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Analysis of multilayered piezoelectric cylinders with noncircular cross-section

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ABSTRACT

The problem of ultraacoustic wave propagation in piezoceramic cylinders remain an interesting one since such materials are widely used in acoustoelectronics and ultrasonic nondestructive evaluation. In the current paper we study the wave propagation in piezoceramic multilayered cylindrical waveguides with noncircular cross-section. Infinite transversely isotropic cylinders considered here have circular or hollow cross-sections with sector cut of any angular measure arbitrary boundary conditions on surfaces. The method is based on exact analytical integration of wave equations of linear electroelastic medium by using wave potentials. Dispersion functions are obtained from boundary conditions in an analytical form of functional determinants and numerical analysis is carried out to illustrate the approach. The effect of various mechanical parameters is studied and the potential applications are discussed. Results are compared with those published earlier in order to check up the accuracy of the proposed approach, which is found to be very accurate and efficient.

INTRODUCTION

Piezocomposite materials are widely used as sensors and actuators in smart structures. In recent years new technological requirements initiated active theoretical studies on ultraacoustic wave propagation in multilayered piezoelectric structures, which, as appeared, often have characteristics different from those of homogeneous waveguides [1, 2]. Multilayered piezoelectric cylinders have also been intensively investigated in order to improve the efficiency of cylindrical devices and to expand their applications.

The problem has been studied from different angles. Some authors focused on analytical methods for wave propagation in two-layer cylindrical waveguides consisting of piezoelectric and metallic layers [3], while other studied multilayered cylinders with piezoelectric layers of different polarization [4, 5] or both static and dynamic behavior of laminated composite cylindrical shells with piezoelectric layers [6]. A large amount of works was devoted to surface waves in piezoelectric cylinders [7–10]. Many techniques involve Hamilton's principle, for example, the problem of wave propagation in the piezoelectric layered section-varying bar based on the matrix formulation of the principle [11], analytical–numerical study on the characteristics of elastic waves in functionally graded piezoelectric cylinders [12] and the free-vibration problem of multilayered shells including piezoelectric layers [13]. Other works in this direction concern radial vibrations of thin two-layer piezo/metal ring [14], wave propagation in the two-layer cylindrical shells using shell bending model under Love's shear-rigidity assumption [15] and end reflection phenomenon in semi-infinitely layered piezoelectric cylinders [16].

Waveguides studied in the above references have classical circular and hollow cross-sections. In this paper we focus on non-axisymmetric waves propagation in multilayered piezoelectric cylinders of sector cross-section. A theoretical analysis is performed using the exact equations of piezoelectricity in quasistatic approximation. The symmetry of polarized piezoceramic allows us to obtain precise solutions of the problem. Since the direction of polarization is longitudinal and the material tensors have the symmetry of class 6mm, the method could be applied to other transversely isotropic materials. Numerical results are provided for the cylinders made of two piezoelectric layers, although the method is not limited only to this case and also could be applied for waveguides with any number of layers (including metallic ones). Results for multilayered cylinders are compared with data obtained for single-layered waveguides and good agreement is found.

GOVERNING EQUATIONS AND BOUNDARY CONDITIONS

We consider hollow multilayered cylinders of sectorial cross-section. Fig. 1 illustrate the two-layer hollow cylinder; inner radius is denoted by R_0, thicknesses of inner and outer layers are denoted by h_1 and h_2, respectively, and angular measure of the waveguide is 2α. Since in general case the waveguide may consist of piezoelectric and non-piezoelectric layers we need to describe the constitutive relationships for both of them. In the following expressions the superscripts ''P'' and ''E'' denote the quantities for piezoelectric and elastic layers, respectively. The constitutive relationships for piezoceramic material of class 6mm are

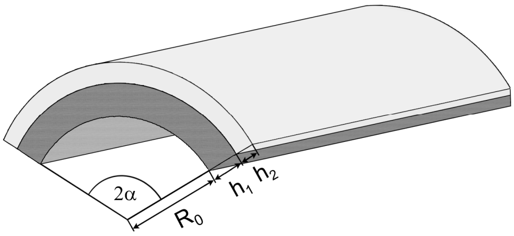

Figure 1. Cross-section of a two-layer hollow sector cylinder

$$\sigma_{rr}^{(P)} = c_{11}S_{rr}^{(P)} + c_{12}S_{\theta\theta}^{(P)} + c_{13}S_{zz}^{(P)} - e_{13}E_z^{(P)},$$
$$\sigma_{\theta\theta}^{(P)} = c_{12}S_{rr}^{(P)} + c_{11}S_{\theta\theta}^{(P)} + c_{13}S_{zz}^{(P)} - e_{13}E_z^{(P)},$$
$$\sigma_{zz}^{(P)} = c_{13}S_{rr}^{(P)} + c_{13}S_{\theta\theta}^{(P)} + c_{33}S_{zz}^{(P)} - e_{13}E_z^{(P)},$$
$$\sigma_{r\theta}^{(P)} = 2c_{66}S_{r\theta}^{(P)},$$
$$\sigma_{rz}^{(P)} = 2c_{44}S_{rz}^{(P)} - e_{15}E_r^{(P)},$$
$$\sigma_{\theta z}^{(P)} = 2c_{44}S_{\theta z}^{(P)} - e_{15}E_\theta^{(P)},$$
$$D_r^{(P)} = 2e_{15}S_{rz}^{(P)} + \varepsilon_{11}E_r^{(P)},$$
$$D_\theta^{(P)} = 2e_{15}S_{\theta z}^{(P)} + \varepsilon_{11}E_\theta^{(P)},$$
$$D_z^{(P)} = e_{13}S_{rr}^{(P)} + e_{13}S_{\theta\theta}^{(P)} + e_{33}S_{zz}^{(P)} + \varepsilon_{33}E_z^{(P)}. \quad (1)$$

And the constitutive relationships for transversely isotropic elastic material are

$$\sigma_{rr}^{(E)} = c_{11}S_{rr}^{(E)} + c_{12}S_{\theta\theta}^{(E)} + c_{13}S_{zz}^{(E)},$$
$$\sigma_{\theta\theta}^{(E)} = c_{12}S_{rr}^{(E)} + c_{11}S_{\theta\theta}^{(E)} + c_{13}S_{zz}^{(E)},$$
$$\sigma_{zz}^{(E)} = c_{13}S_{rr}^{(E)} + c_{13}S_{\theta\theta}^{(E)} + c_{33}S_{zz}^{(E)},$$
$$\sigma_{r\theta}^{(E)} = 2c_{66}S_{r\theta}^{(E)}, \quad \sigma_{rz}^{(E)} = 2c_{44}S_{rz}^{(E)},$$
$$\sigma_{\theta z}^{(E)} = 2c_{44}S_{\theta z}^{(E)}; \quad (2)$$

where $\sigma_{\alpha\beta}$, $S_{\alpha\beta}$ are the components of stress and strain, respectively, and E_α, D_α are the components of electric field intensity and electrical displacement in piezoelectric layer. The equations of motion are of the form

$$\sigma_{ij,j} - \rho\ddot{u}_i = 0, \quad (3)$$

where u_α are the components of displacement and ρ is the material density.

For piezoelectric layers the governing equations of the problem are completed by Gauss's law in the absence of free electric charge density

$$D_{i,i} = 0. \quad (4)$$

In the quasistatic approximation the electric field intensity can be described as the gradient of a scalar electric potential $E = -grad\varphi$, or in cylindrical coordinate system

$$E_r = -\partial_r\varphi, \quad E_\theta = -\frac{1}{r}\partial_\theta\varphi, \quad E_z = -\partial_z\varphi. \quad (5)$$

On the boundary of two piezoelectric layers the conditions of mechanical contact

$$\left(u_r^{(q)}\right)_{\Gamma_q} = \left(u_r^{(q+1)}\right)_{\Gamma_q}, \quad \left(u_\theta^{(q)}\right)_{\Gamma_q} = \left(u_\theta^{(q+1)}\right)_{\Gamma_q},$$
$$\left(u_z^{(q)}\right)_{\Gamma_q} = \left(u_z^{(q+1)}\right)_{\Gamma_q}, \quad \left(\sigma_{rr}^{(q)}\right)_{\Gamma_q} = \left(\sigma_{rr}^{(q+1)}\right)_{\Gamma_q},$$
$$\left(\sigma_{r\theta}^{(q)}\right)_{\Gamma_q} = \left(\sigma_{r\theta}^{(q+1)}\right)_{\Gamma_q}, \quad \left(\sigma_{rz}^{(q)}\right)_{\Gamma_q} = \left(\sigma_{rz}^{(q+1)}\right)_{\Gamma_q}; \quad (6)$$

and the conditions of electrical contact

$$\left(D_r^{(q)}\right)_{\Gamma_q} = \left(D_r^{(q+1)}\right)_{\Gamma_q}, \quad \left(\varphi^{(q)}\right)_{\Gamma_q} = \left(\varphi^{(q+1)}\right)_{\Gamma_q}; \quad (7)$$

are satisfied.
If the q-th layer is piezoelectric and $(q+1)$-th is conducting, then the conditions of mechanical contact (6) and the following condition:

$$(\varphi^{(q)})_{\Gamma_q} = 0; \quad (8)$$

are satisfied at the interface. On the inner and outer cylindrical surfaces several types of boundary conditions can be chosen. In this paper we consider the traction-free boundary condition, which for cylindrical surfaces has the following form

$$(\sigma_{rr})_\Gamma = (\sigma_{r\theta})_\Gamma = (\sigma_{rz})_\Gamma = 0; \quad (9)$$

and the close circuit condition for piezoelectric layers

$$(\varphi)_\Gamma = 0. \quad (10)$$

ANALYTICAL SOLUTION OF THE PROBLEM

Ultrasonic harmonic waves along the axial direction of a cylinder are represented through amplitude functions of cross-section coordinates and exponential dependence on axial coordinate and time

$$\phi(r,\theta,z,t) = \phi^{(0)}(r,\theta)\cdot e^{-i(\omega t - kz)}; \quad (11)$$

where ω is the angular frequency of waves and k is the wavenumber.

After a series of transformations similar to described in [17] amplitude components $\sigma_{ij}^{(0)}$, $u_i^{(0)}$, $D_i^{(0)}$ and $\varphi^{(0)}$ are finally expressed through infinite series with unknown amplitude coefficients A_{nj}, Bessel functions of the first and second kind of radial coordinate and sine/cosine functions of angular coordinate

$$\sigma_{rr}^{(0)} = \sum_{n=0}^{\infty}\sum_{k=1}^{2}\left[\sum_{j=1}^{3} A_{nj}\cdot\left(\beta_{1j}\cdot\left(\frac{2c_{66}\gamma_j}{r}F_{\lambda_n+1}^{(k)}(\gamma_j r) - \right.\right.\right.$$
$$\left. -c_{11}\gamma_j^2 F_{\lambda_n}^{(k)}(\gamma_j r) + \frac{2c_{66}\lambda_n(\lambda_n-1)}{r^2}F_{\lambda_n}^{(k)}(\gamma_j r)\right) +$$
$$+ikc_{13}\beta_{2j}F_{\lambda_n}^{(k)}(\gamma_j r) - ike_{31}\beta_{3j}F_{\lambda_n}^{(k)}(\gamma_j r)\Big) +$$

$$+A_{n4}\frac{2c_{66}\lambda_n}{r}\left(\frac{\lambda_n-1}{r}F^{(k)}_{\lambda_n}(\gamma_4 r)-\gamma_4 F^{(k)}_{\lambda_n+1}(\gamma_4 r)\right)\Bigg]\cos\lambda_n\theta\,,$$

…

$$u_r^{(0)}=\sum_{n=0}^{\infty}\sum_{k=1}^{2}\left[\sum_{j=1}^{3}A_{jn}\beta_{1j}\left(\frac{\lambda_n}{r}F^{(k)}_{\lambda_n}(\gamma_j r)-\gamma_j F^{(k)}_{\lambda_n+1}(\gamma_j r)\right)+\right.$$

$$\left.+A_{4n}\frac{\lambda_n}{r}F^{(k)}_{\lambda_n}(\gamma_4 r)\right]\cos\lambda_n\theta\,,\ \ldots$$

$$D_r^{(0)}=\sum_{n=0}^{\infty}\sum_{k=1}^{2}\left[\sum_{j=1}^{3}A_{jn}\left(ike_{15}\beta_{1j}+e_{15}\beta_{2j}+\varepsilon_{11}\beta_{3j}\right)\cdot\right.$$

$$\cdot\left(\frac{\lambda_n}{r}F^{(k)}_{\lambda_n}(\gamma_j r)-\gamma_j F^{(k)}_{\lambda_n+1}(\gamma_j r)\right)+$$

$$\left.+A_{4n}\frac{ike_{15}\lambda_n}{r}F^{(k)}_{\lambda_n}(\gamma_4 r)\right]\cos\lambda_n\theta\,,\ \ldots$$

$$\varphi^{(0)}=\sum_{n=0}^{\infty}\sum_{k=1}^{2}\left[\sum_{j=1}^{3}A_{jn}\beta_{3j}F^{(k)}_{\lambda_n}(\gamma_j r)\right]\cos\lambda_n\theta\,; \qquad (12)$$

where $F_n^{(1)}(r)\equiv J_n(r)$, $F_n^{(2)}(r)\equiv Y_n(r)$.

Substituting from Eqs. (12) to the boundary conditions results in the system of linear algebraic equations in the unknown wavenumber k and angular frequency Ω for each value of $n=\overline{0,\infty}$ and the dispersion equation of the problem is a non-trivial solution of this system. For the two-layer sector cylinder with both piezoelectric layers it is obtained in the form of the sixteenth order determinant being equal to zero

$$\begin{vmatrix} Q_{1,1} & \cdots & Q_{1,8} & 0 & \cdots & 0 \\ \vdots & & \vdots & \vdots & & \vdots \\ Q_{4,1} & \cdots & Q_{4,8} & 0 & \cdots & 0 \\ Q_{5,1} & \cdots & Q_{5,8} & Q_{5,9} & \cdots & Q_{5,16} \\ \vdots & & \vdots & \vdots & & \vdots \\ Q_{12,1} & \cdots & Q_{12,8} & Q_{12,9} & \cdots & Q_{12,16} \\ 0 & \cdots & 0 & Q_{13,9} & \cdots & Q_{13,16} \\ \vdots & & \vdots & \vdots & & \vdots \\ 0 & \cdots & 0 & Q_{16,9} & \cdots & Q_{16,16} \end{vmatrix}=0\,. \qquad (13)$$

NUMERICAL RESULTS

A few numerical examples for the two-layer piezoelectric cylinder are given below. $PZT\text{-}5A$ and $BaTiO_3$ ceramic with axial polarization were chosen as the material of inner and outer layers, respectively, since they have quite different material parameters. Elastic characteristics for these materials and aluminium (numerical analysis was also carried out for the piezo/metal layered cylinders with sector cross-section) were taken from [15, 18, 19].

Figs. 2 and 3 illustrate the first and second modes of the spectrum with $n=0$ for the cylinder of angular measure $\alpha=\pi/4$ and different thicknesses of the layers. Curves for the single-layered cylinders made of $PZT\text{-}5A$ and $BaTiO_3$ are illustrated in Figs. 2 and 3 by the black lines. It is easily seen that the configuration of layers have substantial effects on dispersion behavior of the waveguide and the increase of the layer's thickness leads to the position of the curves more similar to the case of a single-layered cylinder made of the same material.

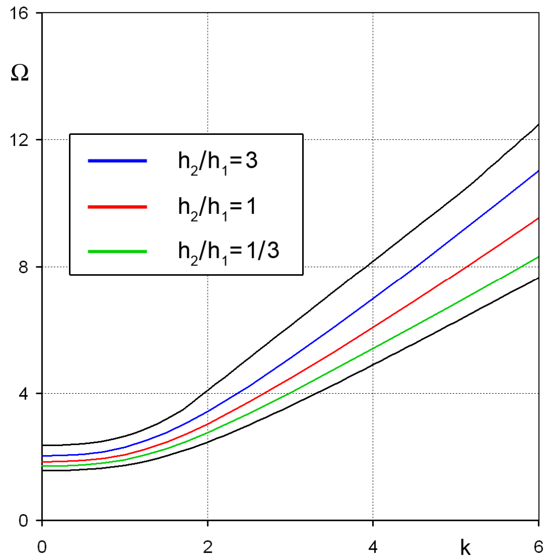

Figure 2. First modes for the two-layer $\alpha=\pi/4$ cylinders

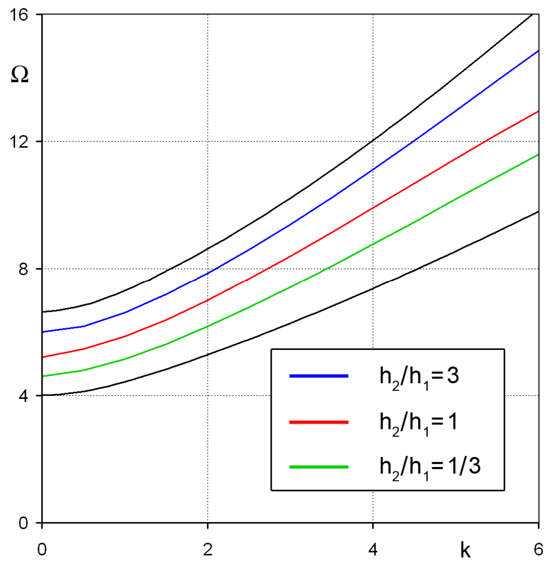

Figure 3. Second modes for the two-layer $\alpha=\pi/4$ cylinders

The first and second modes are linked in the imaginary part of the spectrum forming a loop, which in common for such waveguides. For each ratio of thicknesses this loop is situated between the loops for single-layered $PZT\text{-}5A$ (blue lines) and $BaTiO_3$ (green lines) cylinders shown in Fig. 4. Other results for the waveguides with different angular measure and inner radius show that the structure of wave spectrums strongly depends on geometric and material parameters of the layered structure.

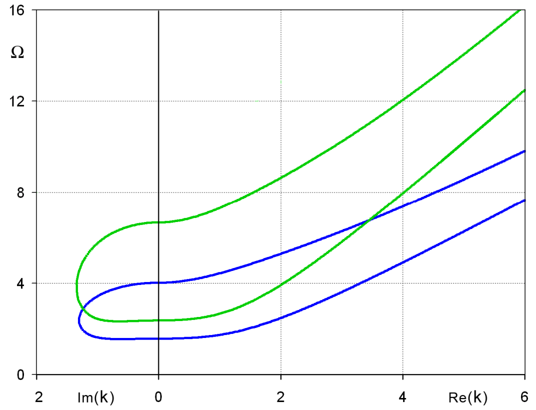

Figure 4. Two lowest modes in the single-layered $\alpha = \pi/4$ cylinders

CONCLUSIONS

In this paper the analytical method for studying ultrasonic wave propagation in laminated piezoelectric cylinders of sector cross-section have been presented. Frequency equations of the problem for the cylinder with N piezoelectric and M non-piezoelectric layers have the form of functional determinant of $8N + 6M$ order. Dispersion curves of the lowest modes for different waveguides are obtained and compared with the curves for single-layered cylinders showing good agreement. Numerical results allow us to understand better the influence of geometrical parameters and laminate configuration on the wave propagation in multilayered cylindrical structures.

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