Temperature Inversion Frequency from Sigma-Theta

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The NSW Environment Protection Authority in their Noise Policy for Industry (NPfI) requires the consideration of certain meteorological conditions that may increase noise levels by focusing sound wave propagation paths at a single point. Such refraction of sound waves will occur during temperature inversion conditions.

One of the principal methods recommended in the NPfI to identify the strength of a temperature gradient is to use a relationship developed by the US Atomic Energy Commission between temperature gradient and atmospheric stability categories. To determine atmospheric stability categories typically requires the measurement of sigma-theta (the standard deviation of wind direction), wind speed and time of day.

This paper reviews how sigma-theta is often analysed to estimate atmospheric stability category for acoustic purposes.

1 INTRODUCTION

Noise-enhancing weather conditions alter the way sound propagates, causing noise to travel further or be perceived more intensely at distant receivers. These conditions are primarily caused by changing temperature and wind with elevation above the ground. As temperatures increase above ground level (AGL) (i.e. referred to as a temperature inversion, which is the opposite of normal daytime adiabatic conditions), or as wind speeds increase above the ground, the top of propagating sound waves travel faster than the bottom, causing the wave to refract back towards the ground (or away from the ground in upwind locations). This refraction or channelling of sound can lead to increased noise levels at sensitive receptors compared to what would be expected under normal (i.e. lapse) conditions.

The combination of these factors can significantly impact the noise environment and as a result plays a crucial role in noise assessments. As temperature inversions can be a significant characteristic of certain locations, these often become a focus for considering noise-enhancing conditions in assessments.

The New South Wales (NSW) Noise Policy for Industry (NPfI) (NSW EPA, 2017) provides methodologies to develop noise criteria, calculate industrial noise and for considering feasible and reasonable noise mitigation. The policy outlines that noise level limits apply under either standard or noise-enhancing conditions, depending on the significance of occurrence of these noise-enhancing conditions.

Two options are available in the NPfI for practitioners to consider meteorological effects when noise modelling is conducted for an assessment; either assume noise-enhancing meteorological conditions occur for all assessment periods, or determine the significance of noise-enhancing conditions. If the latter is selected, where noise-enhancing conditions occur for less than 30% of the time, standard meteorological conditions may be adopted for the assessment.

Challenges may arise when determining the frequency of temperature inversions and hence noise-enhancing conditions. Temperature inversions, which have the potential to effect ground noise sources, are very difficult to measure as they occur within the planetary boundary layer (PBL) which can range up to 1 kilometre (km) AGL, however the most significant conditions for noise enhancement occurs within the first few hundred metres (m) AGL. They can vary significantly in strength with height AGL, but also horizontally in many locations due to terrain effects. Inversions can also form and decay quite rapidly as diurnal heating and cooling occur with day and night transitions and due to cloud cover.

For the purpose of evaluating inversions for noise assessments, they are simplified down to a number representing their strength over 100m (i.e. the change in temperature over the first 100m AGL). This can be directly measured using temperature probes, at ground level (2m or 10m AGL) and at 100m, however this requires a 100m tall structure (i.e. an inversion tower) which is cost prohibitive. Alternate methods exist for directly measuring inversions, for example with a smaller tower and by reducing the height of the measurements by half (i.e. approx. 50m AGL) and assuming the strength of the inversion in the bottom 50m is the same as that between 50 and 100 m AGL. Alternatively, weather stations on natural terrain features can be used in some cases. For example, two weather stations can be constructed on the top and bottom of a hill where appropriate site circumstances occur (NSW EPA, 2014). Even where such towers are present, inversions above the height of the tower, (which can be very significant for receptors several kilometres away) cannot be measured.

Other methods for measuring inversions and wind gradients include balloon flights (radiosondes and tether sondes) and remote sensing such as a Radio Acoustic Sounding System (RASS), Microwave Temperature Profilers (MTP) or Light Detection and Ranging (LIDAR) equipment. Such equipment can cost up to $1M and requires specialised technicians to operative and maintain it. These are all highly specialised or specific options which are generally cost or operationally prohibitive for all but the largest mining or ore processing operations.

Due to the challenges in directly measuring temperature inversions and wind gradients, they are almost always inferred by analysing data which are readily available. The NPfI outlines that inversion conditions can be identified using Pasquill–Gifford (PG) stability categories, as opposed to stipulating temperature lapse rates in degrees per 100 metres. The relationship between stability categories and temperature gradients is shown in Table 1. A positive temperature gradient signifies a temperature inversion; hence inversions occur during E, F and G stability categories. These three categories are considered to represent weak, moderate and strong inversions respectively. For noise assessment purposes, only moderate and strong inversions are considered significant enough to require assessment (NSW EPA, 2017).

Table 1 – Stability categories based on DT/DZ

Stability category	Range of vertical temperature gradient – DT/DZ (degrees Celsius/100m)
A	DT/DZ < –1.9
B	–1.9 DT/DZ < –1.7
C	–1.7 DT/DZ < –1.5
D	–1.5 DT/DZ < –0.5
E	–0.5 DT/DZ < 1.5
F	1.5 DT/DZ < 4
G	4 DT/DZ

Whilst the above temperatures are typical average values, measurement data indicates that in any short (1 to 10 minute period) period temperature gradients may be several times greater.

2 STABILITY CLASS

The NPfI provides three options for determining stability classes:

- direct measurement of temperature lapse rate over a height interval as described above;
- cloud cover wind speed and solar elevation (PG stability classification scheme and Turner scheme); and
- measurements of sigma-theta (the standard deviation of wind direction), wind speed and time of day.

The cloud cover methods (either PG or Turner schemes) require cloud cover observations to be measured within the vicinity of a site. Cloud cover generally does not vary as much spatially compared to other variables such as wind speed, however it can be significantly affected by elevation or distance to the coast. Thus to use the PG or Turner schemes, measurements of cloud cover using a ceilometer need to be collected within the generally vicinity of a site (i.e. within 30km).

Currently in NSW/ACT, only the Bureau of Meteorology (BoM) routinely measures cloud cover, however less than half of its weather monitoring sites include ceilometer measurements (49 of 117 stations). The NSW Department of Climate Change, Energy, Environment and Water (DCCEEW) measures cloud cover at two of its 55 stations. Private weather stations run by industry are unlikely to include ceilometers due to the high cost. However, sigma-theta data are available at essentially every weather station, including all BoM and DCCEEW stations. Consequently, noise practitioners are more likely use the sigma-theta method over the cloud cover method.

2.1 Sigma-theta method

The sigma-theta method is a turbulence-based method which uses the standard deviation of the horizontal wind direction (sigma-theta) in combination with the concurrent (scalar) mean of the wind speed and the time of day to derive a stability class for the period measured.

Sigma-theta quantifies how much the horizontal wind direction fluctuates or oscillates around the mean direction for a set period of time. This oscillation, when measured over a suitably short time period, is due to atmospheric turbulence, and hence from this value an estimation of the atmospheric stability can be calculated. The Meteorological monitoring guidance for regulatory modelling applications (US EPA, 2000) outlines a two-step method for calculating stability class from sigma-theta, as summarised below in Table 2 and Table 3.

An initial estimate of stability class is determined using sigma-theta data per Table 2. This initial estimate is then refined in a second step using Table 3, depending on scalar wind speed and whether it is day or night to determine the stability class.

Table 2 – Lateral turbulence criteria for initial estimate of PG stability category

Initial estimate of PT Stability category	Standard deviation of wind azimuth angle (σ_A)
A	$22.5 \leq \sigma_A$
B	$17.5 \leq \sigma_A < 22.5$
C	$12.5 \leq \sigma_A$ 17.5
D	$7.5 \leq \sigma_A < 12.5$
E	$3.8 \leq \sigma_A < 7.5$
F	$2.1 \leq \sigma_A < 3.8$
G*	$\sigma_A < 2.1$

*G class as adopted by NPfI

US EPA's methodology for sigma-theta PG stability class calculations was developed for the purpose of air dispersion modelling.

The NPfI adopts the US EPA's methodology with two notable changes; the adoption of a G class for sigma-theta values $\sigma_A < 2.1$ to represent stronger inversion conditions, and the lack of any day-time second pass adjustment. It is assumed that no day-time second pass adjustment is made because inversion conditions (stability classes E, F and additionally G) cannot occur in the day per the sigma-theta method; as such, these adjustments do not need to be made in order to calculate the frequency of temperature inversions and hence noise-enhancing conditions.

In reality temperature inversions do occur in daytime hours, generally more often in the cooler seasons. This is one of the limitations arising when using the sigma-theta method.

Table 3 – Wind speed adjustments for determining final estimate of PG stability category

Initial estimate of PT stability category		Category 10-metre wind speed (m/s)	Final estimate of PG stability category
Daytime	A	$u < 3$	A
	A	$3 \le u < 4$	B
	A	$4 \le u < 6$	C
	A	$6 \le u$	D
	B	$u < 4$	B
	B	$4 \le u < 6$	C
	B	$6 \le u$	D
	C	$u < 6$	C
	C	$6 \le u$	D
	D, E, or F	ANY	D
Night	A	$u < 2.9$	F
	A	$2.9 \le u < 3.6$	E
	A	$3.6 \le u$	D
	B	$u < 2.4$	F
	B	$2.4 \le u < 3.0$	E
	B	$3.0 \le u$	D
	C	$u < 2.4$	E
	C	$2.4 \le u$	D
	D	ANY	D
	E	$u < 5$	E
	E	$5 \le u$	D
	F	$u < 3$	F
	F	$3 \le u < 5$	E
	F	$5 \le u$	D
	G*	$u < 2$	G
	G*	$2 \le u < 3$	F
	G*	$3 \le u < 5$	E
	G*	$5 \le u$	D

*G class as adopted by NPfI

3 SIGMA-THETA CALCULATIONS

There are several methods for determining sigma-theta, however the Yamartino method (Yamartino, 1984) is the most widely adopted.

$$\sigma_\theta = arcsin(\epsilon)[1 + 0.1547\epsilon^3] \tag{1}$$

$$\epsilon = \left[1 - \left(\overline{sin\theta_i}^2 + \overline{cos\theta_i}^2\right)\right] \tag{2}$$

Sigma-theta is a measure of oscillations in wind direction in the horizontal plane over a period of time. The longer the period of time, the more likely the standard deviation measurement is to include longer term changes in wind direction (wind meander) rather than turbulence-based oscillations. Hence the longer the period being analysed per the Yamartino method, the more inflated the calculated sigma-theta value is likely to become.

To minimise the artificial inflation of sigma-theta values due to longer measurement periods, it is recommended that a separate averaging calculation is used to create longer period sigma-theta values (US EPA, 2000). For determining say hourly sigma-theta using 15-minute data the following root mean square equation (Equation 3) would apply:

$$\sigma_\theta(1 - hr) = \left(\frac{(\sigma_{\theta1})^2 + (\sigma_{\theta2})^2 + (\sigma_{\theta3})^2 + (\sigma_{\theta4})^2}{4}\right)^{\frac{1}{2}} \tag{3}$$

The US EPA guidance is based on calculating hourly average values using the above equation. Hourly average values have historically been the standard time interval used for regulatory air dispersion modelling, hence the sigma-theta method is concerned with created such hourly sigma-theta values for use in air dispersion modelling.

The National Environment Protection Council (NEPC) has incorporated this averaging recommendation into the Meteorological Measurements guidance (NEPC, 2001), noting that wind direction meander should not be included in 1-hour sigma theta. If 1-hour time period sigma theta is needed, it should be calculated from the square root of the average of 15-minute variances.

It is noted that NEPC has a requirement of at least 360 samples for calculating sigma-theta. This means for a typical sampling rate of 1-second, the requirement of 360 samples for single pass sigma-theta is easily met by 1-second sampling over 10-minutes (NEPC, 2001). This is consistent with the relevant Australian standard (AS/NZS 3580.14) which requires >360 samples for sigma-theta, and the original US EPA guidance which outlines that substantial evidence and experience suggest that 360 data values evenly spaced during the sampling interval will provide estimates of the standard deviation to within 5 or 10% (US EPA, 2000).

It is not clear if the 15-minute sampling period outlined in the US EPA guidance was specifically determined as the most optimal for the Yamartino method or whether shorter periods with sufficient sampling rate (e.g. 10-minutes) would be equally appropriate, as 10 and 15-minute data frequencies are both common in meteorological recording. It could be inferred that if 10-minute sigma-theta values contain an appropriate frequency of values collected (at least 360 samples) it would be more accurate than a 15-minute sample as it would contain less potential for wind meander. It is however clear from these requirements that the adoption of Yamartino sigma-theta calculations to hourly variance data would artificially inflate the sigma-theta.

For noise assessments conducted per the NPfI, noise practitioners would typically obtain hourly sigma-theta data from the BoM or DCCEEW. It is unlikely that the practitioner would know the instrumentation and/or data processing involved in producing this data. For example, the practitioner is unlikely to know the anemometer type (mechanical v ultrasonic) and performance characteristics, or the sampling rate and any averaging algorithms applied to the data.

4 APPLICATION OF SIGMA-THETA PROCEDURE TO A REAL DATASET

Figure 1 presents 1-minute, 5-minute, 15-minute and hourly sigma-theta data for a single dataset at a BoM weather station. It is understood that BoM standard deviation data are based on 1-second sampling rates, meaning that 1-minute and 5-minute data would not contain enough samples to calculate sigma-theta per the NEPC and US EPA guidance. 15-minute and hourly data would contain enough samples, however as can be seen in the figure, as the time period increases, so too does the sigma-theta value. The figure indicates that hourly data can be significantly inflated whereas shorter time periods would provide much smaller sigma-theta values.

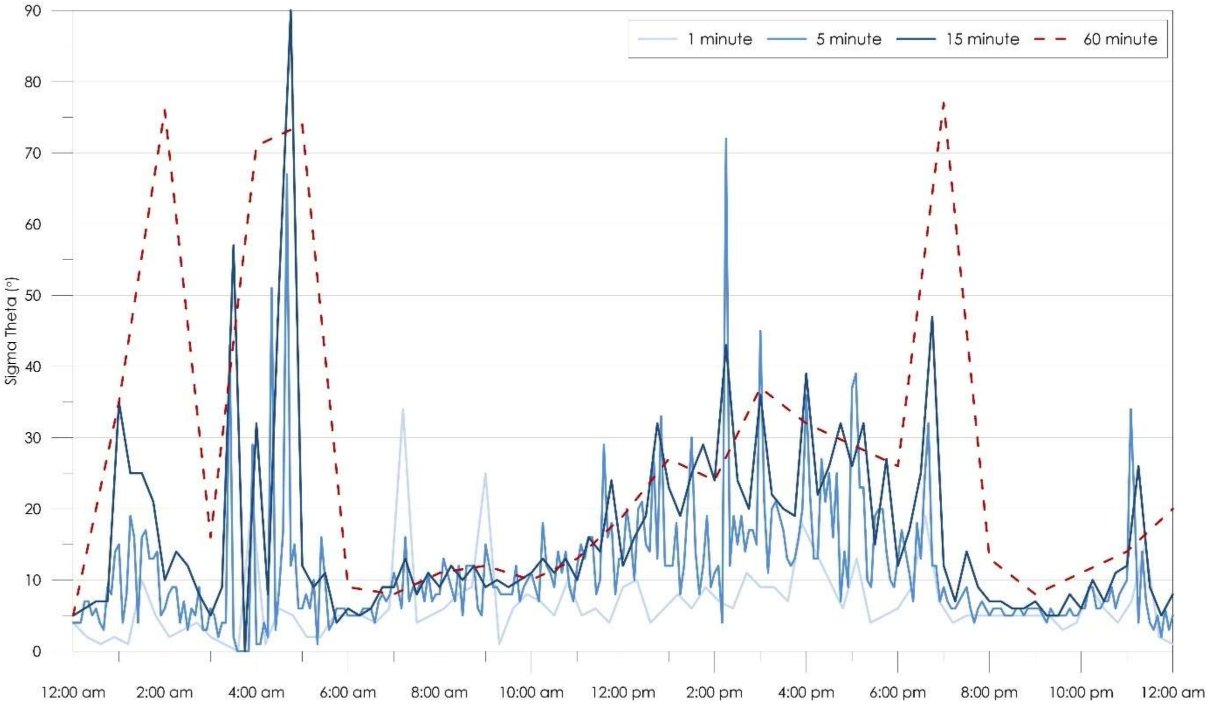

Figure 1 – Timeseries of sigma-theta monitoring for 1-minute, 5-minute, 15-minute and hourly data

Table 3 shows that strong inversion conditions (F and G classes) can only occur during the night. Final estimates of PG stability equal to F and G classes at night can only come about as a result of initial PG stability estimates of A, B, F or G. These initial estimates correspond to sigma-theta values greater that 17.5 degrees (A and B) or less than 3.8 degrees (F and G).

Therefore, depending on the distribution of sigma-theta, using inflated sigma-theta values (for example by using 1-hour BoM data, which is inflated by using the Yamartino method on a longer timescale than the method recommends) may affect the frequency of moderate and strong inversion classes (classes F and G).

Figure 2 compares the sigma-theta distributions when the Yamartino method is applied over a 15-minute period (red) and a one-hour period (blue) for one year's worth of data from a private weather station. Figure 3 presents this comparison for the winter night-time sigma-theta distribution, which is relevant to determining significant noise enhancing conditions as per the NPfI. The red lines show the cut-offs for sigma-theta to contribute to moderate and strong inversions (only sigma theta values outside of the 2 red lines can contribute to F and G inversion classes).

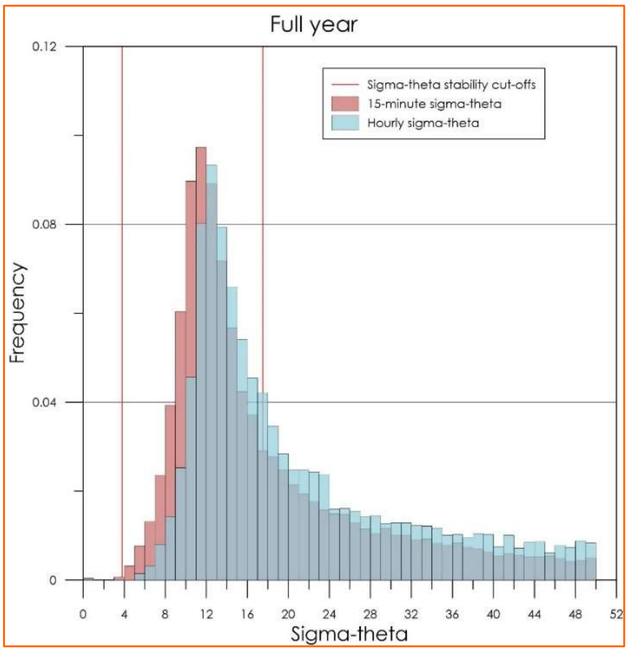

Figure 2 – Distribution of 15-minute (red) and hourly (blue) sigma-theta for full year of data (overlapping data is grey)

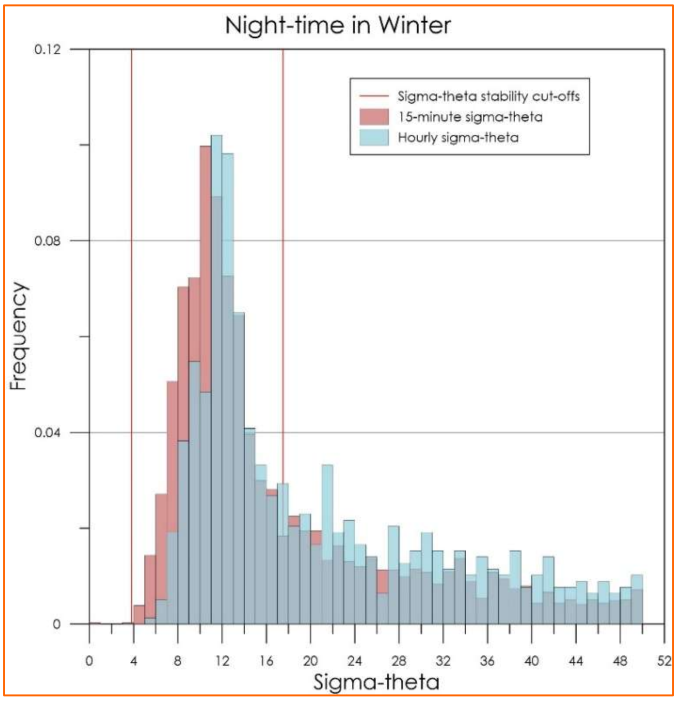

Figure 3 – Distribution of 15-minute (red) and hourly (blue) sigma-theta for winter nights (overlapping data is grey)

Figure 2 and Figure 3 show that, as expected, applying the Yamartino method over a longer period results in inflated sigma-theta values. The sigma-theta values that are outside of the cut-offs (red lines) can contribute to moderate and strong classes, and amount to a higher proportion of the hourly data when compared to the 15-minute data (58% vs 42%).

Table 4 compares the frequencies of moderate and strong inversion classes for 15-minute and hourly data calculated across a full year and during the winter night-time periods with the dataset from Figure 2 and Figure 3. The table indicates that inflated sigma-theta (due to hourly averaging) can significantly increase the frequency of strong inversion classes, particularly during winter nights.

Table 4 – Stability class category comparison – 15-minute and hourly values

Stability class	Full year		Winter nights	
	15-minute data	Hourly data	15-minute data	Hourly data
A	16%	20%	0%	0%
B	6%	6%	0%	0%
C	14%	14%	0%	0%
D	39%	29%	41%	30%
E	8%	6%	19%	11%
F	16%	25%	40%	60%
G	0%	0%	0%	0%
Moderate and strong inversions per NPfI (F+G)	**16%**	**25%**	**40%**	**60%**

5 WHAT DOES THIS MEAN FOR NOISE PRACTITIONERS?

Inflated sigma-theta values, when applied to the sigma-theta method as per the NPfI, can alter the frequency of stability classes. As the method is a two-pass calculation, the final stability class is also dependent on the wind speed at the time of the initial sigma-theta reading, which can lead to unintuitive or unexpected results when the inversion conditions are summed together.

In most situations, inflated sigma-theta values will increase strong inversion stability categories, however in other situations they may decrease, depending on the distribution of the sigma-theta data. In either situation, inflated sigma-theta values will lead to a decrease in the prevalence of G classes and alter the total distribution of stability classes in a manner that is not consistent with original method.

Care should thus be taken where applying the sigma-theta methodology for the purpose of calculating inversion frequency. Where possible:

- Use 15-minute or 10-minute sigma-theta data as these are most likely to contain enough samples whilst minimising the prevalence of sigma-theta inflating data which arises due to wind meander. It is also noted that noise assessments and measurements conducted per the NPfI typically adopt a 15-minute assessment period and it would be logical to calculate sigma-theta over a consistent timescale.
- Calculate stability class based on these 15-minute or 10-minute values. If 1-hour values are needed, calculate the average sigma-theta based on Equation 3.
- When in doubt, assume noise-enhancing meteorological occur for all assessment periods.

6 FURTHER RESEARCH

Suitable data has been collected which could be used to validate the inversion strength classes inferred from stability classes calculated via the sigma-theta method. Over a decade of temperature profile data have been collected from a RASS instrument and an inversion tower in the Hunter Valley, and could be compared with adjacent weather station monitoring data. Tethersonde and radiosonde data have also been collected in this region, which would combine to form a valuable database for a future analysis on this topic.

Traditional mechanical anemometers are starting to be replaced by ultrasonic anemometers. Ultrasonic anemometers do not have any damping effects as they have no moving parts. In theory this should lead to more accurate measurement of sigma-theta. However, it needs to be considered that the calculation method for stability class using sigma-theta data was developed based on data from mechanical instruments with inherent mechanical limitations (overshoot and damping). Figure 4 presents relative frequency plots (density plots) of sigma-theta and wind direction for 16 datasets from weather stations around NSW. The data are collected for various time intervals (1 minute to 1 hour) and collected from stations in different locations with different types of instruments (mechanical and ultrasonic). Of note are some wind direction bands where no directions are recorded (top right gap at 180˚), and where sigma-theta values appear to fall within certain bands for a given direction (top left three fingered shape). However, no clear differences between sigma-theta values measured by mechanical or ultrasonic anemometers were identified in this limited sample. This is a topic for future research.

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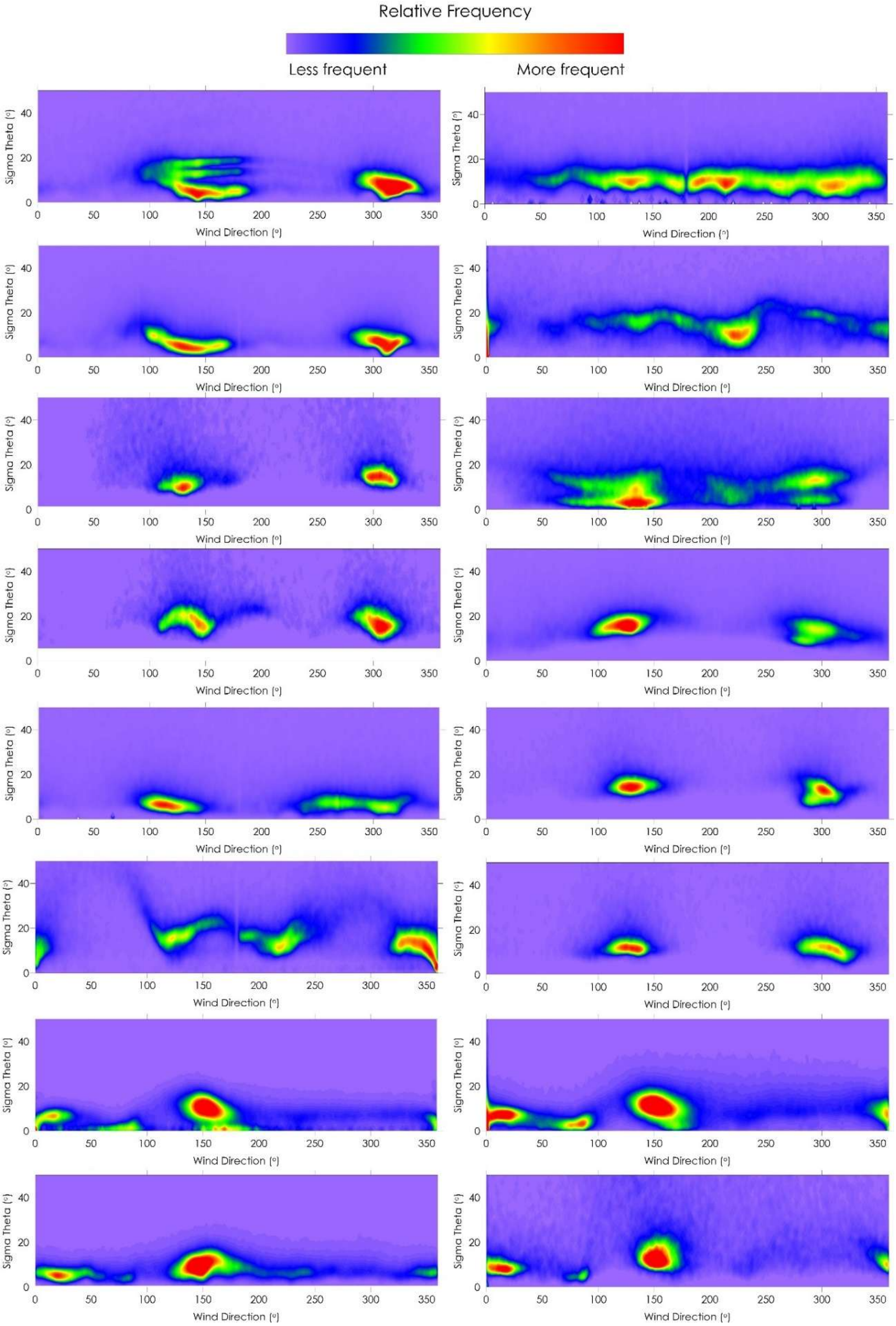

Figure 4 – Wind direction and sigma-theta – relative frequency plots