



# Vibration of a cantilever panel due to turbulence ingestion, part I: A semi-analytical approach

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**Abstract** - Airfoils that are incident to turbulent flow will experience unsteady lift. Typically, the turbulent flow interacting with the leading edge of the airfoil will induce a pressure jump, resulting in lift fluctuations that are stochastic in nature. The phenomenon of turbulence interaction of the leading edge can be modelled by Amiet's theory of an aerodynamic gust. In this work, for the system to be amenable to analytical analysis, the airfoil is modelled as a cantilever thin panel. The transverse motion of the cantilever panel is predicted by employing a thin plate theory as its governing equation. The governing equations are then solved by a modal decomposition in its in-vacuo mode shapes. The in-vacuo mode shapes can be approximated by a free vibration analysis using the semi-analytical Trigonometric Ritz method, where a sum of trigonometric admissible functions estimates the mode shapes. Combining the semi-analytical framework to predict the response of the panel with Amiet's theory of turbulent flow, vibrations of the cantilever panel due to turbulence ingestion can be efficiently resolved. The predicted vibration responses are compared against the Uncorrelated Wall Plane Wave (UWPW) technique for verification. The UWPW approach involves converting the random excitation into a set of deterministic uncorrelated plane waves that approximate the wall pressure field associated with the target random excitation. These wall pressure fields can then be used as an input to excite a structure, either through semi-analytical models or in a finite element model. The presented semi-analytical approach can serve as a framework for investigating vibration responses of more complex geometries with turbulence ingestion.

## 1 SEMI-ANALYTICAL MODEL

Vibration of a randomly excited structure can be characterized in space-frequency  $(x, y, \omega)$  domain by the velocity auto-spectral density (ASD) function  $S_{vv}$  at a particular point of interest  $(x_i, y_i)$  on the surface of the structure  $\Sigma$  by

$$S_{vv}(x_i, y_i, \omega) = \iint_{\Sigma} \iint_{\Sigma} H_v(x_i, y_i, x_1, y_1, \omega) \Psi_{pp}(x_1, y_1, x_2, y_2, \omega) H_v^*(x_i, y_i, x_2, y_2) dx_1 dy_1 dx_2 dy_2, \quad (1)$$

where  $H_v(\vec{x}_i, \vec{x}, \omega)$  is the space-frequency sensitivity function equivalent to the transfer function between the structure's velocity at  $(x_i, y_i)$  due to a unit point load at  $(x, y)$ ,  $\Psi_{pp}$  is the cross-spectral density (CSD) function of the random wall pressure field of interest, and \* denotes the complex conjugate.

As we are interested in a cantilever panel, the sensitivity function required can be determined by applying a similar expression as found in (Maxit, 2016) (which is for a simply-supported panel). The difference will be to replace the mode shape functions with that of a cantilever panel. The mode shape functions can be approximated by the Rayleigh-Ritz method as described in (Nelisse et al., 1998) and (Dozio 2011). While for the random wall pressure field, it is due to turbulence ingestion, and in particular, we will apply the CSD function that corresponds to wall pressure fluctuations due to leading-edge interaction as described by Amiet's theory (Amiet, 1975). The CSD function is expressed as

$$\Psi_{pp}(x_1, x_2, y_2 - y_1, f) = 2\pi U_{\infty} (\pi\rho)^2 \int_{-\infty}^{+\infty} \phi_{ww}(K_x, k_y) g^*(x_1, K_x, k_y) g(x_2, K_x, k_y) e^{-jk_y(y_2 - y_1)} dk_y, \quad (2)$$

where  $U_\infty$  is the mean free stream velocity,  $\rho$  is the fluid density,  $\phi_{ww}(K_x, k_y)$  is the incoming upwash velocity fluctuation spectrum for a particular chordwise wavenumber  $K_x = \omega/U_\infty$ ,  $g(x, K_x, k_y)$  is the airfoil transfer function, and frequency is related to angular frequency  $\omega = 2\pi f$ . The necessary components can be found in (Karimi et al., 2021).

## 2 NUMERICAL EXAMPLE

Table 1 – Numerical example specifications

Parameters	Value
Young's modulus, $E$ (GPa)	70
Poisson's ratio, $\nu$	0.33
Panel density, $\rho_p$ (kg/m <sup>3</sup> )	2700
Structural damping, $\eta$	0.01
Spanwise length, $a$ (m)	1.83
Streamwise length, $b$ (m)	0.61
Thickness, $h$ (m)	0.01
Mean stream velocity, $U_\infty$ (m/s)	30
Fluid density, $\rho$ (kg/m <sup>3</sup> )	1.225
Integral length scale, $\Lambda_f$ (m)	0.0818
Turbulence intensity, $u'/U_\infty$	0.0398

A numerical example based on specifications provided in Table 1 is used for the evaluation of the vibration of a cantilever panel due to turbulence ingestion. For verification, the prediction from the semi-analytical model (TRM) is compared against the uncorrelated wall plane wave (UWPW, See (Maxit 2016) or (Karimi et al, 2021)) approach in Figure 1. The results are in good agreement.

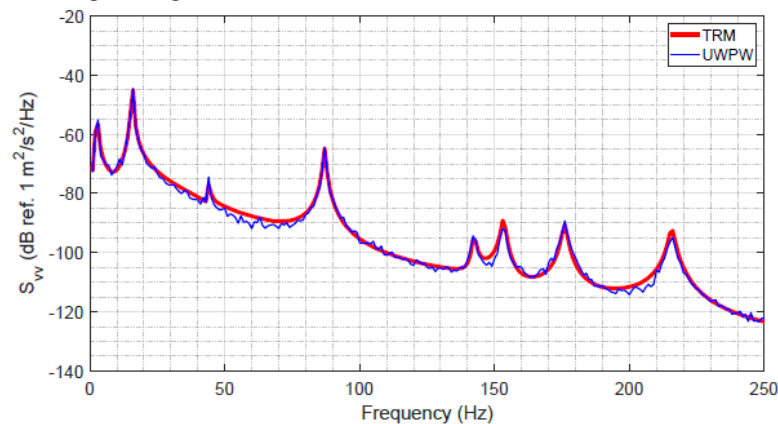


Figure 1: Evaluation of the velocity ASD function at the centre of a cantilever panel due to turbulence ingestion.

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