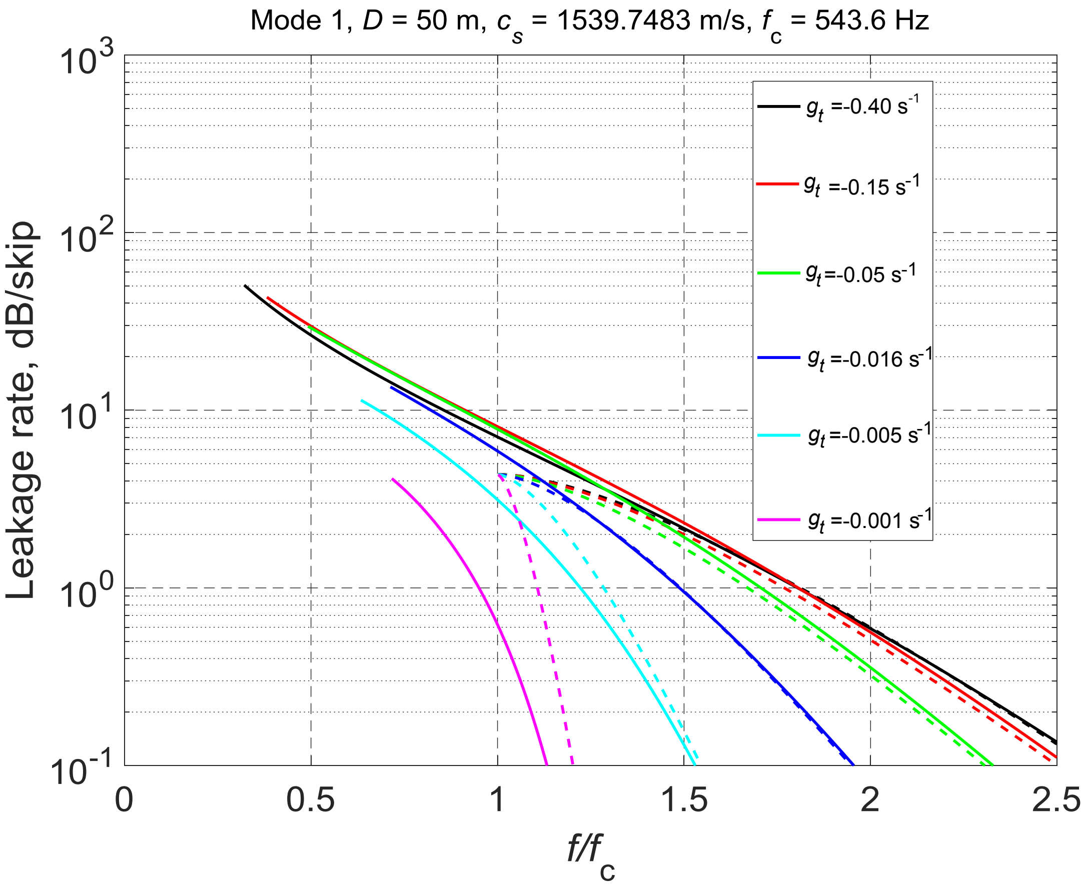
$$A_m(f) \approx 4.34 \left[\frac{f}{f_{c,m}} \right]^{1/3} \exp\left[-\tfrac{4}{3}\left(1 - g/g_t\right) |\varsigma_m|^{3/2} \left(\left(\frac{f}{f_{c,m}} \right)^{2/3} - 1 \right)^{3/2} \right] \text{ dB/skip} \tag{1}$$

where $|\varsigma_1| \approx 2.3381$, $|\varsigma_2| \approx 4.0879$, etc. (e.g. Kerr (1951) page 95) and the "skip distance" r_s m is given by the well-known ray-based expression $r_s \approx 2\sqrt{2c_w D / g}$ m, where D m is the depth of the surface duct, c_w m/s is the speed of sound at the ocean surface. The approximation provided by (1) may be shown to improve as the frequency ratio $f/f_{c,m}$ increases.

From (1) it is anticipated that leakage per skip will be a function of ratios, $f/f_{c,m}$ and g/g_t, and mode number.

2.1 Leakage for 1st mode

Figure 1 shows the leakage rates for the 1st mode, as calculated by (1) as well by a theoretical analysis that determines leakage by a much more exact, iterative, method that is beyond the scope of the present paper. For this figure, the exact results have been determined for a duct of depth 50 m and duct gradient $g = 0.016\ \mathrm{s}^{-1}$. Values of below layer gradient g_t vary from very strong ($-0.40\ \mathrm{s}^{-1}$) to very weak ($-0.001\ \mathrm{s}^{-1}$). The data clearly show the improvement in the result from (1) as frequency ratio increases, as anticipated. Although not shown here, it has been established that the exact leakage rates, on these axes, are virtually unchanged for ducts with the same gradients and, in turn, depths 20 m, 50 m and 100 m. Likewise the leakage rates, on these axes, are virtually unchanged for a 50 m duct with a duct gradient $g = 0.008\ \mathrm{s}^{-1}$, and below layer gradients g_t that maintain the same ratios as for Figure 1. This confirms that leakage is a function of the ratios $f/f_{c,1}$ and g/g_t only.



Source (Authors, 2019)

Figure 1: Theoretical leakage rates for 1st mode for 50 m surface duct of gradient $g = 0.016\ \mathrm{s}^{-1}$ for various values of below layer gradient g_t. Leakage estimates from Equation (1) shown as dashed lines.

2.2 Leakage rates for higher modes

Figure 2 shows the corresponding leakage rates for the 2nd mode. As in the case of the data in Figure 1, the dashed lines corresponding with (1) provide an improved approximation as the frequency ratio increases. Also, as in Figure (1), the leakage data at frequencies below the trapping frequency trend to approximate straight lines on these axes, that is the data trend to the form $A_m(f) \approx X \exp\left(-Y f/f_{c,1}\right)$ dB/skip where X and Y are particular constants. Although not shown here, the corresponding plots for higher modes $m = 3,4,5$ trend to similar leakage rates at frequencies below $f_{c,m}$. However, for frequencies above $f_{c,m}$, for progressively higher modes, the leakage rates trend to a greater drop-off as frequency rises. This is in accord with expectations from (1), as the values $|\varsigma_m|$ increase with mode number, thus reducing the value of the exponential term.

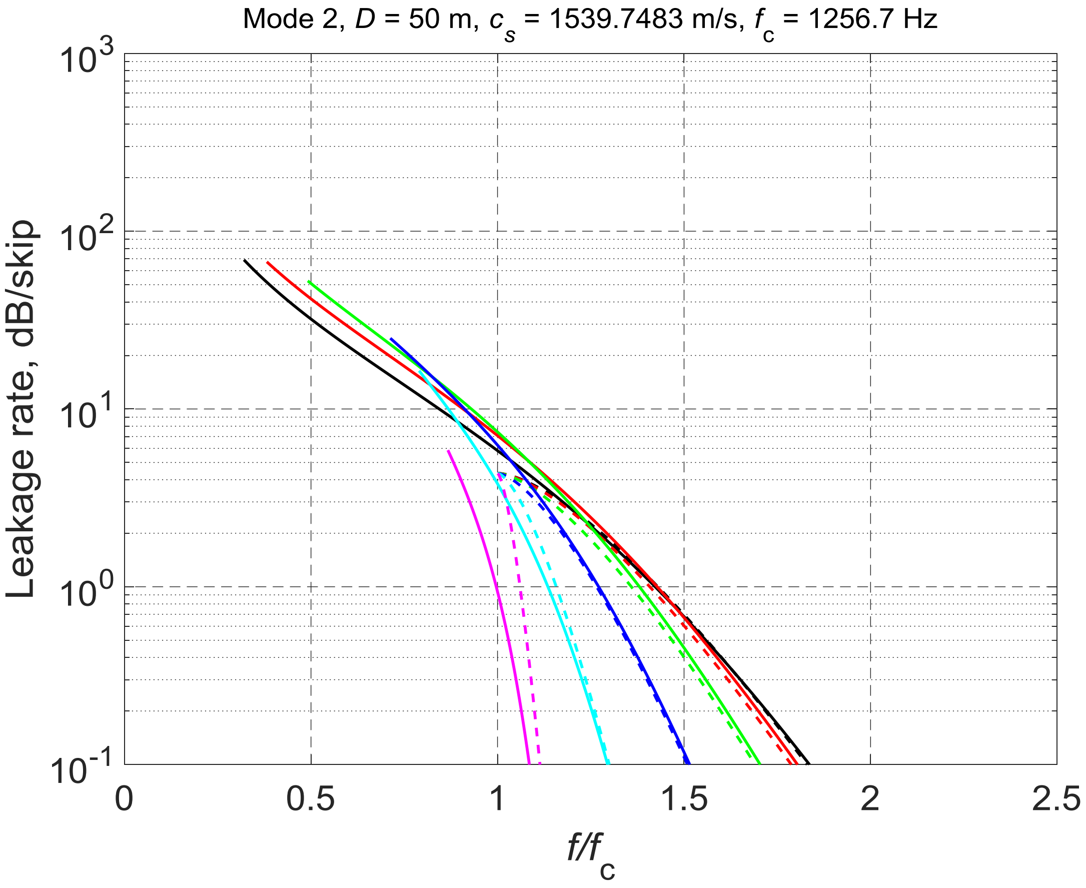


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Figure 2: Theoretical leakage rates for 2nd mode for 50 m surface duct of gradient $g = 0.016\ \mathrm{s}^{-1}$ for various values of below layer gradient g_t. Leakage estimates from Equation (1) shown as dashed lines.

3 ALGORITHMS FOR DUCT LEAKAGE

It is not feasible to obtain an exact closed form expression for leakage loss rate, so a pragmatic approach must be taken to obtain algorithms for practical use. Firstly, the following has been derived, being based on adjustments to (1) for frequencies close to the duct trapping frequency:

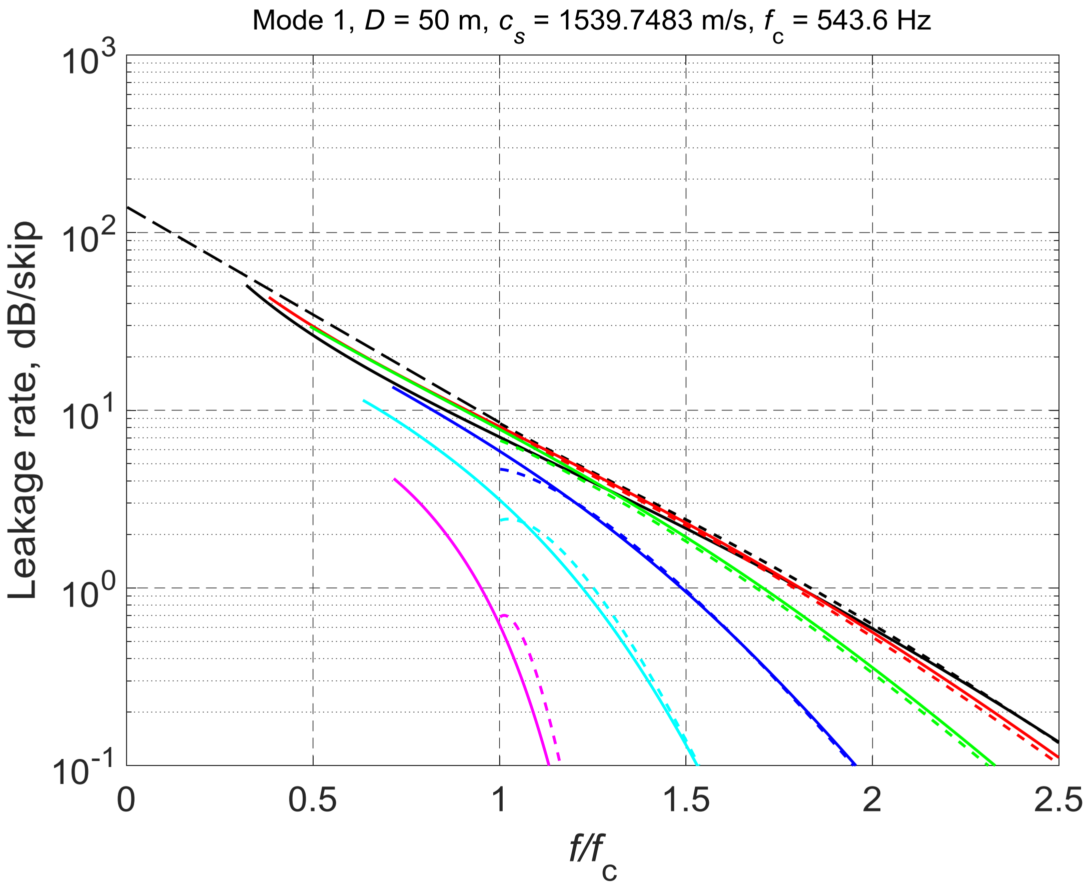
$$A_1(f) \approx 4.34 \left[\frac{f}{f_{c,m}}\right]^{1/3} \exp\left[y\left(\frac{f_{c,m}}{f}\right)^4 - \tfrac{4}{3}\left(1 - g/g_t\right)\left|\varsigma_m\right|^{3/2} \left(\left(\frac{f}{f_{c,m}}\right)^{2/3} - 1\right)^{3/2} \right] \ \mathrm{dB/skip} \qquad (2)$$

where $y = \ln\left(2\left[1 - g/g_t\right]^{-0.9}\right)$ for mode 1, so that $A_m(f)\big|_{f \to f_{c,m}+} \approx 8.69\left(1 - g/g_t\right)^{-0.9}$ dB/skip. This provides an improved approximation for mode 1 values, as shown in Figure (3). Although not shown here, the values of the constants that determine the form of y may be adjusted to provide a better fit in the case of higher modes.

For frequencies below duct trapping, an algorithm of the form $A_m(f) \approx X \exp\left(-Y f / f_{c,1}\right)$ $\mathrm{dB/skip}$ is taken from earlier work by the authors (Jones et al. 2017), this is

$$A_1(f) \approx 136.0\exp\left(-2.79\, f/f_{c,1}\right) \text{ dB/skip}. \quad (3)$$

A line of this form, shown in Figure (3) by the long-dashed black line, is a reasonable fit to data for stronger gradients. Although not shown here, it is possible to make further adjustments to (3) to more closely fit the exact curves for the various below layer gradients, and to make adjustments to (3) to better fit data for higher modes.



Source (Authors, 2019)

Figure 3: Theoretical leakage rates for 1st mode for 50 m surface duct of gradient $g = 0.016\ \text{s}^{-1}$ for various values of below layer gradient g_t. Leakage estimates from Equations (2), (3) shown as dashed lines.

4 CONCLUSIONS

Algorithms have been derived by which the coherent leakage of sound from a mixed layer ocean surface duct may be reliably estimated as a function of frequency, and of the strength of the below-layer sound speed gradient, for the first as well as higher modes. The algorithms are approximate, being based partly on existing theory of ducted radio wave transmission, and partly on a pragmatic but reasonable fit to more exact data.

REFERENCES

Jones, A.D.; Duncan, A.J. and Zhang, Z.Y. 2017, *Coherent Leakage of Sound from Ocean Surface Ducts of Non-linear Sound Speed Profile*, Proceedings of Acoustics 2017, 19-22 November, Perth, Australia.

Kerr, D. E., ed. 1951, *Propagation of Short Radio Waves*, McGraw-Hill.