An experimental study of acoustical properties of tubular tower bells

Jie Pan and Sören Bergmann

School of Mechanical Engineering, The University of Western Australia, Crawley WA, 6009, Australia.

ABSTRACT

This paper presents the measured spectra and identified pitch of thirteen tubular tower bells, which are made of thick-wall brass cylinders. Results show that bell's natural frequencies that dominating its pitch and timbre are well predicted by Flugge's formula for thin-wall cylinders. The spectra of tubular tower bells and thin-wall pipe bells are compared and their tonal differences are attributed to the differences in their modal masses and decay times. The identified pitches of the strike tones of the tubular tower bells are analysed against the frequency distribution and amplitude ratios of the partials in the spectra. The result supports the previous work on the virtual pitches of pipe bells and their determination by the "octave rule". Using this result, we offer an explanation to the question of why both tubular tower bells and pipe bells have similar tonal characteristics of church bells.

INTRODUCTION

Tubular tower bells are made of thick-wall brass cylinders, and often wieght under hundred of kg. They were known as early as 1850, and used as hall clocks and in bell towers in the late nineteenth century before being replaced by conventional bells. They were introduced in England by John Harrington, who patented his design and stiffening device for tuning of the bells [1, 2].

Shown in Figure 1 are twelve tubular tower bells located in Perth's Swan Bell Tower, and bell #13 in the Acoustical Laboratory of the University of Western Australia. The thickness-to-mean diameter ratios of these bells are much larger than that of the pipe bells used in orchestras. Typically, the thickness-to-mean diameter ratio δ of pipe bells is between 0.038 and 0.06, while that of tubular tower bells is about 0.2. Previous work on tonal quality of pipe bells is scarce. The authors have not found any literature on the acoustics of thick wall tubular tower bells.

(a)

(b)

Figure 1 (a) Tubular tower bells located in Perth Swan Bell tower. (b) the 13th tubular tower bell at the University of Western Australia.

Previous work on pipe bells by Hueber [3] and Rossing [4] reported that the "beam-like" bending vibrations of circular cylinders are not harmonic. Rather they produce sound impression from their "nearly harmonious" partials. Hueber's measurement indicated that the fundamental frequency (pitch) of the bell's strike tone was not present physically rather it is formed psychologically. The ratio of the fourth and third natural frequencies of the bending modes shows similar characteristics of a church bell sound if it is at an interval of one-sixth and if the mean radius to length ratio ε of the pipe bell is less than 0.019. The ratio for the tubular tower bells under test is between 0.0162 and 0.0256.

Fletcher and Rossing [5] described the application of Flugge's formula to pipe bells and discussed the model of the virtual pitch.

The questions for understanding the acoustical properties of tubular tower bells are: (1) whether the thick wall feature of the bells requires a specific attention of using the three-dimensional modeling for determining the natural frequencies of the bells? (2) how tonal quality of thick-wall bells differ from that of thin-wall pipe bells and why? (3) if the strike tones of tubular tower bells are also characterized by virtual pitches? and (4) why both tubular tower bells and pipe bells have similar tonal characteristics to church bells?

The answers to those questions not only improve the understanding of bell acoustics but also have application for effective tuning of the bells and selection of appropriate location for suspending the bells.

In this paper, we compare the measured natural frequencies of thirteen tubular tower bells with the predicted values using Flugge's approximate formula [6, 7]. We find that the natural frequencies that dominating the sound impressions of the thick-wall tubular tower bells belong to the "beam-like" bending modes of the cylinders and can be predicted by Flugge's formula with adequate accuracy. As a result, the three-dimensional modeling of the bells becomes unnecessary.

The measured frequency response functions (defined by the ratio of acceleration at one end of bell and impact force at the other end) of the pipe bells and tubular tower bells are then compared. The analysis of their differences shed some lights on the understanding of the bell's tonal quality in terms of the bells physical properties such as its modal mass and modal quality factors. The paper also briefly discusses the experimental determination of the pitch of bell's strike tones. The result confirms that the previous finding of the "octave rule" in determining the virtual pitch of the pipe bells also applies to tubular tower bells.
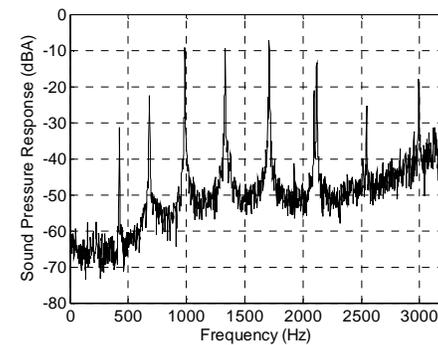
NATURAL FREQUENCIES

The dimensions of the thirteen tubular tower bells are listed in Table A1 of the appendix. Each bell is suspended by a nylon rope through two holes on the central plane of the bell, at approximately $10cm$ from the top end. The natural frequencies of each bell were obtained by measuring the acceleration response at the top end of the bell to an impact force at the bottom end. The acceleration frequency response function of the thirteenth bell is shown in Figure 2. Measurement of distributed vibration of the bells at the peak frequencies of the frequency response function demonstrated that all the peak frequencies in the frequency range of investigation correspond to the bending modes with peripheral modal number of $n = 1$. The measured natural frequencies of the thirteen bells are listed in Table A2.

As the third peak was the first observable partial of the bell in the A-weighted sound pressure response (see Figure 3), the measured peak frequencies $f_{1,3}$ with respect to the length L, outer diameter D_o and wall thickness t of the thirteen bells are compared with the predicted values using Flugge's formula [6, 7] (see Figure 4). The natural frequencies of the remaining bells are presented in terms of the frequency ratios with respect to the third natural frequency (see Figure 5).
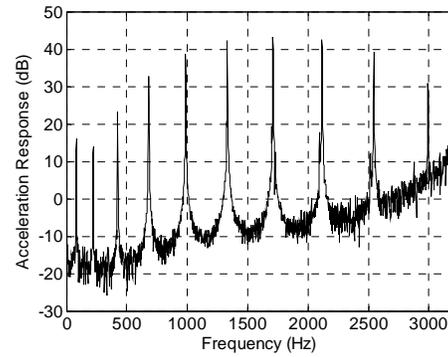
Figure 2 Acceleration frequency response of the 13th bell (measured at the top end of the bell) to an impact force at the bottom end.

Figure 3 Sound pressure frequency response of the 13th bell (measured at 1.5m away from the bell) to an impact force at the bottom end.
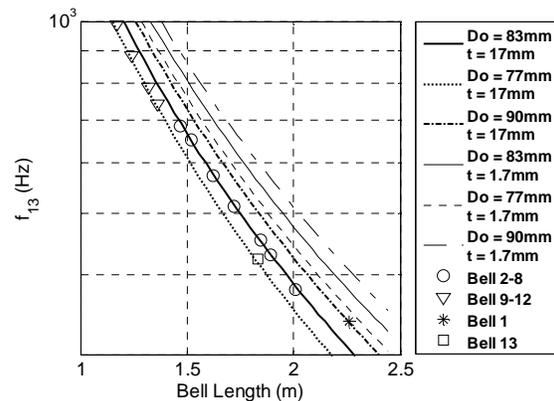

Figure 4 Measured and predicted frequencies of the third natural frequency, $f_{1,3}$, of tubular tower bells.

From Figure 4, we observe that:

(1) The measured $f_{1,3}$ of the thirteen thick-wall tubular tower bells agree very well with Flugge's formula prediction.

(2) Theoretical curves from Flugge's formula allow the design of frequency intervals between $f_{1,3}$ of different bells based on their parameters (t, D_o and L).

(3) As expected, the decrease of the bell's wall thickness increases the natural frequency.

Figure 5 shows the measured and predicted frequency ratios (with respect to $f_{1,3}$) of each bell as functions of the bell parameters. Although an excellent agreement was observed between predicted and measured frequency ratios for the $m = 1, 2$ modes, we didn't include them in the figure as they are located in the lower frequency range and contribute little to the bell's sound impression because of A-weighted feature of human hearing. We then concentrated on the ratios of the $m \geq 4$ modes of all the bells. The predicted frequency ratios in Figure 5 are for one wall thickness ($t = 17mm$) as the first twelve bells all have the same thicknesses. Three predicted ratios are presented for one bell mode and they correspond to three different outer diameters. The solid curves represent $D_o = 83mm$, dotted curves $D_o = 77mm$, and dash-dotted curves $D_o = 90mm$. The bell number is associated with the frequency ratios through the corresponding bell length in Figure 5 and length/bell number relation in Table A1.
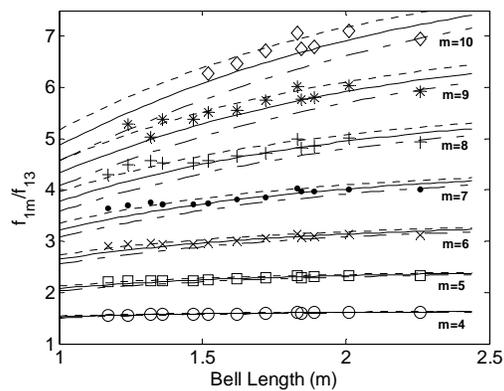
Figure 5 Natural frequency ratios of the bending modes ($m \geq 4$) of tubular tower bells with respect to $f_{1,3}$. Parameters used for prediction are $t = 17mm$, solid curves: $D_o = 83mm$, dotted curves: $D_o = 77mm$, and dash-dotted curves: $D_o = 90mm$. The measured frequency ratios (symbols) for each bell are noted by its length, and the corresponding bell number is found in Table A1.

The frequency ratios in Figure 5 provide the following observations:
(1) The measured and predicted frequency ratios agree very well for the first seven bell modes ($m \leq 7$);
(2) Frequency ratios of modes of the thick-wall bells begin to depart from the predicted value of Flugge's formula when $m = 8$. This discrepancy increases with the increase of the half-wave number m of the bending modes. Such a discrepancy indicates that the modal structure in the radial direction (along the thickness direction) of the bell may start to play important roles in determining the natural frequencies of the tubular tower bell when $m \geq 8$.
(3) The frequency ratios decrease gradually as the bell length decreases. The rate of decrease increases with m.

The most relevant discussion concerning the effect of wall thickness on the natural frequencies of hollow cylinders was found in Guicking and Boisch [8] and Gazis' [9, 10] work. Guicking and Boisch [8] indicated that when $\delta \geq 0.125$ the effect of shear deformation and rotational inertia have to be included in the calculation of the cylinder's natural frequencies. When both the peripheral mode number n and axial mode number m are small, the thin-wall Flugge-like frequency formula is a relevant approximation of thick-walled cylinders, whilst for large n, the equivalent plate frequencies are useful approximations. In the transition range of these two cases, the natural frequencies obtained are based on the assumptions that the influence of shear deformation and rotational inertia is roughly equal for both cylinders and plates. Guicking and Boisch used experimental results to support the claimed accuracy of predictions. However, the two cylinders used in their experiments had δ of 0.0425 and 0.1091, which failed to satisfy the condition of their concern for the cylinders with $\delta \geq 0.125$, where the effect of shear deformation and rotational inertia are not negligible.

Gazis [9, 10] investigated three-dimensional propagating waves in hollow circular cylinders of infinite length. His solution holds for all thickness/diameter ratios. The comparison of his dispersion curves (phase velocities as a function of frequency) with those based on a Timoshenko shell theory [11] demonstrated a very close agreement for the lowest (flexural and $n = 1$) propagating mode even for relatively thick shells (e.g. $\delta = 0.125$). We haven't found any comparison of Gazis' result with experimental data for hollow cylinders with $\delta \geq 0.125$ or with Flugge's thin shell result.

The results in Figure 5 add the following contributions to the previous work on the vibration of thick wall hollow circular cylinders:
(1) Experimental evidence showing that Flugge's thin shell result cannot accurately predict the natural frequencies of higher order modes $m \geq 8$ for cylinders with δ as large as 0.2.
(2) For bells with ε less than 0.0256, the radiated sound in the frequency range where bell's pitch and tonal quality are determined is dominated by the beam-like bending modes with $n = 1$. Modal features in the radial direction of the bell still have insignificant contribution to the natural frequencies in this frequency range.

FREQUENCY RESPONSE FUNCTIONS

If a tubular tower bell and a pipe bell produce the same pitch, what is the difference in their tonal qualities? We attempted to answer this question by comparing the measured acceleration frequency responses (with respect to a unit force excitation) of a tubular tower bell (Bell 13) and a pipe bell (shown in Figure 6) from School of Music, UWA. Both bells produce the same pitch of E_4. The dimensions of brass pipe bell are $t = 2.5mm$, $D_o = 39mm$ and $L = 1.44m$. Figure 7 shows that the first 7 peak frequencies (below $2kHz$) of the pipe bell are similar to those of the tubular tower bell. It also shows that the frequencies of the eighth - tenth modes of the pipe bell are respectively higher than that of the tubular tower bell. Such frequency differences of the higher order modes, together with the difference in the relative peak levels and modal decay times, contribute to the timbre difference of the two bells.

Figure 6 Pipe bell used for a comparison study with tubular tower bell #13.
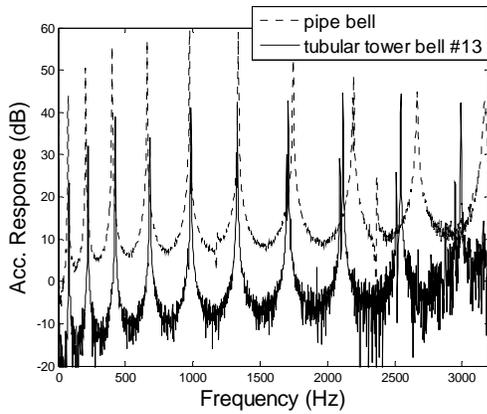
Figure 7 Acceleration frequency response functions of the tubular tower bell #13 and the pipe bell.

The natural frequencies of the two bells were predicted using Flugge's approximate formula. In Figure 8, the predicted and measured natural frequencies of the two bells clearly show the gradual departure between the natural frequencies of the same mode as mode number increases.
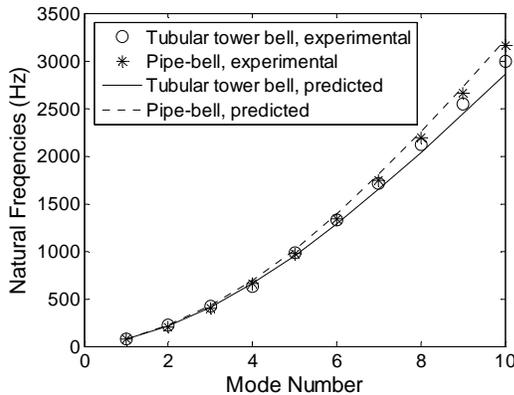
Figure 8 Measured and predicted natural frequencies of the tubular tower bell #13 and the pipe bell of the same pitch (E_4).

Significant differences in the levels of the bell's frequency responses are observed in Figure 7. The peak levels of the

pipe bell are up to $20dB$ higher than the tubular tower bell's. This indicates that the pipe bell's vibration and sound are much more sensitive to the same excitation force than those of the tubular tower bell. Tubular tower bells therefore require greater excitation impact force to produce the sound levels similar to that of pipe bells.

To gain a qualitative understanding of the difference in the peak levels, we approximate the bell's response at each natural frequency only by using the resonant mode corresponding the natural frequency, and ignoring the contribution of other modes. As a result the level difference between the $(1, m)$ modes of the tubular tower bell and the pipe bells is simplified as:

$$\Delta L_{(1,m)} \approx 20 \log_{10}(\frac{Q_{1,m}^{P} M_{1,m}^{T}}{Q_{1,m}^{T} M_{1,m}^{P}}), \qquad (1)$$

where $Q_{1,m}^{P}$ and $M_{1,m}^{P}$ are the quality factor and mass of the $(1, m)$ pipe bell mode, and $Q_{1,m}^{T}$ and $M_{1,m}^{T}$ are those of the tubular tower bell mode. If the bells are both made of the same material the modal mass ratio in Equation (1) equals their volume ratio. In this case,

$$20 \log_{10} \frac{M_{1,m}^{T}}{M_{1,m}^{P}} = 20 \log_{10}(15.6) \approx 24(dB), \quad (2)$$

which indicates that The large modal mass difference of the two bells is the dominant source for the considerable difference between the peak levels of the bells.

The measured decay times of eight modes (starting from mode $(1, 3)$) of the two bells are shown in Figure 9. Distinct differences between the modal decay times of the two bells are observed:

    (1) The decay times of $(1, 4)$ and $(1, 5)$ modes of the pipe bell are approximately twice as long as those of the tubular tower bell;

    (2) The decay times of $(1, 7)$, $(1, 8)$, $(1, 9)$ and $(1, 10)$ modes of the pipe bell are less than one-half of those of the tubular bell;

    (3) The decay times of $(1, 3)$ and $(1, 6)$ modes of the pipe bell are similar to that of the tubular bell.

Using the measured modal decay times (Figure 9) and the estimated modal mass ratio, the peak level differences of the modes $(1, 3)$ to $(1, 10)$ are predicted using Equation (2) and presented in Figure 10 together with the measured peak level difference for comparison. The comparison shows that Equation (2) can at least qualitatively explain the peak level difference between frequency responses of the two bells. The discrepancy between the measured and predicted peak level differences in Figure 10 may be due to the assumption that the effect of other non-resonant modes on the peak response is negligible. Nonetheless, Equation (2) demonstrates that the significantly larger modal mass is the main cause of the reduced sensitivity of the tubular tower bell's vibration and sound.
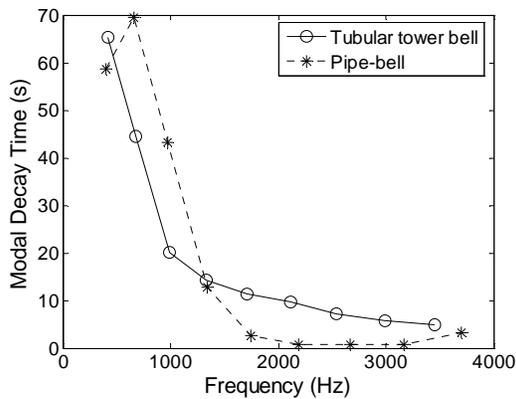
Figure 9 Measured modal decay times (starting from mode $(1,3)$) of the tubular tower bell #13 and the pipe bell.

Figure 7 also shows that the peak response of the $m > 7$ modes of the pipe bell decreases significantly, whilst that of the tubular bell does not. This evidence and the fact that the higher order modes ($m > 7$) of the pipe bells have very small modal decay time as shown in Figure 9 indicate that the sound impression of the pipe bell is mainly dominated by the $(1,3)$-$(1,7)$ modes. However the sound quality of the tubular tower bell is contributed by all the modes including the higher frequency modes with $m > 7$. The different properties of the relative peak levels and modal decay time between the two bells explains why the subjective impression of the pipe bell's pitch is easier to identify; its sound being clearer and more pleasant when compared with the tonal quality of tubular tower bells.
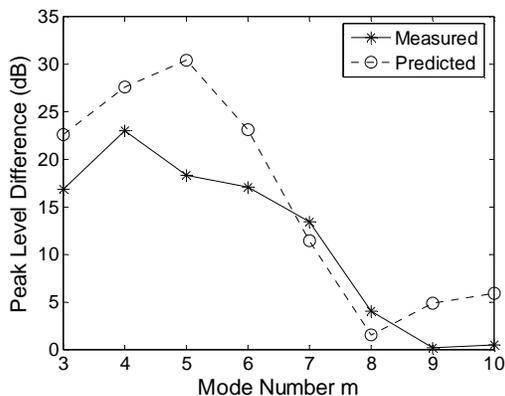
Figure 10 Level differences between the peak frequency responses of the tubular tower bell #13 and the pipe bell.

PITCH DETERMINATION

The perceived pitch of the tubular tower bell's strike tone (bell sound after an impact excitation) corresponds to a frequency which is not present in the bell's sound spectrum. Such frequency was determined by the authors and musicians producing a viola tone matching the bell tone as closely as possible, then measuring the fundamental frequency of the recorded viola sound, which was used as the physical measure of the bell's pitch.

The viola was used to reproduce the sound of the bells because it covers adequately lower frequency range where the

bell's pitch may be located. Two violin players with trained ears assisted with the viola sound production. The pitches of all the 13 tubular bells were determined and listed under f_{Viola} in Table 1.

Table 1 Bell pitch (Hz) identified subjectively using a viola and by prediction.

Bell Number	$f_{Viola}(Hz)$	$f_p(Hz)$
1	270	271.5
2	305	305.5
3	343	344.0
4	359	363.0
5	408	408.0
6	459	454.5
7	519	514.0
8	551	538.5
9	590	584.0
10	611	621.0
11	701	690.5
12	788	771.5
13	341	342.0

To correlate the pitch f_{Viola} to the frequencies of the partials in the centre of the audible range, we adopted the Rayleigh's empirical 'octave rule' [13] for the strike note of church bells and proposed that the pitch of the strike note of tubular tower bells is one octave below that of the fourth partial:

$$f_{p1} = f_{1,4} / 2 . \qquad (3)$$

Such hypothesis agrees with Hueber's experimental work on pipe bells [4].

The predicted pitches based on Equation (3) are also listed in Table 1 for comparison. Very close agreement between the measured and predicted pitches was found for all the thirteen tubular tower bells. For the first twelve bells, the pitches were compared with a viola sound in sequence. As a result, the interval between the pitch being measured and the previously measured pitch, provided some "hint" in the pitch determination. It is nearly impossible to tell the pitch of a bell (especially higher bells) without comparison to other tubular bells because all the single frequencies were heard instead of the striking tone. The pitch of the 13th bell was determined in isolation without any need to compare its pitch with other bells. Some difficulty in the pitch determination was encountered when the bell was excited at certain locations. This observation will presently be explained in further detail.

Walliser's model [16, 17], which is based on Schouten's experimental work, presents a straight forward method in determining the residue pitch of complex tones. The method includes two steps:
(1) To estimate the approximate pitch corresponding to the frequency difference between neighboring partials;
(2) To determine a subjective subharmonic of the lowest present partial, so that the pitch of this subharmonic lies as close as possible to the estimated pitch.

Terhardt [17] modified Walliser's model by pointing out that a residue pitch will always be a subharmonic of a dominant partial rather than simply the lowest partial. He also observed that the dominant frequency range of pitch identification is from 500Hz to 1500Hz. Terhardt's method is also used for the pitch identification of the tubular tower bells.

The frequencies of the fourth partial $f_{1,4}$ are located in the dominant frequency range for all the bells, even the $f_{1,4}$ of the 12th bell is at $1543 Hz$ which is near the boundary of the range. The magnitude of sound pressure at $f_{1,4}$ is either dominating, or is at least at the comparable level with that of higher partials. Furthermore, the decay time of the bell sound at this frequency is at least twice as long as that of higher partials, which means that the dominant effect of this partial will not diminish with time when compared with that of higher partials. Thus the predicted pitches for those bells also correspond with those obtained by Equation (5). In this case, the factor of 2 between $f_{1,4}$ and the virtual pitch was obtained to produce the subharmonic (the pitch) which is closest to the estimated pitch from the averaged frequency difference using $f_{1,5} - f_{1,4}$, $f_{1,6} - f_{1,5}$ and $f_{1,7} - f_{1,6}$.

The third natural frequency of bells 8 to 11 enters the dominant frequency range. For the twelfth bell, even the second natural frequency enters the range. In addition, the higher partials above $f_{1,5}$ gradually move outside the range. In these cases, the information from the frequency difference of those higher partial reduces, which increases the relative loudness of the sound at the 3rd (and 2nd for the 12th bell) natural frequency, producing some confusing impressions in the pitch determination. However, when played in sequence with the other bells, the residue pitches are audible and determinable by Equation (3) with reasonable accuracy.

The pitch determination of bell strike tones is a subject of psychoacoustics. It is felt that more detailed experiment assisted with a synthesized harmonic complex or a pure tone should be conducted in the pitch comparison test. The effect of the decay rate of each harmonics in the dominant frequency range should also be included in the pitch determination as strike notes are essentially transient signals. Nevertheless, the experimental results of pitch identification and Equation (3) are used as a guide for bells' tuning before better understanding of the physical basis of the pitch is achieved.

For ideally tuned church bells, presence of the partial of "tierce", which is a minor third ($6:5$) above the prime, characterizes the sound impression of church bells. The measured natural frequencies in Table A2 show that the ratio of the fourth and third natural frequencies ($f_{1,4} : f_{1,3}$) of the bending modes has approximately an interval of one-sixth ($f_{1,4} : f_{1,3} = 5:3$). If the pitch of the strike tone of a tubular tower bell can be determined as $f_p = f_{1,4} / 2$, (see Equation (5)) as demonstrated in this section, then the frequency ratio of one-sixth between $f_{1,4}$ and $f_{1,3}$, and the frequency relation between f_p and $f_{1,4}$ give rise to a ratio of a minor third ($f_{1,3} : f_p = 2^{3/12}$) between $f_{1,3}$ and the virtual pitch f_p. Therefore we believe that it is the minor third frequency interval between the third partial ("tierce") and the virtual pitch ("prime") of the bell that gives the tubular tower bells the similar tonal characteristics of church bells.

CONCLUSIONS

This paper has provided experimental evidences to support the following conclusions:

(1) Frequencies of the first 8 partials of the thick-wall tubular tower bells can be accurately predicted using Flugge's formula. As those frequencies cover the dominant frequency range of sound impression, the pitch and tonal quality of the bells are mainly determined by the frequencies and amplitude of those partials.

(2) Although they may produce the same pitch, the tonal quality of tubular tower bells is different from that of pipe bells. Tubular tower bells are less sensitive to impact force due to their heavy modal masses. Therefore greater impact force is required to produce the same level of the bell's sound and vibration. The higher order modes of tubular tower bells above $f_{1,5}$ have longer decay times than those of pipe bells. As a result, the pipe bells sound clearer than tubular tower bells do.

(3) Consistent with the predictions by the "octave rule" and Terhardt's model, the pitch of tubular tower bells are usually one octave lower than the frequency of the fourth partial.

ACKNOWLEDGMENT

APPENDIX

Table A1 Dimensions of tubular tower bells.

Bell No.	1	2	3	4	5	6	7
L (mm)	2260	2010	1890	1845	1720	1620	1520
D_o (mm)	90	83	83	83	83	83	83
t (mm)	17	17	17	17	17	17	17
Bell No.	8	9	10	11	12		13
L (mm)	1470	1360	1320	1240	1170		1830
D_o (mm)	83	77	77	77	77		76
t (mm)	17	17	17	17	17		17.5

Table A2 Natural frequencies of the 13 tubular tower bells, where m is the mode numbers along the axial direction.

m	B1	B2	B3	B4	B5	B6	B7
1	66.0	74.0	84.0	89.0	101.0	113.0	130.0
2	177.0	199.0	226.0	239.0	270.0	304.0	345.0
3	338.0	380.0	430.0	454.0	512.0	572.0	651.0
4	543.0	611.0	688.0	726.0	816.0	909.0	1028.0
5	787.0	884.0	991.0	1044.0	1172.0	1296.0	1458.0
6	1058.0	1190.0	1329.0	1399.0	1569.0	1725.0	1931.0
7	1355.0	1524.0	1709.0	1798.0	1975.0	2184.0	2435.0
8	1673.0	1908.0	2088.0	2192.0	2420.0	2670.0	2967.0
9	2007.0	2295.0	2498.0	2620.0	2940.0	3179.0	3588.0
10	2356.0	2698.0	2927.0	3065.0	3434.0	3702.0	4088.0
m	B8	B9	B10	B11	B12		B13
1	136.0	148.0	157.0	178.0	200.0		80.0
2	364.0	394.0	419.0	472.0	529.0		220.0
3	684.0	741.0	788.0	883.0	986.0		424.0
4	1077.0	1168.0	1242.0	1381.0	1543.0		684.0
5	1526.0	1654.0	1765.0	1968.0	2173.0		988.0
6	2018.0	2188.0	2340.0	2601.0	2863.0		1332.0
7	2544.0	2758.0	2956.0	3275.0	3593.0		1712.0
8	3100.0	3359.0	3600.0	3973.0	4243.0		2116.0
9	3678.0	3986.0	3973.0	4674.0			2548.0
10							2996.0

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