On the acoustic properties of a hole in a wall of finite thickness

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ABSTRACT

Holes and gaps are often found at the cracks in walls or the conjunctions between structural elements, as a result of manufacturing deficiencies or functional purposes. The airborne noise transmitted through those channels could make a considerable effect upon the sound insulation performance of the structures and increase the noise level in the receiving field. Despite the extensive efforts made in the past to study the sound transmission characteristics of a hole in a wall of finite thickness, there remain questions to which proper answer should be given. For example, the negative sound transmission loss value in the frequency regions associated with the resonance frequency of the fluid-loaded hole is an apparent violation of the power conservation. Such a result may confuse the understanding of the transmission process because the sound, instead of being reduced, tends to be amplified in the course of transmission. The occurrence of this phenomenon would be explained in this paper using a simple one-dimensional model. The sound intensity field at the incidence side shows that the hole essentially draws energy from a region much greater than its physical dimension around the resonance frequencies of the fluid-loaded hole. At off-resonance frequencies, the way in which energy flows into the hole is found to depend upon whether the coupled system is stiffness-like or mass-like. The model is then extended to the case in which the hole is filled with an acoustic seal. Results show that the acoustic seal would improve the sound insulation to a large extent at the off-resonance frequencies, while poor insulation performance is noticed around the resonances, the frequencies of which are determined by the depth of the hole.

1. INTRODUCTION

Sound transmission through a hole in a wall of finite thickness is an essential acoustic problem pertaining to various applications. The pioneer work on studying the influence of the hole can be traced back to Gomperts (1964), who deduced a simple formula to predict the transmission of sound through circular and slit-shaped holes. Experiments demonstrated the reliability of the model at the off-resonance frequencies. Around the resonance frequencies, the model failed in rendering good prediction accuracy as the viscosity effect of the hole was ignored. The model was then improved (Gompeters, 1965) by combining with the viscosity formula developed by Ingerslev and Nielsen (1944), yielding better accuracy for ka up to 0.5 (k being the wavenumber and a the radius for circular holes or the breadth for the slits). Wilson and Soroka [6] developed a model for a circular hole that extended the validity of the model to a greater range. In a subsequent study, Sauter and Soroka (1970) found that the difference in transmission between a circular hole and a rectangular hole was marginal and it was concluded that the model can be applied to a rectangular hole by replacing the radius a with an equivalent radius of the hole cross section. A survey made by Morfey (1969) of the low-frequency acoustic properties of circular, rectangular and elliptic holes demonstrated also the weak dependence of the acoustic response on the hope shape. Oldham and Zhao (1993) examined the models of Gomperts and Soroka using the sound intensity technique with the results showing that the models were valid over a large frequency range. However, discrepancy between the models and the measurement was found to arise with the decrease of the cross-sectional area and the increase of the hole thickness since the viscosity effect is substantial for small and long holes. The damping loss in the hole was characterized in terms of the characteristic impedance and the complex propagation constant in the model of Sgard et al (2007), with which the diffuse field sound transmission was predicted and compared with the normal incidence sound transmission to which the models of Gomperts and Soroka are applicable.

Despite the considerable efforts dedicated to the relevant field, there remain questions to which proper answers should be given for better understanding the mechanism underlying the sound transmission process. For example, it is known that the transmission curve would exhibit an oscillatory feature with maxima and minima due to resonances of the fluid-loaded hole. Negative values of transmission loss may be encountered around the resonance frequencies. This phenomenon, which apparently violates the power conservation, may lead to a misunderstanding of the transmission characteristics of the system and a comprehensive explanation to this, however, appears to be lacking in the literature. This paper will fill this gap by resorting to a simplified system model and the relation between the hole parameters and the sound transmission will be investigated in terms of the wave impedance of the coupled system. The advantage of qualifying the coupled system, which consists of the hole and the fluid acting upon its two ends, as the wave impedance, is that the system characteristics could be examined independently without the constraint of external excitation. Indeed, in the work of Gervais and Gervais (2007), measurement of door gap noise showed that the spectrum shape is almost independent of the flow velocity, indicating a weak coupling between the fluid and the acoustic field since the gap is too narrow to allow the vortices in the mixing layer to develop. The door gap, in this case, functions as an acoustic filter to the acoustic sources generated at the opening and it was suggested that the door gap could be modeled independent of the source for simplifying the modeling process. The developed model will then be employed to study the sound insulation characteristics of a hole filled with an acoustic seal, which may be reckoned as a two-dimensional representation of a door weatherstrip. Past research on this is either quantitative (Mechel, 1986) or finite element method based (Park *et al*, 2003). It is shown in this paper that the system behavior at the resonance and the off-resonance frequencies could readily be seen from the simplified wave impedance.

2. Mathematical modelling

The system under investigation consists of a circular hole flushed in an infinite baffle of finite thickness that separates an air media into two domains. Inside the hole, an acoustic seal is inserted, occupying a space from $x = d_2$ to $x = d_2+d_3$. The dynamic property of the seal may be characterized by two mass layers, with surface densities m_{23} ($x = d_2$) and m_{34} ($x = d_2+d_3$), vibrating in a piston-like motion at its two ends. The approximation holds for the vibration of the seal in the low-frequency range where the diameter of the hole is much smaller than the acoustic wavelength (Mechel, 1986). The vibration of the air at the ends of the hole, i.e. $x = 0$ and $x = d_2+d_3+d_4$, is assumed also to exhibit piston-like motion at low frequencies. The two mass layers and the two virtual mass layers (m_{12} and m_{45}) divide the whole acoustic domain into five sub-domains. A dual-digit subscript is used to indicate a quantity located at the interface between a pair of domains. The time harmonic factor $e^{-j\omega t}$ is understood and omitted hereafter. The cross section of the configuration is shown schematically in Fig. 1. The symmetry of configuration enables the modeling process to be implemented in a two-dimensional coordinate system with the origin set at the center of the entry of the hole.

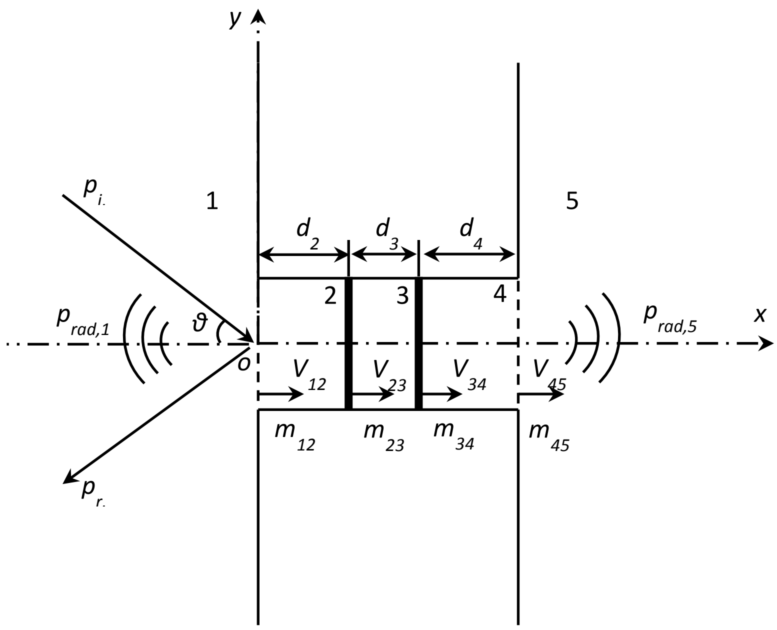


Figure 1: A schematic representation of the cross section of the system under investigation.

According to Fahy & Gardonio (2007), the velocity at the entry surface of the hole, V_{12}, subject to a block pressure $2P_{in}$ could be obtained from the following expression

$$Z_c V_{12} = 2p_{in} \tag{1}$$

where P_{in} is the pressure of the incident wave and Z_c is the wave impedance of the fluid-loaded hole. The impedance-translation theorem states that, for normal plane wave transmitted through a homogeneous fluid layer with a characteristic acoustic impedance of Z_0, the local acoustic impedance at any two points x and L-x along the transmission line is related by

$$Z(L-x) = Z_0 \frac{Z(x) + jZ_0 \tan(k_0 L)}{Z_0 + jZ \tan(k_0 L)} \tag{2}$$

Equation (2) suggests that the local acoustic impedance at one end of the fluid layer can be determined from the acoustic impedance information at the other end. Referring to Fig. 1, for an empty hole ($d_2 = d_4 = 0$ and $m_{23} = m_{34} = 0$), the local acoustic impedance can be obtained by substituting the radiation impedance $Z_{r,5}$, being the local acoustic impedance at the hole exit, into Eq. (A1), yielding

$$Z_c = Z_{r,1} + Z_0 \frac{Z_{r,5} + jZ_0 \tan(k_0 d_3)}{Z_0 + jZ_{r,5} \tan(k_0 d_3)} \tag{3}$$

For the hole fully occupied by an acoustic seal, i.e. $d_2 = d_4 = 0$ and $m_{23} = m_{34} \neq 0$, the local acoustic impedance at the hole exit is the sum of radiation impedance $Z_{r,5}$ and impedance of the mass layer $j\omega m_{34}$. Substituting this into Eq. (2) and taking the impedance of the mass layer $j\omega m_{23}$ at the hole entry into account, the local impedance is

$$Z_c = Z_{r,1} + j\omega m_{23} + Z_0 \frac{(j\omega m_{34} + Z_{r,5}) + jZ_0 \tan(kd_3)}{Z_0 + j(j\omega m_{34} + Z_{r,5}) \tan(kd_3)}. \tag{4}$$

In above equations, $Z_{r,1}$ and $Z_{r,5}$ are the radiation impedances of the baffled circular hole at its two ends, which are expressed as (Pierce, 1981)

$$Z_{r,1} = Z_{r,5} = Z_0 \left[1 - \frac{J_1(2k_0 a)}{k_0 a} + j\frac{S_1(2k_0 a)}{k_0 a}\right], \tag{5}$$

where a is the radius of the hole and k_0 is the wavenumber. The solved V_{12} and V_{45} could be used to calculate the sound transmission loss (TL), a quantity that is used to evaluate the sound insulation characteristics of the hole, throughout the paper. It is defined as

$$TL(\theta_i) = 10 log_{10}\left(\frac{1}{\tau(\theta_i)}\right), \tag{6}$$

where $\tau(\vartheta_i)$ is the sound power transmission coefficient for a plane wave incident at angle ϑ_i and is defined as the ratio of the power transmitted through the hole Π_t, to the power incident on the hole Π_i, which are, respectively,

$$\mathrm{W}_t(\theta_i) = \frac{1}{2} S Re(Z_{r,5}) |V_{45}|^2, \tag{7}$$

and

$$\mathrm{W}_i(\theta_i) = \frac{1}{2} S cos\theta_i |P_{in}|^2 / Z_0. \tag{8}$$

Thus

$$\tau(\theta_i) = \frac{W_t(\theta_i)}{W_e(\theta_i)}. \tag{9}$$

In Eq. (7), Re(-) denotes the real part of the argument. For clarity of the analysis, a normal plane wave incidence would be used in what follows.

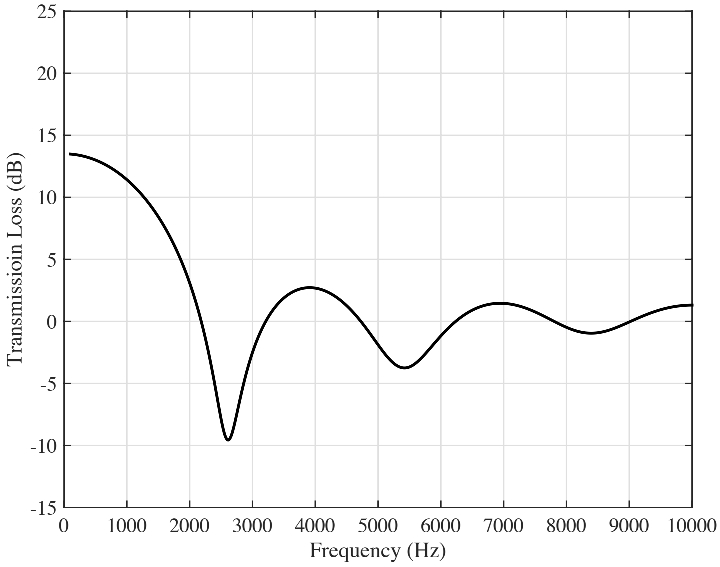


Figure 2: Transmission loss for an empty hole subject to a normal plane wave incidence. (a = 0.01m, $d_2 = d_4$ = 0m, d_3 = 0.05m)

3. Sound transmission through an empty hole

The sound transmission characteristics of an empty hole would be studied first. In both the analysis for the empty hole, and that for the hole with an acoustic seal to be presented in the next section, the radius of the hole is fixed, having a = 0.01m, and the incident pressure amplitude is chosen to be unity. The sound transmission loss for an empty hole of 0.05m thickness is depicted in Fig. 2. It could be seen that, in the low-frequency range, the hole is shown to have high sound transmission loss and the value tends towards a constant as the frequency decreases. In the mid-frequency range, the TL is smaller than that in the low-frequency range. Furthermore, an obvious fluctuation is observed with minima and maxima in the frequency spectrum, and the degree of the fluctuation, in terms of the TL magnitude, decreases with the increase of frequency. While in the high-frequency range, the variation is moderate and the TL approaches zero.

3.1 The negative sound transmission loss

Figure 2 also shows regions in which the transmission loss is negative. This, at first glance, is unexpected as the power conservation is violated. A negative TL value indicates a transmission coefficient larger than unity. By definition, the transmission coefficient is the fraction of the incident power transmitted through the hole. A coefficient larger than unity implies that the power transmitted through the hole exceeds that incident upon the hole.

The reason for this phenomenon is attributed to the definition of the transmission coefficient for which the nominal incident power is used as the denominator in the ratio (Maxit *et al* 2012, Yang *et al* 2013). According to Eq. (8), the incident power is a quantity in proportion to the hole area and the square of the incident pressure amplitude. For circumstances in which the hole is excited by a plane wave incidence at a fixed angle, the incident power will be a constant. This frequency independent quantity is not the in situ power injected into the hole. By integrating the sound intensity across the surface area of the hole at the entry side, the power injected into the hole could be obtained as

$$W_{inj} = \frac{1}{2}\int_S (p_{12})^* V_{12} ds \approx \frac{1}{2} S (p_{12})^* V_{12}, \tag{10}$$

where the asterisk denotes conjugate.

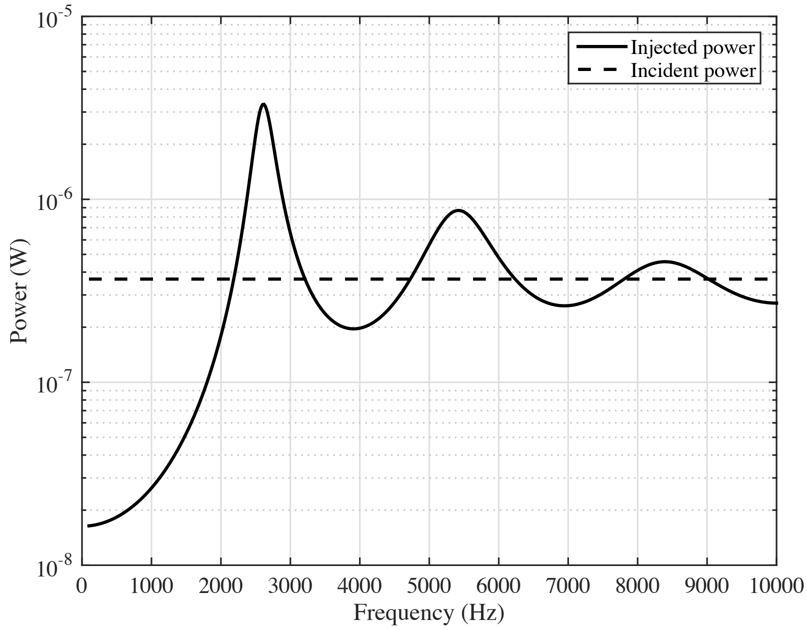


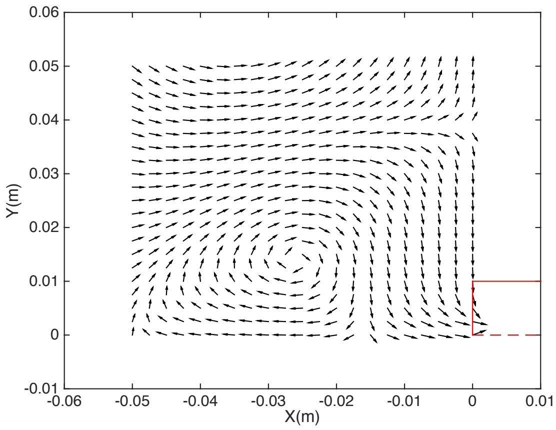
Figure 3: Comparison of the injected power W_{inj} and the incident power Win for an empty hole subject to a normal plane wave incidence. (a = 0.01m, d_2 = d_4 = 0m, d_3 = 0.05m)

Figure 3 compares the power injected into the hole, calculated by using Eq. (10), with the nominal incident power, calculated by using Eq. (8). An apparent difference could be noticed between the two curves. The injected power shows a frequency dependent feature with the value higher than the incident power in certain regions, corresponding to frequencies at which the TL is negative, and smaller in other regions, corresponding to frequencies at which the TL is positive. It reveals that the hole essentially draws the energy of an amount different from that of the incident energy. This is because of the impedance discontinuity at the edges of the hole where diffraction occurs to influence the extent to which energy flows into the hole. To visualize this effect, the sound intensity field at the incident side of the hole is plotted, respectively, at 2620Hz and 3920Hz in Fig. 4. The former frequency corresponds to a peak in the injected power curve whilst the latter corresponds to a dip in the curve. In the figures, the vector lengths are set with the magnitude in proportion to the sound intensity level in dB normalized by a range of 40dB. The figure shows that, at 2620Hz, energy is extracted from a region greatly exceeding the physical area of the hole, causing more energy to flow into the hole. In contrast, at 3920Hz, the region from which the energy flows into the hole is smaller than that at 2620Hz and is comparable with the area of the opening. Additionally, a portion of the energy that proceeds towards the hole from afar is seen to circulate off due to the presence of the energy vortex in the vicinity of the hole, yielding less injected energy.

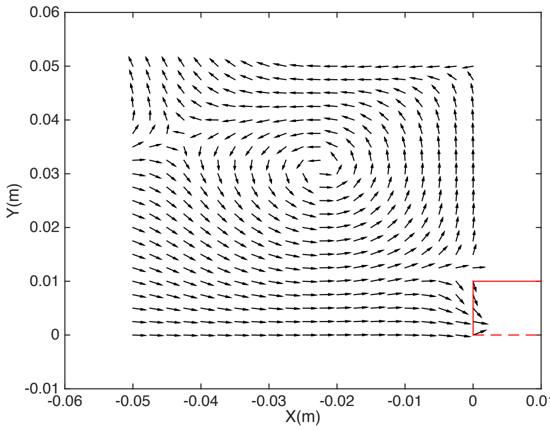
3.2 The dependence of the injected energy on the wave impedance of the fluid-loaded hole

The resistance and reactance of the wave impedance of the coupled system, normalized by Z_0, are depicted, respectively, in Fig. 5. When the frequency increases, the resistance is shown to follow a growing trend with profound local maxima whose magnitude is smaller when it occurs at higher frequency. For the reactance, the curve oscillates between positive and negative due to the tangent term involved in the wave impedance expression, implying that the presence of the air channel in the hole would lead the coupled system to behave either stiffness-like or mass-like, which is frequency dependent.

A comparison between Fig. 5 and Fig. 3 allows one to identify the influence of the wave impedance of the coupled system on the amount of the energy that is injected into the hole. It is observed that the maximum energy injection occurs approximately at frequencies where the reactance of the wave impedance vanishes and the resistance of the wave impedance is not close to local maxima, as denoted by circles in Fig. 5. In circumstances associated with this wave impedance condition, the coupled system undergoes resonance where the fluid-loaded hole vibrates with large velocity amplitude and relatively in phase with the surface pressure at the hole entry, enabling the energy to be conducted into the hole in an effective manner. Furthermore, as the coupled system is purely resistive at the resonance frequencies, the velocity would be a value inversely proportional to the magnitude of the resistance of the wave impedance. In connection with this, the degree to which the energy can be injected into the hole would depend upon the resistance of the wave impedance. The smaller the value is, the larger the velocity amplitude would be, and thus the more energy flows into the hole.



(a)



(b)

Figure 4: Sound intensity at the incident side of the hole. The cross section of the hole is drawn in red. (a) 2620Hz; (b) 3920Hz. (a = 0.01m, $d_2 = d_4$ = 0m, d_3 = 0.05m)

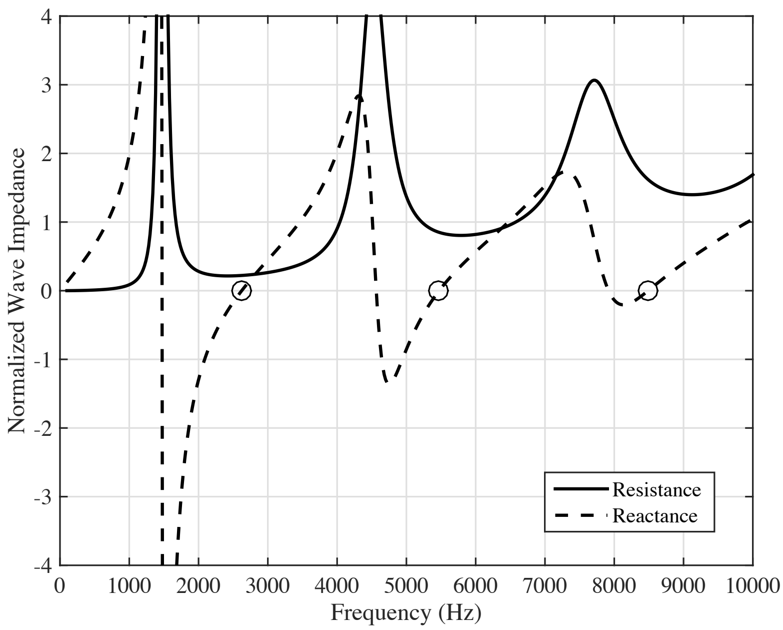


Figure 5: Normalized wave impedance of the coupled system. The circles correspond to the peaks of the injected power in Fig. 3. (a = 0.01m, $d_2 = d_4$ = 0, d_3 = 0.05m)

At non-resonance frequencies, the reactance of the wave impedance is non-zero and works in conjunction with the resistance to determine the sound transmission value. Besides, the reactance term is found to play a role in determining the way in which the energy flows into the hole. An illustrative example of this phenomenon is given in Fig. 6, showing the sound intensity at the incidence side of the hole. The frequencies are chosen in a way that the reactance is positive (mass-like) at 3400Hz whilst negative (stiffness-like) at 2220Hz. The sound intensity field is plotted across a larger zone in order to capture this phenomenon better. When the reactance is positive, the energy is seen to advance towards the hole from the region right in front of the hole. In contrast, when the reactance is negative, the energy in front of the hole is found to exhibit an obvious backward-going pattern. In this case, the energy injected into the hole essentially originates from a channel-like field angled from the normal to the hole surface. The incoming energy, when approaches the hole surface, is circulated, due to the presence of the energy vortex in the vicinity of the hole, and partially flows into the hole and partially joins the backward-going energy to flow out.

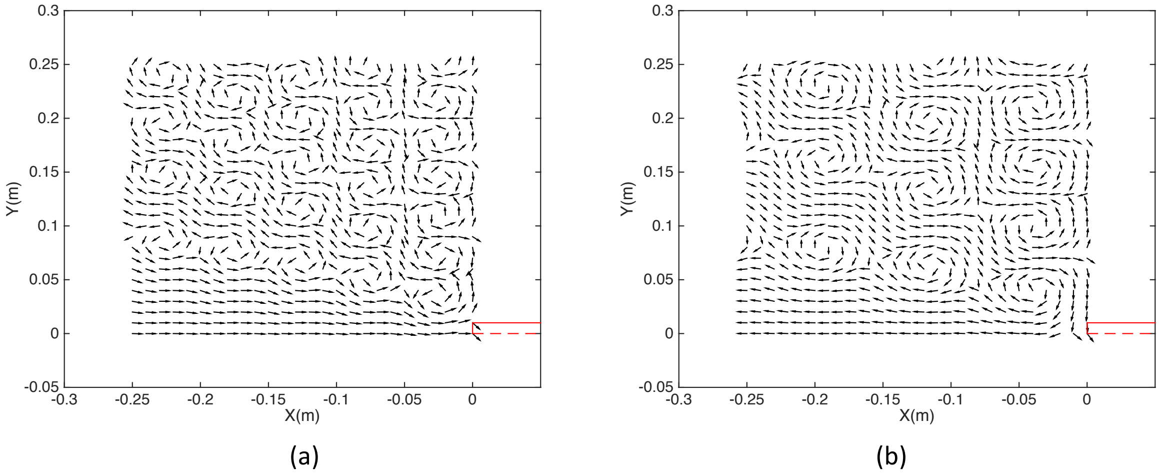


Figure 6: Sound intensity at the incident side of the hole. (a) 3400Hz; (b) 2220Hz. The cross section of the hole is drawn in red. (a = 0.01m, d_2 = d_4 = 0m, d_3 = 0.05m)

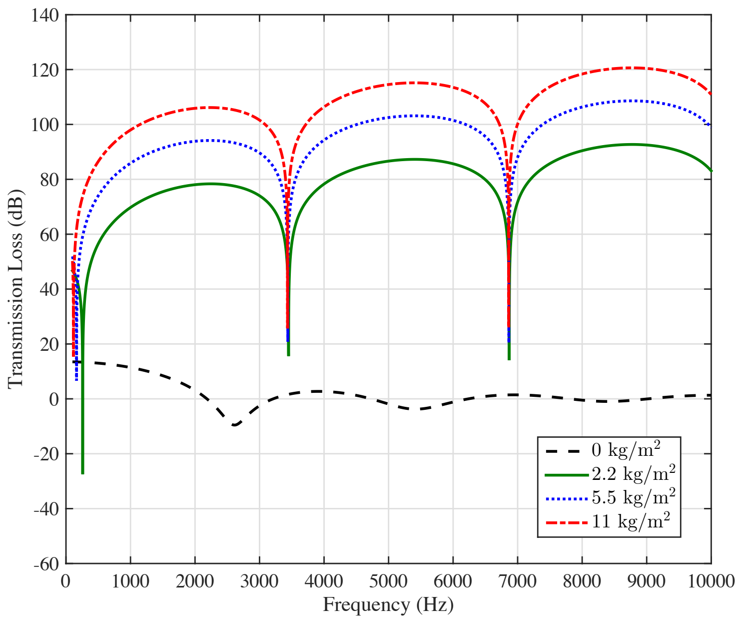


Figure 7: Transmission loss for a hole without and with an acoustic seal of different surface densities subject to a normal plane wave incidence. (a = 0.01m, d_2 = d_4 = 0m, d_3 = 0.05m)

4. Sound transmission through a hole with an acoustic seal

The sound transmission through a baffled hole in which an acoustic seal is inserted is investigated in this section. Emphasis is made on the change brought by the seal, which is modeled as a one-dimensional air channel terminated by a pair of mass layers at its two ends, to the acoustic characteristics of the hole, and on the influence of the seal parameters, i.e. surface density and thickness, on the sound insulation performance. In what follows, the thickness of the air channel in the seal, unless otherwise stated, is chosen to be 0.05m.

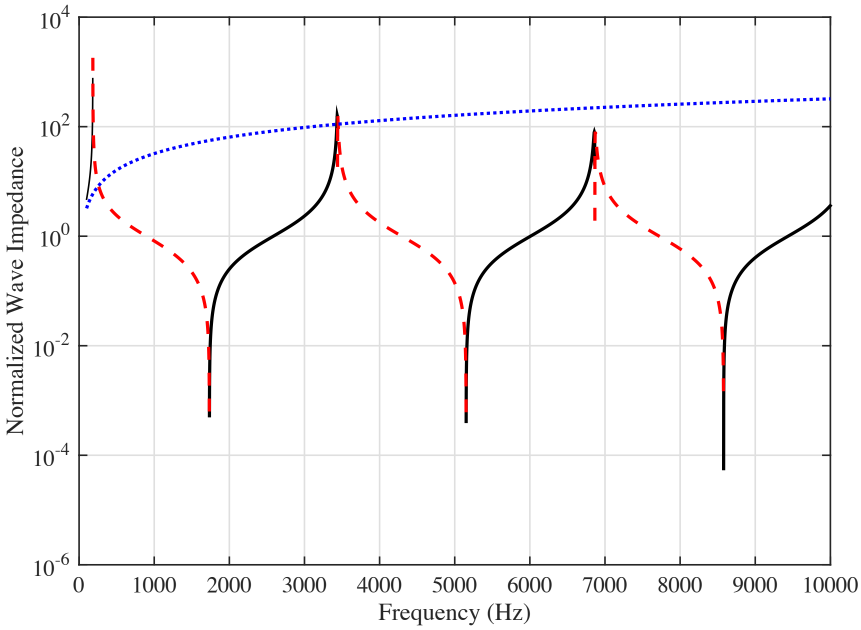


Figure 8: The second term (dot) and the third term (solid for positive value and dash for negative value) in Eq. (12) as a function of frequency.

4.1 A hole fully occupied by an acoustic seal

The TLs for the hole with an acoustic seal, placed at the two ends of the hole, of different surface densities, i.e. 2.2 kg/m^2, 5.5kg/m^2 and 11kg/m^2 are depicted in Fig. 7. Also shown, for comparison, is the TL for the hole without a seal. An observation over the curves shows the effect of the acoustic seal on the sound transmission in two distinguished frequency regions. Above 400Hz, a significant improvement to the TL is found over a broad frequency region, indicating the beneficial effect of the acoustic seal on the sound insulation performance, and a further enhancement of the TL is obtained as the surface density increases. Contrary to the advantageous effect, the use of the acoustic seal also causes sharp dips, and the locations of the dips are found be independent of the surface density, occurring approximately at $k_0d_3=n\pi$, where n is an integer.

The involvement of the acoustic seal in the hole is to modify the wave impedance through the brought by the mass layers. By virtue of the large surface density of an acoustic seal in practice and for frequencies that are not too low, the two terms $j\omega m_{23}$ and $j\omega m_{34}$ in Eq. (4) would possess values much greater than the radiation impedances $Z_{r,5}$ and the characteristics impedance of air Z_0. In this situation, an approximate expression for the wave impedance, after some algebra, could be obtained as

$$Z_c \approx Z_{r,1} + j\omega m_{23} - jZ_0 \cot(kd_3), \tag{11}$$

which holds for frequencies at which the tangent terms are away from reaching infinity. Figure 8 compares the second and the third terms of Eq. (11) as a function of frequency. The dominant effect of the mass layer m_{23} in the wave impedance is evident in the off-resonance frequency regions. Also, the cotangent form of variation of the third term could readily be seen. Referring to the third term, the mass layer m_{34}, which is located at the exit of the hole, behaves like a rigid termination to the hole. For frequencies in this region, the hole filed with an acoustic seal is analogous to a system consisting of a vibrating piston backed by an air cavity. Energy extracts by the hole is reflected back when encountering the rigid termination, resulting in lower net energy transmission.

The system exhibits a different acoustic behavior when the tangent terms in Eq. (11) equal zero. In this situation, the wave impedance could be written as

$$Z_c = Z_{r,1} + j\omega m_{23} + j\omega m_{34} + Z_{r,5}, \tag{12}$$

which is the sum of the radiation impedance and the impedance of the mass layers at the two ends of the hole. The above equation implies that, when $k_0 d_3 = n\pi$, the system is equivalent to a fluid-loaded mass layers in the absent of the air stiffness effect provided by the air channel. At these frequencies, the mass layers vibrate together with the air in between as a whole system. Below 400Hz, a profound dip dominates in the frequency region and the dip frequency varies with respect to the surface density. The improvement to TL in this frequency region is constrained due to the presence of the dip. An explicit expression for the first resonance frequency is not straightforward. As a rule of thumb, increasing the surface density would shift the dip to lower frequencies.

5. Conclusions

This paper studies the transmission of sound through an empty hole and that with an acoustic seal in a wall of finite thickness. An explanation is given to the negative value occurring in the oscillatory TL curve. This is because that the energy is extracted to the hole from a region much larger than the physical dimension of the hole itself. As a result, the nominal incident power used in the definition of transmission coefficient fails in quantifying the amount of the energy injected into the hole. The oscillation feature of the TL curve is attributed to the finite thickness of the hole and the dips correspond to the system resonance frequencies. At the off-resonance frequencies, the way in which energy flows into the hole depends upon whether the coupled system is stiffness- or mass- like.

Two regions could be identified in the TL curve of the hole with an acoustic seal. In the low-frequency region, sound is transmitted through the hole in an effective manner as the system undergoes resonance, the frequency of which is determined by the hole thickness and the surface density of the mass layers. Beyond this frequency region, a simplification of the wave impedance is made to provide insightful understanding of the acoustic behavior of the system in different scenarios: at off-resonance frequencies, the system is equivalent to that of a mass layer backed by an air cavity with rigid termination. Energy extracted into the hole is reflected back by the rigid end, yielding smaller net transmitted energy and thereby larger transmission loss; around the resonance frequencies, the two mass layers at the ends of the hole vibrate as a whole, resulting in smaller transmission loss.

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