Transition from perturbation to Kirchhoff approximation for modelling coherent reflection of underwater sound from rough sea surfaces

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Abstract - Wind generates rough sea surfaces and subsurface air bubbles. Both rough surfaces and bubbles scatter incident sound, leading to loss of sound energy in the specular reflection. In this paper, we ignore the effect of bubbles and formulate a simple formula to model the coherent components of reflection losses from scattering by the rough air-sea interfaces. The formulation is based on transitioning two well-known approximations for treating scattering from rough interfaces. The first one is perturbation approximation, which is suitable for small grazing angles. The second one is Kirchhoff approximation, which is suitable for large grazing angles. The new simple formula is suitable for all grazing angles and compares well with results from more complex numerical integration methods.

1 INTRODUCTION

Wind roughens sea surfaces and generates waves. Wave-breaking produces air bubbles which can be transported downwards by Langmuir circulation currents (Hanson, 1993). The interaction of underwater sound with sea surfaces involves several coupled mechanisms (Urick, 1982; Christian, 1992): (1) rough air-sea interfaces scatter incident sounds into coherent and incoherent components; (2) the sub-surface air bubbles absorb and scatter sound energy; and (3) the air-water mixed layer, with decreasing air/water volume ratio with depth, refracts sound energy upwards towards the surface. These effects vary with acoustic frequency and wind speed. Accounting for these effects is nontrivial for most sonar acoustic models. In this paper, we ignore bubble effects and limit our consideration to reflection losses due to scattering from random roughness of the sea surfaces. Shadowing effects of the rough surfaces are also ignored and we consider the coherent (mean) reflection coefficient only.

The need for a suitable surface reflection loss model to represent wind effects on underwater sound transmission is well recognised. Severe discrepancies exist among various models and as an interim measure, a modified form of the Eckart model (Eckart, 1953) was recommended, although its deficiency was recognised at the time (Eller, 1985).

To model the coherent reflection loss from a rough ocean surface without consideration of bubbles, two closed-form formulas are often used. One can be derived from perturbation approximation (Brekhovskikh & Lysanov, 2003) and is more suitable for smaller grazing angles. The other can be derived from Kirchhoff approximation (Eckart, 1953) and is more suitable for greater grazing angles. The method of small slope approximation is applicable to all grazing angles but generally requires numerical evaluation of integrals. In this work, we simplify and adapt earlier results by other authors to obtain a simple expression that transit from the perturbation to the Kirchhoff formulas as grazing angle increases. The new formula matches well the surface loss tables generated using small slope approximations for the Reverberation Modelling Workshop (Thorsos & Perkins, 2008). It is quick and easy to use and provides an alternative to the use of tables and numerical integration.

2 EXISTING APPROXIMATIONS FOR COHERENT REFLECTION LOSS

2.1 The Eckart Model and Kirchhoff Approximation

Kirchhoff approximation is a particular approach of solving general scattering problems including scattering from finite objects, deterministic and random rough surfaces. [It is equivalent to the “tangent plane method” in Russian work, e.g. Brekhovskikh & Lysanov (2003)]. The Eckart model (Eller, 1985) refers to a closed-from formula derived by Eckart (1953) under the Kirchhoff approximation. This formula or its equivalents is also often called the Kirchhoff formula or Kirchhoff approximation.

The Eckart model or Kirchhoff formula for the surface loss per reflection can be written in $Neper$[1] as,

$$SL\ (Np) = 2.25 \times 10^{-3} (f/c)^2 U^4 \sin^2(\theta) \tag{1}$$

where f is the acoustic frequency in Hz and c is the sound speed in m/s, U is the root-mean-square surface roughness in m, θ is grazing angle in $radians$.

While the model fits laboratory measurements at relatively large grazing angles [e.g. the work referenced in Ch.13 of Medwin & Clay (1998) and Ch.1 of Brekovskikh & Lysanov (2003)], computation of the coherent reflection coefficient show that at small grazing angles less than a few degrees, the model predicts smaller losses than perturbation methods and the second order small slope approximation (Kuo, 1988; Williams, Thorsos, & Elam, 2004). Numerical modelling (McDaniel, 1981) using normal modes show that the model underestimates the attenuation of the important lower-order modes compared to the boundary perturbation approach.

It is also relevant to note that in predicting the bistatic incoherent cross section near forward specular direction, the accuracy of the Kirchhoff approximation deteriorates as the incident grazing angle is lowered below 10 degrees. For example, for a frequency of 200 Hz and a wind speed of 10 m/s, the Kirchhoff results are substantially in error (> 3 dB) when the incident grazing angle is 5 degrees (Thorsos, 1990).

The need for accurate prediction of surface losses at small grazing angles of a few degrees is particularly important for surface duct propagation where the trapped energy is little affected by bottom loss and propagates at grazing angles of a few degrees. For example, in an isothermal, isohaline, mixed-layer surface duct, the limiting ray (the ray that vertexes at the bottom of the duct) is incident on the sea surface at a grazing angle of 1.5 degrees for a shallow duct of 30 m and 3.3 degrees for a deep duct of 150 m.

The Modified-Eckart surface loss has a correction term to the Eckart model, which was recommended as an interim surface loss model, although it was recognised at the time that both Eckart and Modified-Eckart under-predicts surface loss at small grazing angles (Eller, 1985).

2.2 The Perturbation Approximation

Following Brekhovskikh & Lysanov (2003) and Ainslie (2005), simple expressions for sea surface losses, valid when the loss is small, may be written as,

$$SL\ (Np) = 1.72 \times 10^{-3} (f/c)^{\frac{3}{2}} U^3 \sin(\theta) \tag{2}$$

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[1] For convenience, we use Neper instead of dB for surface losses here. One neper ($1\ Np$) represents a magnitude (e.g., voltage, pressure) ratio of e, and a power ratio of e^2. Hence, $1\ Neper = 20 \log_{10}(e) \approx 8.686\ dB$. Definitions and discussions can be found in Thompson & Taylor (2008).

It has been reported that measurements of propagation loss in surface ducts are in good agreement with a simple propagation-loss model that uses the Scheifele, Mellen, and Browning absorption coefficient and surface loss predictions that are equivalent to the formula presented here (Cotaras, Morash, & Franklin, 1993).

2.3 The Small Slope Approximation

The small slope approximation is applicable at both low and high grazing angles. The lowest-order small slope expression yields the same results as the Eckart or Kirchhoff approximation. The second order small slope expression yields results that go smoothly from low to high grazing angles, matching the perturbation results for small roughness parameters and/or small grazing angles, [e.g. Williams, Thorsos, & Elam (2004)].

Recent numerical simulations (Williams, Thorsos, & Elam, 2004) show that (1) the Kirchhoff approximation underestimates reflection losses at small grazing angles; (2) the perturbation approximation is accurate but only at small grazing angles or small roughness; and (3) the (second order) small slope approximation is the most accurate among the three. However, the small slope approximation requires 1D or 2D numerical integration of the spatial spectra, which are less simple to use than the closed-form expressions from perturbation or Kirchhoff approximations.

2.4 The Improved Kirchhoff Formula by Chapman

To improve the accuracy of the conventional Kirchhoff formula at small grazing angles, Chapman (1983) derived an improved Kirchhoff formula by applying boundary perturbation method to a rough surface described by a Gaussian spectrum. His formula can be written as,

$$SL\ (Np) = 4\ \sigma^2 k^{3/2} L^{-1/2} \cos(\theta/2)\, sin(\theta)\, F(\varepsilon) \tag{3}$$

where σ is the rms surface roughness, $k = \frac{2\pi f}{c}$ is the acoustic wavenumber, L is the spatial correlation length of the Gaussian rough surface, ε is related to the ratio of correlation length/acoustic wavelength and the grazing angle,

$$\varepsilon = kL \sin^2(\theta/2) \tag{4}$$

The function $F(\varepsilon)$ is expressed by the parabolic cylinder function $V(1,\ 2^{\frac{1}{2}}\,\varepsilon)$

$$F(\varepsilon) = 2^{-3/4}\, \pi^{1/2}\, exp\left(\frac{-\varepsilon^2}{2}\right)\, V(1,\ 2^{\frac{1}{2}}\,\varepsilon) \tag{5}$$

To compute function $F(\varepsilon)$, Chapman (1983) suggested using numerical integration or tabulated values of the parabolic cylinder function.

The modified Kirchhoff formula requires the correlation length of the rough surface as an input, in addition to the rms surface height, whereas the conventional Kirchhoff formula uses only the latter. Strictly speaking, the modified Kirchhoff formula applies to rough surfaces described by Gaussian spectra only. Chapman applied it to sea surfaces described by Pierson-Moskovitz spectra based on the assumption that the reflection losses can be effectively modelled by that from a Gaussian spectrum having the same rms wave height and correlation length. He found the accuracy of the formula was adequate except at low frequencies and low wind speeds where the surface losses are small and dominated by bottom effects.

The correlation functions of sea surfaces are non-Gaussian and have azimuthal dependence. The effective correlation lengths for the Pierson-Moskovitz spectrum in the downwind and crosswind directions have been estimated to be (Fortuin & Boer, 1971),

$$L_{dw} = 23.4\sigma \tag{6}$$

$$L_{cw} = 46.7\sigma \tag{7}$$

Since the modified Kirchhoff formula requires a single correlation length, the mean relation $L = 35.1\sigma$ was chosen in Chapman's calculations.

3 THE NEW SIMPLE ANALYTICAL FORMULA

The present investigation is motivated by the desire for a closed-form formula that matches the results of the second order small slope approximations (SSA2) and also that is simple to use for acoustic propagation, reverberation and sonar performance predictions.

3.1 Simplification of Chapman's Formula

Chapman's (1983) improved Kirchhoff formula requires the computation of the special function $F(\varepsilon)$ and the selection of a suitable input correlation length. While numerical integration or spline algorithms can be used to fit tabulated values of the parabolic cylinder function $V(1,\ 2^{\frac{1}{2}}\,\varepsilon)$, we found that simple approximations were adequate. While using the mean value in the downwind and crosswind directions is a natural choice, it is somewhat arbitrary and does not necessarily produce an acoustically equivalent effect.

We adapt and simplify Chapman's modified Kirchhoff formula in two aspects:

1. We obtain a simple approximation to the parabolic cylinder function, and show later that its accuracy is adequate.
2. We derive an expression for an acoustically equivalent correlation length by matching the reflection losses of the modified Kirchhoff formula at the important small grazing angles with those derived from the perturbation approximation.

Using these adaptations, the new formula we are proposing is based on the Chapman formula, Eq. (3), with the function $F(\varepsilon)$ given by the following approximation,

$$F(\varepsilon) = \begin{cases} a(1 + b\varepsilon), & \varepsilon < \varepsilon_c \\ \varepsilon^{1/2}, & \varepsilon > \varepsilon_c \end{cases} \tag{8}$$

where $a = 0.3457$ and $b = 1.4793$. The changeover point ε_c can be set as the intercept of the two simple functions. Solving the equation,

$$a(1 + b\varepsilon) = \varepsilon^{1/2} \tag{9}$$

yields $\varepsilon_c \approx 2.275$.

Because Chapman's improved formula is derived for rough surfaces described by Gaussian correlation functions whereas the correlation functions of sea surfaces are non-Gaussian, we need to select an input correlation length which produces an acoustically equivalent effect, i.e., the same reflection loss. Since the perturbation approximation is known to be accurate at small grazing angles, we "invert" for the equivalent correlation length by matching the reflection losses of the modified Kirchhoff formula with those derived from the perturbation approximation at small grazing angles. We obtain,

$$L = 0.131\, U^2\, [1 + bkL\, sin^2(\theta/2)]^2\ \ cos^2(\theta/2) \tag{10}$$

At small angles such that,

$$bkL\, sin^2(\theta/2) \approx \left(\frac{bkL\theta^2}{4}\right) \ll 1$$

$$cos^2(\theta/2) \approx 1 - \theta^2/8 \approx 1 \tag{11}$$

We obtain the first order (in small θ) approximation to the equivalent correlation length in terms of rms roughness,

$$L = 0.131\, U^2 = 24.3\, \sigma \tag{12}$$

We can see from Eqs.(6)-(7) that the equivalent correlation length lies between the values in the downwind and crosswind directions, but only slightly greater than that in the downward direction.

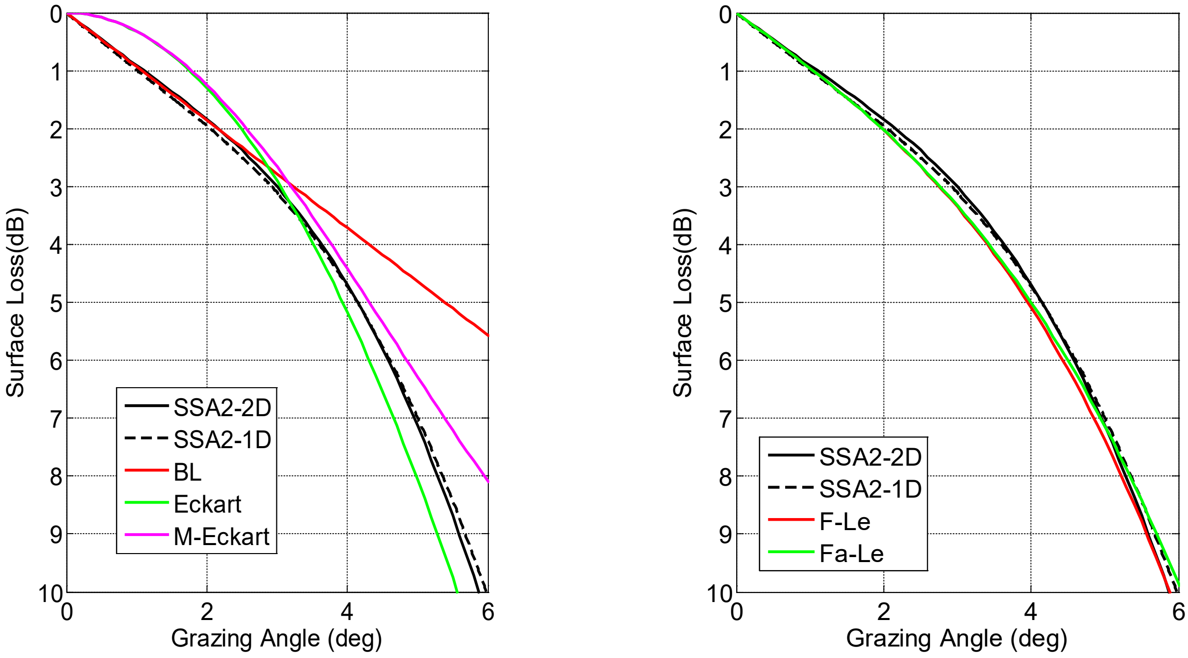
3.2 Comparison of Modelled Coherent Reflection Losses

To ascertain the accuracy of the new formula, we compare with results of numerical integration of the second order small slope integral for a case in the Office of Naval Research Reverberation Modelling Workshop (Thorsos & Perkins, 2008). In this case, the rough sea surfaces are specified by Pierson-Moskowitz spectra for fully developed seas with wind speeds (at a height of 19.5 m) of 10 m/s. Bubble effects were excluded. Computed values of coherent reflection coefficients, generated using the second order small slope approximation (SSA2), are provided by the Workshop for 1D and 2D (isotropic) rough surfaces at frequencies of 3500 Hz, 1000 Hz, and 250 Hz.

Existing analytical formulas from perturbation and Kirchhoff approximations are valid for either small or large grazing angles, as shown by comparisons of surface losses as a function of grazing angle between existing models and the values from the second order small slope integration (Figure 1a). In the figure, the plots for SSA2-1D and SSA2-2D (solid and dashed black lines) refer to the SSA2 for 1D and 2D Pierson–Moskowitz spatial spectra, respectively. The “BL” results are from Eq.(2), the formula derived by Brekhovskikh and Lysanov (2003) using the perturbation approximation for the Pierson–Moskowitz spectra. The Eckart and modified Eckart results were computed from the formulas in Eller (1985), which were derived from Kirchhoff approximation with rms heights from Pierson–Moskowitz spectra with and without correction term, respectively.

In Figure 1a, the small differences between SSA2-2D and SSA2-1D results are due to the approximations made when deriving the 1D spectra from the 2D isotropic spectra. The BL formula results from perturbation approximation match the SSA2 results very well at small grazing angles. This is interesting considering that the BL formula is derived from analytical approximation of the integral of the temporal Pierson-Moskowitz spectra whereas the SSA2 results were from numerical integration of the corresponding spatial spectra. The Eckart formula under-predicts the reflection loss at smaller grazing angles, and over-predicts the reflection loss at greater grazing angles.

In Figure 1b, the improved Kirchhoff’s formula by Chapman (1983) and the new simple analytical formula derived in this paper are compared against the results of numerical integration of the second order small slope integral. The “F-Le” results were computed from Chapman’s improved Kirchhoff formula, Eq.(3), with equivalent correlation length provided by Eq.(12) and the special function $F(\varepsilon)$ was computed exactly. The “Fa-Le” results were computed from our new analytical formula, with the special function $F(\varepsilon)$ approximated by the simple function in Eq.(8). At the important small grazing angles for surface ducted propagation, both Chapman’s improved Kirchhoff formula (F-Le) and the new formula are much closer to the benchmark SSA2 results than the conventional Kirchhoff formula (i.e. the Eckart model). The new simple formula gives results that are as good as Chapman’s original complex formula and provides good fits to all grazing angles.



(a) (b)

Figure 1 – Coherent surface loss (in dB) versus grazing angle for wind speeds of 10 m/s and acoustic frequency of 3500 Hz. (a) Comparisons of existing models (Eckart, M-Eckart, BL perturbation approximation) with second order small slope integration for 1D and 2D rough surfaces (SSA2-1D and SSA2-2D). (b) Comparison of Chapman's formula (F-Le) and the new simple formula (Fa-Le) with the second order small slope integration.

It short, the new closed-form formula matches closely the coherent reflection coefficient values from numerical integration of the second order small slope integral. It provides a simpler alternative to numerical integrations, and is much easier to use for modelling acoustic propagation.

4 SUMMARY AND CONCLUSION

Two well-known formulas are often used for modelling coherent reflection losses from rough sea interfaces. The first one is the perturbation formula, which is accurate for small grazing angles. The second one is the Kirchhoff/Eckart formula, which is accurate for large grazing angles. To obtain results that are valid for all grazing angles, one often resorts to numerical methods such as the small slope approximation, which is often cumbersome to use in acoustic propagation modelling.

In this paper, based on earlier work by Chapman (1983), we derived a simple closed-form formula that is accurate for all grazing angles. The formulation transits to the perturbation formula at small grazing angles, transits to the Kirchhoff formula at large grazing angles, and matches well with results from more complex numerical integration methods.

The new formula is much easier and simpler to use for modelling acoustic propagation and sonar performance than numerical integration of surface integrals, while providing a similar accuracy to numerical integration of the second order small slope approximation.

The new formula is for coherent reflection loss only and has some of the usual assumptions as the usual Kirchhoff and perturbation approximations, Firstly, the effects of bubbles are not considered. Secondly, the effect of shadowing of the rough surfaces is also ignored. Numerical simulations using parabolic equations (PE) show that both the coherent and incoherent scattered fields can be significant in some situations (Perkins & Thorsos, 2006).

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