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Numerical Computations of a Recorder Fluid

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ABSTRACT

In this talk the numerical simulation of the sound spectrum and the propagation of the acoustic noise inside and around a three-dimensional recorder model is presented. The fluid inside and close to the recorder is meshed by Lagrangian tetrahedral finite elements. Complex conjugated Astley-Leis infinite elements are used to obtain results in the far field of the recorder.

When playing a recorder, the air column inside the instrument starts to oscillate due to the inserted air flow. The musician is able to influence the frequency of a note by varying the blowing pressure and therewith a fine-tuning of the sound is possible. The sound propagation in fluids with a non-uniform flow can be described by the Galbrun equation. We present the influence of the flow on the eigenfrequencies.

Furthermore, it is possible to represent the excitation mechanism for a sound propagation inside and around the recorder with quadrupole sources, which occur in the surroundings of the labium. The numerical results are compared to measurements on the recorder.

INTRODUCTION

In acoustics, the general approach for a modal analysis of a time-harmonic fluid is to solve Helmholtz equation. But when thinking of a recorder, we have a moving fluid inside the recorder due to the air flow that is inserted in the instrument by the musician. This flow profile has to be taken into consideration for the modal analysis. Galbrun equation can be used to describe the sound propagation in fluids with non-uniform flow. We present the influence of the flow on the eigenfrequencies.

In experiments, we use microphones to measure the pressure as time-dependent variable in different positions inside and around the recorder. We want to describe the excitation mechanism of the sound propagation with quadrupole sources and use the results of the measurement to verify the numeric computations.

The fluid inside and close to the recorder is described by finite elements, while infinite elements describe the far field. The Sommerfeld radiation condition ensures that only outward propagating components exist at large distance from the radiating body. As finite elements second order Lagrange tetrahedral elements are used and as infinite elements complex conjugated Astley-Leis elements [1] are implemented.

SOURCE PROBLEM

The considered boundary value problem consists of Helmholtz equation, Neumann boundary condition for the recorder boundary and Sommerfeld radiation condition. The matrix formulation of this boundary value problem is

$$(\boldsymbol{K} - ik\boldsymbol{D} - k^2\boldsymbol{M})\boldsymbol{p} = \boldsymbol{b} \ . \tag{1}$$

The pressure $\boldsymbol{p}$ consists of the complementary pressure and the particular pressure

$$\boldsymbol{p} = \boldsymbol{p}^c + \boldsymbol{p}^p \ . \tag{2}$$

The complementary pressure is also known as homogeneous solution, the solution of a source free domain. The particular pressure presents the incident pressure field. The right-hand-side vector is computed by

$$\boldsymbol{b} = -sk\boldsymbol{F}\boldsymbol{v}_a^p + (\boldsymbol{K} - ik\boldsymbol{D} - k^2\boldsymbol{M})\boldsymbol{p}^p \ , \tag{3}$$

with $s = i\rho_0 c$ and $\boldsymbol{F}$ as the boundary mass matrix [5].

According to Lighthill's analogy, quadrupole sources are used to present the excitation mechanisms. The particular pressure for a quadrupole source is computed with [3]

$$p^p = \frac{ik\rho_0 c}{8\pi} Q\, cos(2\vartheta)\, sin^2(\varphi) \left(\frac{2}{r^3} - \frac{2ik}{r^2} - \frac{k^2}{r} \right) e^{i(kr+\gamma)} \ , \tag{4}$$

with Q being the strength of the quadrupole, and the particular velocity is

$$v_a^p = \frac{1}{sk}\frac{\partial p}{\partial n} \ . \tag{5}$$

Experiments are used to verify the numeric results of the source problem. In these experiments, four microphones are placed close around the recorder and a fifth microphone is placed inside the recorder close to the labium. With these microphones the pressure depending on the time is measured. With a Fourier transformation of this pressure the pressure in the frequency domain is obtained. In an optimization process position, strength, orientation and phase of the quadrupoles are determined to match the results of the experiments in the positions of the microphones.

Table 1 shows the deviation between measured and numerically computed pressure, with

$$error = \frac{|p_M - p_C|}{|p_M|} \ . \tag{6}$$

It can be seen that especially for the first microphone good results are obtained, while for the fifth microphone, which is the one inside the recorder, the results do not match at all. One

Table 1: Comparison of measured and computed pressure.

	measurement [Pa]	computation [Pa]	error [%]
1	$-4.213+1.911i$	$-3.966+2.218i$	8.532
2	$-1.394+0.814i$	$-1.177+0.406i$	28.603
3	$-1.956+1.249i$	$-1.958+0.764i$	19.610
4	$-8.859+3.136i$	$-8.518+5.805i$	28.633
5	$-66.11-79.83i$	$-248.13+180.91i$	/

problem there might be that the pressure measured with the fifth microphone was not obtained in the same experiments as the results for microphones one through four, but measured separately. Another negative effect on the result could be the fact that the microphone is placed inside the recorder, close to labium and wind channel, in an area where reflections from the walls and turbulences occur. Possibly, the effect of the eigenfrequency disturbes the measurement inside the recoder and positioning a microphone there is not useful.

MODAL ANALYSIS WITHOUT VOLUME FLOW

For the modal analysis of the stationary fluid inside and around a recorder, the boundary value problem as described in eq. (1) is used. The recorder itself is condisered as sound hard. Therefore, the right-hand-side vector $\boldsymbol{b}$ is zero.

The resulting state space formulation of the eigenvalue problem is [6]

$$\left(\begin{bmatrix} \boldsymbol{I} & \boldsymbol{0} \\ \boldsymbol{0} & -\boldsymbol{K} \end{bmatrix} - \lambda \begin{bmatrix} \boldsymbol{0} & \boldsymbol{I} \\ \boldsymbol{M} & \boldsymbol{D} \end{bmatrix}\right)\begin{bmatrix} \boldsymbol{\Phi} \\ \boldsymbol{\Psi} \end{bmatrix} = \begin{bmatrix} \boldsymbol{0} \\ \boldsymbol{0} \end{bmatrix}, \quad (7)$$

with $\boldsymbol{\Phi} = \lambda \boldsymbol{\Psi}$ and $\lambda = -ik$.

The three-dimensional finite element model of the recorder fluid is build in Ansys 11.0 and read into a noncommercial code that was developed at our institute. In this Fortran 90 code the infinite elements are added before starting with the computations.

The modal analysis is accomplished for all playable notes, except the ones with half-open tone holes. Figure 1 shows first eigenfrequency and first harmonic of note b$''$.

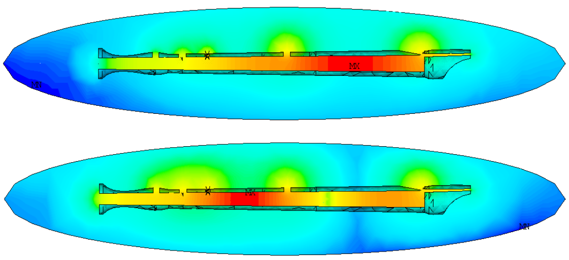

Figure 1: First eigenfrequency (above) and first harmonic (below) of b$''$.

The recorder that is used for these computations is a soprano recorder with german fingering and tuning to 442 Hz. The computed eigenfrequencies are compared to the according values from the MIDI-table. After converting the values of the MIDI-table to 442 Hz, the frequency of note b$''$ can be obtained with 936.565 Hz. In Figure 2 the convergence behaviour of this note is shown.

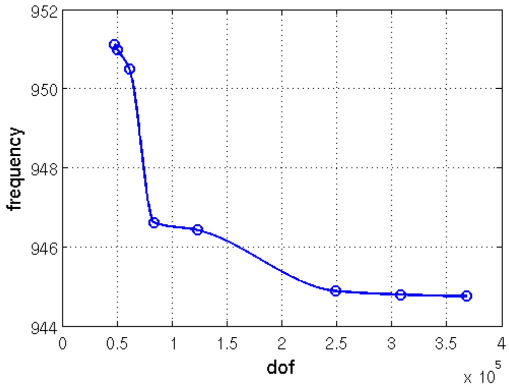

Figure 2: Convergence of b$''$.

At first sight, the numeric results show a good convergence behaviour, but even for the finest mesh the deviation between computed frequency and the one from the MIDI-table is still about 8 Hz. For a music instrument, this deviation is too high. We assume that one reason for this high deviation is that we didn't consider the volume flow inside the recorder for those computations. Therefore we will discuss the relation between frequency and volume flow in the following section.

MODAL ANALYSIS WITH VOLUME FLOW

In this section we consider the sound propagation of moving fluids. For now, the computations are made for interior domains.

Fluids with a non-uniform flow can be described by the Galbrun equation [4, 7]

$$\begin{aligned} &-\int_\Omega \frac{1}{\rho_0 c_0^2} p^* p - \int_\Omega \nabla p^* \cdot \boldsymbol{w} + \int_\Omega \boldsymbol{w}^* \cdot \nabla p \\ &-\int_\Omega \rho_0 (\boldsymbol{v}_0 \cdot \nabla \boldsymbol{w}^*) \cdot (\boldsymbol{v}_0 \cdot \nabla \boldsymbol{w}) - \omega^2 \int_\Omega \rho_0 \boldsymbol{w}^* \cdot \boldsymbol{w} \\ &-i\omega \int_\Omega \rho_0 \boldsymbol{w}^* \cdot (\boldsymbol{v}_0 \cdot \nabla \boldsymbol{w}) + i\omega \int_\Omega \rho_0 (\boldsymbol{v}_0 \cdot \nabla \boldsymbol{w}^*) \cdot \boldsymbol{w} \\ &+\int_\Gamma \boldsymbol{w}^* \cdot \left[\rho_0 (\boldsymbol{v}_0 \cdot \boldsymbol{n}) \frac{d\boldsymbol{w}}{dt}\right] - \int_\Gamma p^* (\boldsymbol{w} \cdot \boldsymbol{n}) = 0 \, . \qquad (8) \end{aligned}$$

This equation is a mixed formulation of pressure p and displacement $\boldsymbol{w}$, $\boldsymbol{v}_0$ represents the velocity of the fluid.

The computations are carried out for two different finite elements. In the first case we use tetrahedral MINI elements and in the second case tetrahedral Taylor-Hood elements.

The effect of the flow on the eigenfrequencies is analysed on a three-dimensional duct with $l = 3.4m, \quad b = h = 0.2m$. We assume a constant flow over the cross-section. For this example an one-dimensional solution can be found with

$$f_n = \frac{c_0 n}{2l}(1 - Ma^2) \quad \text{with} \quad Ma = \frac{v_0}{c_0} \, . \qquad (9)$$

Figure 3 shows the first three eigenfrequencies of the duct over the Mach number (Ma). The numeric computations, using Galbrun equation, verify the assumption that the frequencies decrease by increasing Mach number. Table 2 presents the results of the computations obtained with MINI elements compared to the Helmholtz solution.

CONCLUSION

This paper showed that it is possible to describe the excitation mechanism inside and around a recorder with quadrupole sources.

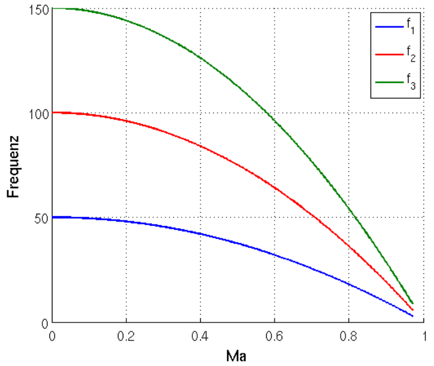

Figure 3: 1., 2. and 3. eigenfrequency over Mach number.

Table 2: Eigenfrequencies depending on Mach number, obtained with MINI elements.

Helmholtz	$v_{0_z} = 0\frac{m}{s}$ Ma=0	$v_{0_z} = 10\frac{m}{s}$ Ma=0.02941	$v_{0_z} = 100\frac{m}{s}$ Ma=0.29412
50.00003	50.02247	49.96569	45.66757
100.00093	100.16144	100.02416	91.28340
150.00674	150.56087	150.34645	136.78154

Furthermore, a modal analysis was accomplished for a static fluid as well as for a moving fluid. It was shown that a volume flow decreases the values of the eigenfrequencies.

In a next step, we want to include the characteristic volume flow profile inside the recorder to examine the influence of the flow on the sound.

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