Effects of finite cavities within a finite double-leaf panel on sound transmission loss

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ABSTRACT
The paper is about modelling a lightweight double plate structure having two plates attached by a set of beams placed at a fixed distance, i.e. a periodic structure. The structure is of finite dimension and is considered to be simply supported on its four edges. The cavities formed by the beams and the plates are treated separately instead of considering a single cavity between the two plates. The plate displacements are expressed in terms of a sine series and the pressure fields inside the cavities are expressed with cosine series. The model solves for the coefficients of these sine and cosine terms. Kirchhoff’s thin elastic plate equations for two plates along with the continuity conditions at each plate and cavity connection are set as the governing equations. Orthogonality of the Fourier sine and cosine terms is implemented to solve the system of equations. Sound transmission loss is then calculated from the displacements of the radiating plate using the Rayleigh integral. Finally, the transmission losses are averaged over the incident angles. Similar model is available where panels although finite, are considered to be infinite. The transmission loss calculated using both the finite panel model and the infinite panel model agree with measurement data in the high frequency region. While in the low frequency region, the agreement is better with the finite panel model. Using this finite panel model the effect of finite dimensions of the panel is analysed. In addition, influence of the cavities to the overall sound transmission is described.

1. INTRODUCTION

The lightweight panels are widely used in the building industries. Often partition walls between rooms are made of lightweight panels. Wood is one of the materials used to construct these lightweight panels. The abundance of wood in nature is also a reason for the wide use of these panels. Moreover, these lightweight panels are easy to handle. The whole panel is often constructed in the factory and then transported to the site when necessary. However, the drawback of these types of panels is their poor acoustic characteristics. The lightweight panel gets excited with a little amount of energy and thus starts vibrating. This vibration then propagates through the panel and finally radiates sound. The aim of this paper is to understand the factors involved in the transmission of sound through the panel.

A typical lightweight panel is constructed by attaching two plates to a set of beams. The beams are usually identical and placed at a fixed interval as shown in Figure 1. In this paper this typical panel will be studied. The beams split the space between plates into number of small cavities. Often these small cavities are ignored, and models as in (Chung & Emms, 2008; Lin & Garrellick, 1977), are developed by considering the space between the plates as a single cavity. Although, in a recent paper by Brunskog (Brunskog, 2005), these small multiple cavities are considered, the panel considered was of infinite extent. However, the assumption of a single cavity is appropriate when two sets of beams are considered where each set of beams are connected to a single plate only, as in (Mosharrof, Brunskog, Ljunggren, & Ågren, 2010). In this paper, a finite panel model is developed so that these multiple cavities and the finiteness of the panel are taken into account. The finite model is verified by the measurement data given in Brunskog (Brunskog, 2005). In this paper, the effect of the finiteness of the panel in sound transmission loss, (TL) is analysed by comparing TL calculated for various panel dimensions. Although the finite model is capable of calculating the TL for panels with any dimension, only the square panels are considered here. Obviously the smaller panels will transmit sound differently than the larger panels. The boundary conditions have strong influence over a smaller panel and as the dimension increases the boundary effect gradually decreases. The goal here is to understand how the TL varies with the variation in the panel dimensions. Another goal is to find out any frequency region where the panel dimension has significant effect. In this paper, the TL calculated for various panels are also compared with the TL for an infinite panel to find at what dimension the panel can be treated as an infinite one. This way, the effect of finiteness of the panel in sound transmission is understood. The effect of number of beams or number of cavities is examined as well by offsetting the beams by half the cavity length.
The effect of the multiple cavities is understood by comparing the TL, calculated for two cases: 1) when the multiple cavities are considered, and 2) when a single cavity is considered. The TL is calculated for both these cases keeping the parameter values same. Comparison between the TL calculated for the above mentioned cases are done for four different panels. The goal is to identify the frequency range and/or the panel dimension at which the effect of the multiple cavity is unavoidable.

Figure 1: Cross section of double-leaf floor panel

The transmission of sound through the panel is dependent on the material properties of the plates and beams. A stiffer plate or beam will transmit sound differently than a less stiff plates or beams. In addition to these material properties, there are some geometric parameters, such as spacing between the beams or depth of the cavity. These geometric parameters influence the transmission of the sound as well. The transmission will change if the cavity depth/spacing between the cavities is changed and same is true for all the parameters. The overall sound transmission is the result of the cumulative effect of all these parameters. Understanding of the relative influence of each parameters is crucial for making an optimum design of the panel. In this paper the relative influence of each of these parameters on the overall sound transmission is examined. There exist other works concerning parametric study of similar structures. Garrelik and Lin (Lin & Garrelick, 1977) examined the relative influence of the number of frames attached to plates on sound pressure level and Brunskog (Brunskog & Hammer, 2003) made a parametric study for the infinite panel.

2. METHODOLOGY

A model is developed for the panel described above, having two plates connected by a set of beams. The whole structure is assumed to be simply supported on four edges. The brief overview on the development of the model will be given in the following section, followed by results obtained and the discussions. Using the developed model, the transmission loss is calculated for different values of the parameters and thus the influence of the parameters are understood. The parameters considered here are material parameters, such as density/mass of the plates, bending stiffness of the plates and the cavity depth. Each of these parameters are assigned with a default value as in the next paragraph. Any parameter is varied by multiplying the factors [0.5 1 1.5 2] with its default value and the transmission loss is calculated for all these different values of the parameter. During this process of calculating the transmission loss and varying any parameter, all other parameters are kept constant and are assigned with their default values. This process is repeated for all parameters.

The material data for calculation is taken from a recent paper (Brunskog, 2005), where the floor dimension of the test structure is not mentioned. Here, the panel dimension is set as a square one with 3m on each side to best fit the experimental data. The mass per unit area and the loss factor for both upper and lower plate are considered as 10.9 kg/m² and 0.06 respectively. While bending stiffness for upper and lower plate are taken as 520 N/m² and 521 N/m² respectively. The beams are 0.045 m thick having density 550 kg/m³, Young’s modulus 9.8 Gpa and loss factor 0.03. Beams are placed 0.6m apart and cavities are set 0.095 m deep. The spring constant of the springs considered for coupling the plates and beams, is set to 1.10\(^{-10}\) N/m². The speed of sound in air is 340 m/sec and the density of air is 1.29 kg/m³. Similar to (Brunskog, 2005), damping factors for surrounding air and the air trapped inside the cavities are taken as 1.10\(^{-8}\) and 1.10\(^{-3}\) respectively.

3. DEVELOPMENT OF THE MODEL

The panel considered here is exactly same as the one considered in (Brunskog, 2005), except that a finite panel is modelled here. Here, the finiteness of the panel is considered as well as the multiple cavities that are formed between the plates and the beams. Similar to the previous models (Brunskog, 2005; Chung & Emms, 2008), the Kirchhoff’s thin plate equations for two plates are considered here as two governing equations. The plate
equations are written as
\[
D_i \left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) w_i - m_i \omega^2 w_i = \text{Forces}
\]
where \( D_i \), \( w_i \), and \( m_i \) are the mass per unit length, bending stiffness and the displacement of the plates where \( i = 1,2 \) corresponds to upper and lower plate respectively. The bending stiffness is defined as, 
\[
D_i = E_i h_i^3 / 12 (1 - \nu_i^2)
\]
where \( E \) is the Young’s modulus of a plate, \( h \) is plate thickness, \( \nu \) is the Poisson’s ratio. Forces acting on the plates are the reactions from the beams and the pressure from the cavities. In addition to these forces, excitation is given to the upper plate in the form of a plane wave as, 
\[
p_i(x,y,z) = \rho_0 e^{i (k_x x + k_y y + k_z z - \omega t)}
\]
where \( k_x = k \sin \theta_1 \cos \phi_1 \), \( k_y = k \sin \theta_1 \sin \phi_1 \) and \( k_z = k \cos \theta_1 \) are the wave numbers in the \( x \), \( y \) and \( z \) direction respectively, and where \( k \) is the wave number of sound in air. The incident wave will be reflected back with the same angle as the reflected wave. Thus the pressure acting on the upper plate due to this excitation pressure is the sum of the incident and the reflected wave.

Continuity conditions at the solid and fluid connections for each cavity are set as the governing equations as well. The air inside the cavities interact with the upper plate and the lower plate. Thus for the \( j^\text{th} \) cavity the continuity equations, as in (Brunskog, 2005), are
\[
\frac{\partial p_{j,c}^{z,0}}{\partial x} \bigg|_{x=0} = \omega^2 \rho_0 w_i \theta (y, j, l, (j_c + 1)l),
\]
\[
\frac{\partial p_{j,c}^{z,d}}{\partial x} \bigg|_{x=d} = \omega^2 \rho_0 w_2 \theta (y, j, l, (j_c + 1)l),
\]
where \( p_{j,c}^{z} \) is the pressure of air inside the \( j^\text{th} \) cavity, for \( j_c = 0,1,2,\ldots \) and \( \rho_0 \) is the density of the air. The function \( \theta \) is defined as
\[
\theta (y, j, l, (j_c + 1)l) = \begin{cases} 1 & j_c l \leq y \leq (j_c + 1)l, \\ 0 & \text{Elsewhere} \end{cases}
\]

3.1 Fourier series method:

The system of equations is solved by using the Fourier series expansion method (Chung & Emms, 2008). The plate displacements are expressed using the two dimensional Fourier sine series, which states,
\[
w_i = \sum_{l=1}^{L} C_{il} \phi_l(x) \Psi_l(y),
\]
where \( C_{il} \) is the set of the Fourier coefficients that need to be determined and \( i = 1,2 \) corresponds to the upper and lower plate respectively. With the sine series the simply supported boundary condition has been satisfied.

Pressure inside each cavity is defined as a two dimensional Fourier cosine series in \( x \) and \( y \) direction. The \( z \) component of the pressure field in each cavity is the combination of two waves reflecting back and forth from two plates. The pressure field in each cavity follows the Helmholtz’s equation. After solving the Helmholtz’s equation, the final form of the pressure field is derived, as done in (Brunskog, 2005; Chung & Emms, 2008). Here the pressure field in each cavity is derived separately and for the \( j^\text{th} \) cavity the pressure field is expressed as
\[
p_{j,c}^{z} = \sum_{u,v} (T_{u,v}^{1/j} e^{y_{u,v}' + \gamma_{u,v}' t} + T_{u,v}^{2/J} e^{-y_{u,v}' - \gamma_{u,v}' t}) a_{u,v}(x) \beta_{u,v}(y),
\]
where \( y_{u,v}' = \sqrt{k_{u,v}^2 + k_{u,v}' - k^2} \) is the propagation constant, and \( T_{u,v}^{1/j} \) and \( T_{u,v}^{2/J} \) are the sets of two coefficients to be determined. These sets of coefficients are different for different cavities. However, the number of terms in both these sine and cosine series is limited to a finite value assuring that both the series converge.

3.2 Calculating the Force terms

The force terms in equation (1) are the total air pressure from the cavities and the total reaction from the beams. The brief calculation of the force terms is given below.

Total cavity pressures acting on the plates are derived by summing cavity pressure given in equation (6) for the total number of cavities. Therefore, the pressure acting on the upper and the lower plates are,
\[
p_{1}^{z} = \sum_{j=0}^{J} p_{j,c}^{z}(x, y, 0),
\]
\[
p_{2}^{z} = \sum_{j=0}^{J} p_{j,c}^{z}(x, y, d),
\]
Respectively.

Each beam exerts reaction forces to the plates and The Euler beam equation is considered for the beams. The connection between plates and beams is considered to be a spring type line connection as in (Brunskog, 2005). The Euler beam equation is considered here, and the equation for these set of forces are given in (Brunskog, 2005), are
\[ \begin{bmatrix} F_1 \cr F_2 \end{bmatrix} = \begin{bmatrix} G + K & K \\ K & K \end{bmatrix} \begin{bmatrix} w_i(x, j) \\ -w_i(x, j) \end{bmatrix}, \]

where \( F_1 \) and \( F_2 \) are the force acting to the upper and lower plate respectively by the \( j \)th beam, and \( j = 1, 2, \ldots J \). \( K \) is the stiffness of the spring considered for coupling the plates and the beams. The differential beam operator \( G \), derived from the Euler beam equation, is defined as \( G = \left( E_b l_b \frac{d^4}{dx^4} - \rho_b A_b \omega^2 \right) \). \( E_b \), \( l_b \), \( \rho_b \), \( A_b \) are the Young's modulus, moment of inertia, density and cross-sectional area of beams respectively. Now, total reaction force acting to the plates will be the sum of all these reactions from the individual beams. Therefore, total reaction from the beams acting to the upper and lower plates are \( F_1 = \Sigma_1 F_1 \delta(y - jl) \) and \( F_2 = \Sigma_1 F_2 \delta(y - jl) \), respectively.

### 3.3 Solving the system of equations

Inserting the force terms into the governing equations (12), (14) and (15) and making use of the orthogonality of the Fourier sine and cosine terms, a system of linear equation is formed and given in the matrix notation as

\[ [M_{\text{Coefficient}}] \begin{bmatrix} C_1 \\ C_2 \\ \cdots \\ C_J \end{bmatrix} = \begin{bmatrix} P_{uu} \\ 0 \\ 0 \\ 0 \end{bmatrix}, \]

where

\[ \begin{align*}
C_1 &= \begin{bmatrix} C_{11} \\ C_{12} \\ \vdots \\ C_{1J} \end{bmatrix}, \\
C_2 &= \begin{bmatrix} C_{21} \\ C_{22} \\ \vdots \\ C_{2J} \end{bmatrix}, \\
\cdots \\
C_J &= \begin{bmatrix} C_{J1} \\ C_{J2} \\ \vdots \\ C_{JJ} \end{bmatrix},
\end{align*} \]

\[ \vec{T}_1 = \begin{bmatrix} T_1^{1.0} \\ T_1^{1.1} \\ \vdots \\ T_1^{1.2} \end{bmatrix}, \quad \vec{T}_2 = \begin{bmatrix} T_2^{1.0} \\ T_2^{1.1} \\ \vdots \\ T_2^{1.2} \end{bmatrix}, \quad \text{and} \quad \vec{P}_{uu} = \begin{bmatrix} 2p_1 \int_0^1 e^{-ikx} \Phi_1(x) dx \int_0^\beta e^{-iky} \Psi_1(y) dy \\ 2p_1 \int_0^1 e^{-ikx} \Phi_1(x) dx \int_0^\beta e^{-iky} \Psi_2(y) dy \\ \vdots \\ 2p_1 \int_0^1 e^{-ikx} \Phi_J(x) dx \int_0^\beta e^{-iky} \Psi_J(y) dy \end{bmatrix}. \]

The \( M_{\text{Coefficient}} \) is the matrix formed by the coefficients of \( C_1, C_2, \vec{T}_1 \), and \( \vec{T}_2 \). Solving for this system of equations the unknown Fourier coefficients are determined.

### 3.4 Calculating the transmission loss

Having the vibration profile of the radiating plate, it is possible to calculate the amount of sound transmitted. The transmission loss as given in [Cremer, Heckl, & Ungar, 1988], is defined by

\[ TL = 10 \log \left( \frac{1}{\tau} \right), \]

where \( \tau \) is the transmission coefficient. The transmission coefficient is defined as the ratio of the transmitted power to the incident power as,

\[ \tau = \frac{\int_0^{\beta} \int_0^{\theta_{\text{off}}^A} \sin \phi \cos \phi \rho d\theta d\phi}{\int_0^{\theta_{\text{off}}^A} \sin \phi \cos \phi \rho d\theta d\phi} \quad (12) \]

For finite panels the radiating area is of great importance. In fact, the transmission loss is defined for all possible incident angles \( (\theta_i, \phi_i) \). Therefore, the transmission loss is integrated for all angles and then averaged using equation (12) as used in [Brunskog, 2005]. The transmission loss is then averaged to one-third octave frequency bands.

When the incident angle is 90°, i.e. the incident wave is parallel to the incident plate, no incidence take place. This results in an infinite transmission loss in the calculation. Therefore, the upper limit of the incident angle is cut off at a value smaller than 90°. The value for this cut off angle is chosen so that the calculated result best fit with the measured ones and in this paper it is 78°.

### 4. RESULT AND DISCUSSION

In this section, the effect of finiteness of the panel is studied by calculating sound transmission loss for different panel dimension and comparing them with the transmission loss calculated for the infinite panel. In addition to this, splitting up the single large cavity into number of smaller cavities is also examined. The transmission loss is calculated for various panel dimensions considering the multiple cavities and without considering the multiple cavities. This way it is possible to understand at which dimension and at which frequency range the effect of these cavities are significant. Finally, the parametric analysis of a 3 m² panel is conducted.
4.1 Effect of finite size

Figure 2 shows comparison between the transmission losses calculated at four different square panels along with the transmission loss calculated assuming an infinite panel. The side lengths of the square panels considered are 1.2m, 1.8m, 2.4m and 3m. In each case, length of all the cavities is assumed to be 0.6m. From the figure it seems that the size of the panel influences the sound transmission loss mainly in the high frequency region. Moreover, as the panel dimension increases, the transmission loss also increases and tends to reach an ultimate point. Beyond a panel size, increasing the size of the panel does not seem to affect the TL much. Thus a little difference is noticed in the high frequency region, between the 2.4 m² panel, the 3 m² panel and the infinite panel. In Figure 3 as well, a little difference is seen in the high frequency region between the TL corresponding to a large finite panel and an infinite panel. On the other hand, as the panel dimension decreases the disagreement between finite and infinite model becomes clear. Because for a small panel, the boundary conditions and the geometry of the panel becomes dominant. Therefore, for the small size panels, the finite panel model must be used for calculating the sound transmission loss.

Figure 2: Transmission loss calculated for finite panels with various dimensions when cavities are of same sizes.

Figure 3: Transmission loss comparison; finite panel model, infinite panel model and measurement data.
Comparison between the transmission losses calculated from the finite model for 3 and 2.4 m² panels, from the infinite model and the measurement data are shown in Figure 3. At a high frequency region, i.e. above 1600 Hz, both the transmission loss calculated by finite and infinite models is almost unchanged. This probably is because, the boundary conditions do not affect the high frequency vibration much. Thus the finite panel model and infinite panel model don’t make much difference in the high frequency region. However, there are disagreements in the low frequency region. At low frequency region, the system is influenced by the boundary conditions and the geometry of the panel. It also depends on the receiving chamber. Below 315 Hz the finite model predicts the transmission loss more accurately, except that there is an unusual jump at 160 Hz. Between 315 Hz to 1.6 kHz, the infinite model has better results. For the finite model the panel dimension is of great importance and in (Brunskog, 2005), the dimension of the test panel is not mentioned. With the exact dimensions, the disagreement is expected to be reduced.

In the finite panel model, the detail of a real panel can be exactly replicated. Any inhomogeneity in the panel can be adjusted in the finite model. For example, length of the cavities at the edges are sometimes different from length of the cavities that are at the middle. This modification can easily be modified in the finite model. Transmission loss is calculated by offsetting the beams by 0.3m as compared to 3 m² panel shown in Figure 3. This transmission loss along with the ones in Figure 3 is compared in Figure 4. As seen in the figure, changing the beam location has a considerable impact in the sound transmission loss. With this modification, the prediction in the high frequency region is much accurate. This indicates that the exact location of the beam is of high importance to make accurate prediction of the sound transmission.

![Figure 4: Transmission loss comparison; finite panel model by offsetting the beams, infinite panel model and measurement data](image)

### 4.2 Effect of multiple cavities

The comparison between the transmission loss calculated with and without considering the cavities are presented in Figure 5. The comparison between the TL calculated for four different panels is shown in the figure. As seen in Figure 5, splitting the cavities does not seem to affect the smaller panels. As the size of the panel increases cavities begin to have their effects. The possible explanation is that for smaller panels, the edges are close to each other and the boundary condition at the edges is the dominant constraint for the cavities. The beams at the middle do not offer any significant constraint to the cavity compare to the constraint offered by the boundaries. Moreover, the beams are placed in one direction only and along the x axis it is like a single cavity. As the dimension of the panel increases, the edges start getting apart and thus the effect of the boundaries at edges starts getting weaker. Thus for the larger panels, the effect of beams, i.e. splitting up the cavities are more significant.
4.3 Parametric analysis

The analysis is done for a 3 m$^2$ panel. The results are presented and discussed in this section.

Figure 5: Transmission loss calculated for different finite panels with and without considering multiple cavities.

Figure 6: Transmission loss calculated for different upper plate mass
Figure 7: Transmission loss calculated for different upper plate mass considering single cavity and multiple cavities. Single cavity ( ), multiple cavities ( ).

Figure 6 shows the comparison between the transmission losses calculated at four different upper plate mass per unit areas $m_1$, when multiple cavity is considered. The overall trend of the graphs is that the sound transmission loss increases with the increase of the plate mass until 1.25 kHz. For $m_1 = 10.9 \text{ kg/m}^2$, a peak is noticed around 1.6 kHz, and for $m_1 = 2 \times 10.9 \text{ kg/m}^2$, this peak is shifted leftward to 1.25 kHz. Because of this shifting of the peaks, the graphs lose their trend around 1.6 kHz. Nevertheless, after these peaks, i.e. beyond 2.5 kHz, the same trend as before is noticed, that transmission loss increases with the increase of the plate mass.

Figure 8: Transmission loss calculated for different upper plate stiffness $D_1$.

Figure 7 compares the TL calculated by considering the multiple cavities and without considering the cavities for four different masses. From the figure it seems that the splitting of the cavities has large effects on the lighter panels, and as the panel gets heavier the effects get reduced.
Figure 9 shows the comparison between the TL calculated at four different lower plate stiffness $D_2$. It is hard to find any pattern among the graphs. The peaks occur at different frequencies for different lower plate stiffness and the peaks are scattered around all frequency range. Varying the lower plate stiffness does not seem to have much impact on increasing or decreasing the transmission loss. The Peaks arising at different frequencies probably correspond to the structural resonances.

The transmission loss for a $3 \text{ m}^2$ panel at different cavity depths $d$, is calculated in Figure 10. The overall trend at the high frequency region is that the transmission loss increases with the increase of the cavity depth.
Although, the graph corresponding to 0.19m deep cavity shows different trend than others after 1.6 kHz, this difference is because of the shifting of the peak. On the other hand, the low frequency region almost remains the same as the cavity depth varies. However, the cavity depth has a direct relationship with mass-air-mass resonance frequency, usually occurs at low frequency region and in this case those resonances are outside the frequency range in the graph.

5. CONCLUSIONS

The finiteness of the panel has large effects in the sound transmission loss for smaller panels. For higher frequencies, as the panel gets larger the effect gets weaker and beyond a certain dimension the size effect is negligible. Therefore, for a considerably large panel (3 m for this case), the finite and infinite model does not make any different in the high frequency region. However, for a smaller panel, the finite panel model must be used. Moreover, the finite panel can be modified for little variations in design, such as cavities with different lengths. For accurate prediction, incorporating the details of the panel is crucial.

The effect of the multiple small cavities is significant in the sound transmission through a large panel. The effect is mostly in the low frequency region.

Sound transmission loss increases with the increase of upper and/or lower plate mass. As the panel gets heavy, the effect of multiple cavities seems to gradually disappear in the high frequency region, while in the low frequency region, the effect is still there. The stiffness of the upper plate mainly affects the high frequency region, where sound transmission loss decreases with the increase of the upper plate stiffness. The lower plate stiffness has very little to do with changing the level of transmission loss. The lower plate stiffness mainly impacts on the structural resonances of the panel. Finally, the sound transmission loss increases with the increase of the cavity depth in the high frequency region.

REFERENCES


