Modelling Low-Frequency Sound Propagation in a Ducted Atmosphere Using a Semi-Analytic Finite Element Method

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Abstract - Winds and temperature inversions can cause acoustic refraction, enabling the efficient propagation of low-frequency sound over long distances within acoustic waveguides. In this study, the semi-analytic finite element (SAFE) method is employed to predict this effect in stratified, inhomogeneous, moving air for range-independent noise propagation over reflective half-planes. We present a solution for a point source radiating in an atmosphere with a power-law wind profile. The results are compared to direct numerical solutions of the two-dimensional linearized Euler equations. Additionally, we examine the numerical convergence of the computed acoustic modes and demonstrate how to select modes associated with acoustic ducting near the ground. For large, range-independent domains, the proposed procedure proves to be much more efficient than the tested direct numerical computations. Thus, the method offers a viable approach for benchmarking other pressure field prediction techniques.

1 INTRODUCTION

Low-frequency sound is inherently subject to low atmospheric dampening and can propagate over great lengths. Temperature inversions and winds can refract sound in the atmosphere and channel it inside atmospheric wave guides (‘atmospheric ducts’).Noise from Low-frequency sources, such as wind turbines, biogas power plants, and heat pumps can, therefore, propagate over large distances towards dwellings, where it can cause complaints (Burke et al., 2020; Crichton & Petrie, 2015).

To model downward refracting atmospheres, temperature and wind profiles need to be included into the governing equations, and analytic solutions exist only under further assumptions (Waxler, 2002; Waxler et al., 2017) . In contrast, direct numerical computations, e.g. the linearized Euler Equations (LEE), can solve cases with atmospheric ducting (Colas et al., 2023). However, they are computationally expensive because of the large domains involved. Instead, alternatives that solve simplified equations are frequently used for modelling sound propagation in large domains. For example, ray tracing(Shang et al., 2019), the parabolic equations (Dallois et al., 2001), or the fast field program(Raspet et al., 1985). An alternative for range-independent domains is the semi-analytic finite-element (SAFE) method (Kirby, 2020). The SAFE method expands the acoustic field into vertical atmospheric acoustic modes propagating in the horizontal direction. This can be implemented in an efficient scheme. In addition, the modes can offer physical insight. Waxler et al. first suggested using normal modes for infrasound propagation (Waxler, 2002) by numerically solving for modes governed by an approximate wave equation. On contrast, the SAFE method is based a more complete governing equations with arbitrary temperature and wind profiles that only dismiss gravity waves below the Brunt–Väisälä frequency.

So far, the SAFE method has been tested against analytic solutions for the no-flow and uniform flow cases and a numerical solution of a linearly increasing wind profile (Kirby, 2020). Recently, we have also presented validation data comparing it to solutions of the LEE (Castro Mota et al.). In this paper, we present a low-frequency propagation through a power law wind profile, which is a relevant condition for wind-turbine noise. It is shown that the log-profile creates efficient ducts with a height of only a few hundred meters. Those ducts can be described sufficiently with less than 10 modes, making far-field approximations very efficient.

2 METHODS

2.1 SAFE Method

The SAFE method is derived from the exact fluid dynamic equations for range-independent atmospheres (Ostashev & Wilson, 2016)

$$\begin{bmatrix} \frac{1}{c^2} D_t^3 - D_t\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2} + 2\tilde{g}\ \frac{\partial}{\partial z} + \tilde{g}^2 - \frac{2\tilde{g}}{c}\frac{\partial c}{\partial z}\right) + 2\frac{\partial u_0}{\partial z}\frac{\partial}{\partial x}\left(\tilde{g} + \frac{\partial}{\partial z}\right) + \\ \left(2\tilde{g} + \frac{1}{\rho_0}\frac{\partial \rho_0}{\partial z}\right) D_t\left(\tilde{g} + \frac{\partial}{\partial z}\right) \end{bmatrix} p^{\prime} = \rho_0 D_t^2 Q, \qquad (1)$$

where $p^{\prime}$ is the acoustic pressure, $\rho^{\prime}$ is the acoustic density, ρ_0 is background air density, u_0 is the background velocity in the range direction (x), $\tilde{g}$ is the acceleration of gravity divided by the squared speed of sound ($\tilde{g} = \frac{g}{c^2}$), Q is a mass source, $D_t = (\frac{\partial}{\partial t} + u_0\frac{\partial}{\partial t})$, x denotes the range coordinate, z the height coordinate and t denotes time. This two-dimensional equation results from assuming a stratified atmosphere in a linearized two-dimensional Euler equation and by assuming the Brunt–Väisälä frequency is much lower than the excitation frequency, i.e. neglecting gravity waves

$$\begin{cases} \frac{d\rho'}{dt} + (V' \cdot \nabla)\rho_0 + \rho_0 \nabla \cdot V' + \rho' \nabla \cdot V_0 = \rho_0 Q \\ \frac{dV'}{dt} + (V' \cdot \nabla)V_0 + \frac{1}{\rho_0}\nabla p' - \frac{\rho' \nabla p_0}{\rho} = F \\ \frac{ds'}{dt} + (V' \cdot \nabla)s_0 = 0 \end{cases}, \qquad (2)$$

with F being a force source, p_o being background air pressure, V_0 and V' indicate the background field and the acoustic field, respectively, s' is acoustic entropy and s_0 is the background entropy.

The pressure field may be expressed as a sum of range propagating vertical modes, following the Ansatz

$$p^{\prime}(x,z,t) = \sum_{n=1}^{\infty} A_n p_n(z) e^{iwt - ik_0\gamma_n x}, \quad (3)$$

where A_n is the complex-valued mode amplitude of the n-th mode, p_n are the mode shapes, k_0 is the reference wave number and γ_n are the eigenvalues associated with the mode shapes. Equation 1 then turns into an eigenvalue problem in the vertical coordinate

$$\begin{bmatrix} k^2[1-\gamma_n M]^3 + [1+\gamma_n M]\left\{\left(\tilde{g}^2 - \frac{2\tilde{g}}{c}\frac{\partial c}{\partial z}\right) + 2\tilde{g}\frac{\partial}{\partial z} + \frac{\partial^2}{\partial z^2} - k_0^2\gamma_n^2\right\} + \\ +2\gamma_n\frac{\partial M}{\partial z}\left(\tilde{g} + \frac{\partial}{\partial z}\right) - \left[2\tilde{g} + \frac{1}{\rho_0}\frac{\rho_0}{\partial z}\right][1-\gamma_n M]\left(\tilde{g} + \frac{\partial}{\partial z}\right) \end{bmatrix} p_n = 0 \qquad (4)$$

with M being the Mach number and k the local wave number (Kirby, 2020).

Equation 4 can be solved using the finite-element method and an eigenproblem solver. For the test case shown here, we use an in-house FE code, based on linear hat elements and the *eig* function from the scientific Python package *numpy*. In a last step, the modes' amplitudes are computed by employing a vertical mode matching scheme that was derived to mimic a harmonic point source (Kirby, 2020).

2.2 Linearized Euler Equations in COMSOL Multiphysics

Time harmonic, two-dimensional versions of the linear Euler Equations presented in Eq. 2 are implemented in the acoustic module of the commercial FE solver COMSOL Multiphysics (‘Linear Euler, Frequency Domain interface’). This predefined interface was used throughout the paper. The geometry was a rectangular atmosphere with a size of 900m x 25.000m containing a small circular surface mass source with a frequency of 15Hz placed at a height of 100m (x=0m, z=100m). The background wind profile was given by a power law function

$$u_0(z) = 5 + 5 \cdot z^{\frac{1}{7}}. \qquad (5)$$

The rest of the background field was set as $v_0 = 0$, $t_0 = 297.15\ K$, $\rho_0 = 1.23\ Kg/m^3$, and $p_0 = 1\ atm$. Cylindrical spreading was assumed. We discretised the domain with an unstructured mesh consisting of quadrilateral and triangular elements, progressively resolving the acoustic field with finer grids until convergence in the far field was reached (between 18 to 20 elements per the wavelength). The ground was modelled with a hard-wall boundary condition, whereas the other sides of the geometry were terminated with perfectly matched layers with dimensions of at least two wavelengths.

3 RESULTS

The case computed is described in section 2.2 of this article and was computed both with the SAFE and LEE methods.

For the results from the SAFE method, convergence of the modes and their respective eigenvalues was checked by plotting the transmission loss obtained at a height of 1.5m for an increasing number of elements (N) used in the FEM. All available acoustic modes were used in the procedure. Figure 1 shows that the transmission loss converged well for both, the upstream and downstream direction.

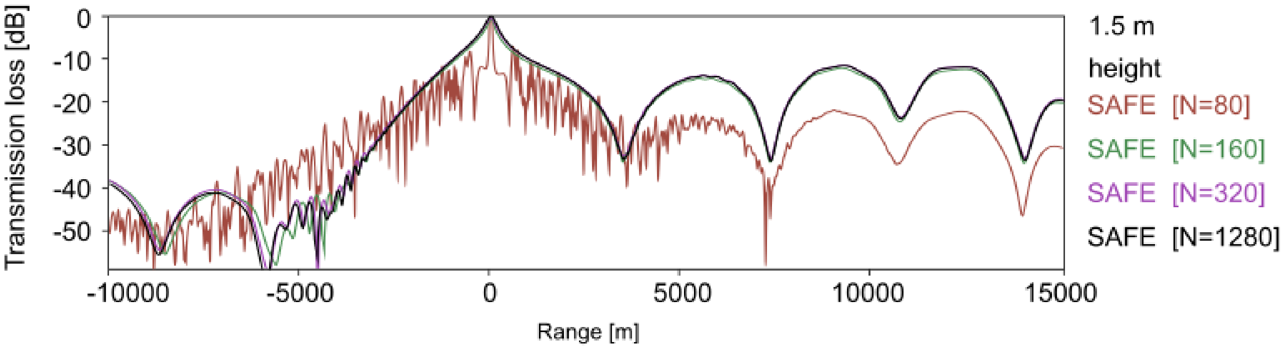

Figure 1 - Convergence study for the transmission loss along a height of 1.5m computed with (brown) 80, (green) 160, (purple) 320 (purple) and (black) 1280 linear hat finite-elements.15Hz point source radiating into an atmosphere with a power-law wind profile.

The results from the SAFE method compared to the LEE are shown in Figure 2. The contour plots for the LEE and the SAFE method (N=1280 and 3842 modes) are presented in Figure 2 a and b, respectively. The qualitative agreement between both plots was good, indicating high similarity of the results obtained with both methods.

Figure 2 d and e show the transmission loss along a horizontal line. The transmission loss in downstream and upstream direction was calculated by using the modes with a negative and positive real part, respectively. We furthermore distinguished between solutions using all propagating modes (red lines) and only the five modes in downstream direction with the smallest imaginary part, i.e. the lowest attenuated modes (black lines). In the current case with a solid (non-absorbing) ground, this separation filters the ducted modes. Alternative filtering for more complex cases can be found in (Waxler et al., 2017).

Figure 2.d and e show the transmission loss at 400m and 10m height. We found a good agreement between the SAFE method (red line) and the LEE solution (blue line) at both heights. When only the five least attenuated modes are used for the field prediction in the SAFE method, we still found a good agreement in the far-field for the forementioned heights, indicating, that the acoustic ducting is captured well. In the near field, however, the prediction using a small set of modes fails. This is attributed to the complexity of the near field, including, evanescent and un-ducted modes, hence requiring a large set of modes for accurate prediction.

Figure 2 c) shows the contour plot of the field in downstream direction of the point source, if only the five ducted modes are used for the prediction. The results are qualitatively in good agreement with the LEE simulation close to the ground. It means, that the field up to approximately 700 m was dominated by atmospheric ducting, and that this ducting can be modelled accurately with five modes or less.

4 CONCLUSIONS

In this paper, we modelled low-frequency sound propagation in an atmosphere with a logarithmic wind profile. This case is relevant, for example for noise propagation from wind turbines. The results showed that the SAFE method, depicted a highly similar acoustic-field to that of the LEE if all modes are included. The location of the acoustic turning points as well as their magnitude showed good resemblance for both methods. Small differences in the fields should be attributed to uncertainties in the wave numbers, which should be studied in detail in the future. An advantage of the SAFE method is its computational efficiency, as the computations shown in this paper were run on a standard laptop within a few hours. In contrast, the LEE computations required a state-of-the-art cluster computer. The solution of the SAFE method can furthermore be used to study the physics of a noise problem. As an example, we showed how to separate the ducted field, which could efficiently be described by five modes. Using a low number of modes for far-field computations could be useful for efficient noise map prediction tools.

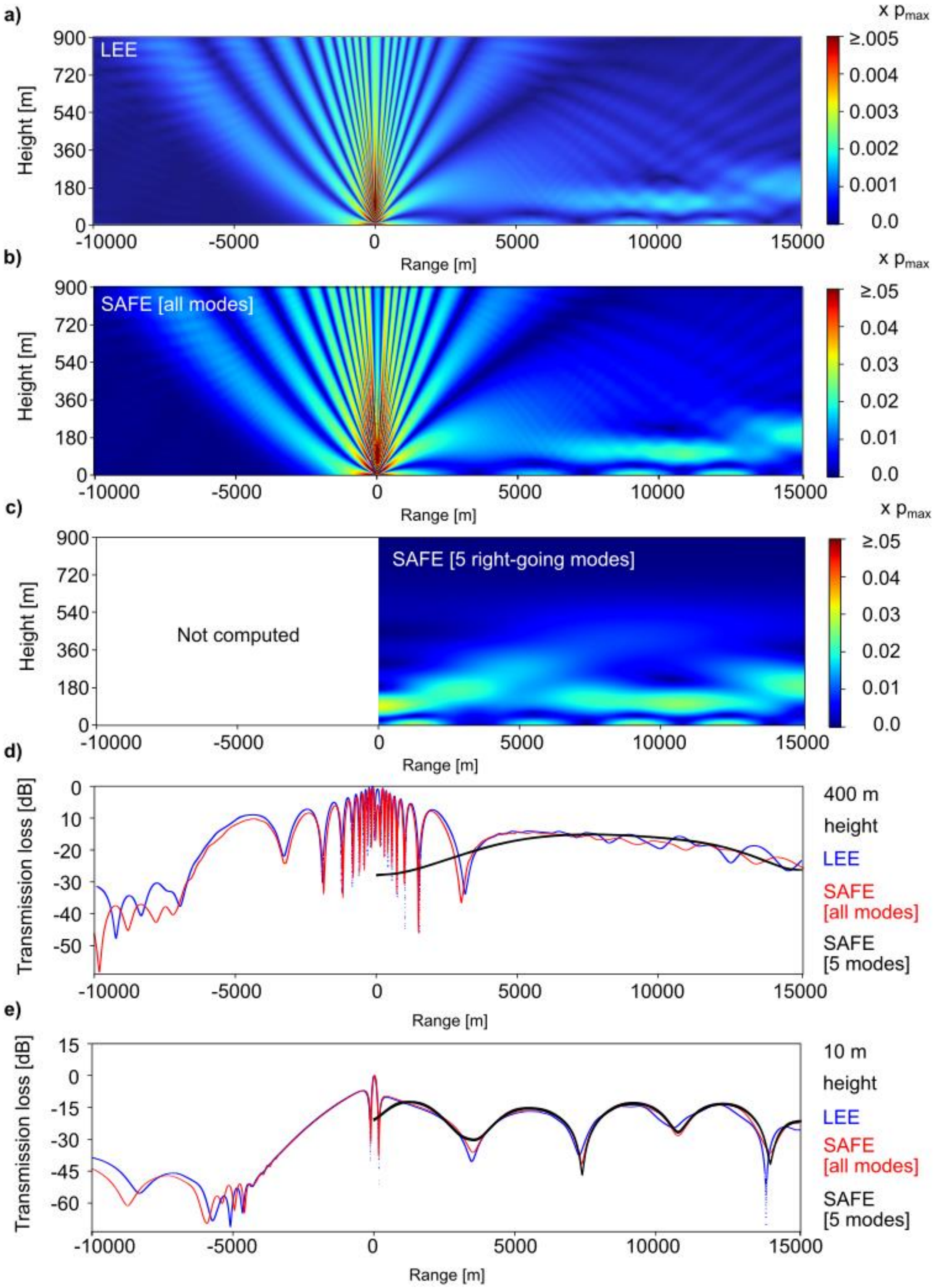

Figure 2 - Normalized pressure contour plot for (a) LEE, (b) SAFE (N=1280, MR=3841, ML=1921) and (c) SAFE with 5 of the least attenuated modes (N=1280, MR=5, ML=0). Transmission loss along a height of (d) 400 and (e) 10 m for (blue line) LEE, (red line) SAFE (N=1280, MR=3841, ML=1921) and (black line) SAFE with 5 of the least attenuated modes (N=1280, MR=5, ML=0).

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