Proceedings of 20th International Congress on Acoustics, ICA 2010

23–27 August 2010, Sydney, Australia

Sound production and radiation of resonator-controlled edge tones

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PACS: 43.28.Py, 43.28.Ra, 43.75.Qr

ABSTRACT

Nonlinear interactions between a free jet and an acoustic field play a major role in the sound production of musical instruments like recorders, flutes and organ pipes, but also in different technological applications. Solving the compressible and unsteady Navier-Stokes equations allows to resolve both the sound production mechanism and also the sound propagation in the nearly linear resonator and the far field. Thereby different length scales between the vortex shedding at the labium and the acoustic wave lengths and the resulting computational effort avoid an efficient optimization of the resonator. To overcome these limitations we separate sound production and propagation. Here the resonator is assumed linear and described in the frequency domain by the Helmholtz equation. The sound production mechanism is modeled by acoustic sources. These sources result from unsteady, compressible RANS calculations. According to the vortex theory by Howe the sound production is modeled by acoustic dipole sources that result from interactions between the jet and the acoustic field. This approach allows the efficient calculation of different resonator geometries, without the necessity to solve the complete sound production mechanism again. We present first studies based on this approach.

INTRODUCTION

In different technological applications but also in musical acoustics nonlinear interactions between a free jet and an acoustic resonator play a major role in the sound production. While for technological applications the detection and reduction of sound sources in terms of a noise reduction are of primary interest, in musical acoustics the modification and tuning of the timbre play the most important role. For all these applications solving the full Navier-Stokes equations allows to simulate the sound production and propagation coupled together. Due to different time and length scales of the fluid flow and the acoustics typically the resulting computational effort only allows to simulate a small number of different configurations. The common approach to overcome this limitation is to estimate the sound production and propagation separated from each other. For this we investigate the sound production by unsteady, compressible RANS calculations in the time domain and the propagation and radiation in the frequency domain by solving the Helmholtz equation. Thereby the time domain results provide acoustic dipole sources that define the source terms in the Helmholtz solver. Under the assumption that small changes in the resonator doesn't influence the sound production significantly the coupling of both solvers can be done unidirectional. That means, we estimate the sound production only once and perform the resonator optimization in the frequency range.

MODAL ANALYSIS

The linear-acoustic resonator is prescribed in the frequency domain. The time-harmonic formulation of the acoustic pressure writes

$$p(\boldsymbol{x},t) = \mathrm{Re}\left\{ p(\boldsymbol{x}) e^{-i\omega t} \right\}, \tag{1}$$

which leads to the following formulation of the Helmholtz equation

$$-\Delta p(\boldsymbol{x}) - k^2 p(\boldsymbol{x}) = 0\,, \quad \boldsymbol{x} \in \Omega\,, \tag{2}$$

with Neumann conditions at solid boundaries

$$\frac{\partial p(\boldsymbol{x})}{\partial \boldsymbol{n}} = 0\,, \quad \boldsymbol{x} \in \Gamma \tag{3}$$

and the Sommerfeld radiation condition

$$R\left\{ \frac{\partial p}{\partial R} - ikp \right\} \to 0 \quad \text{if} \quad R \to \infty\,. \tag{4}$$

This system of equations is solved by a second order Finite Element scheme. The discretization bases on P_2-tetrahedra and the Sommerfeld radiation condition is imposed by Infinite Finite Elements (Astley 1998). For example in Fig. 2 and 3 the eigenvectors of a soprano recorder are shown.

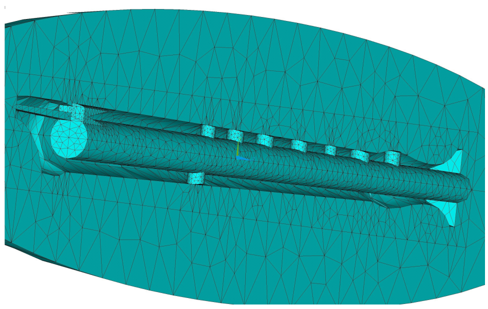

Figure 1: Modal analysis of a soprano recorder. Computational grid. Number of elements ≈ 30.000.

Sound production

The compressible, unsteady and three-dimensional Navier-Stokes equations are solved using a second order Finite Volume scheme (Ansys CFX™). The last tone hole of the recorder was moved into the symmetry plane in order to achieve a full symmetric model that allows to halve the computational effort. The final

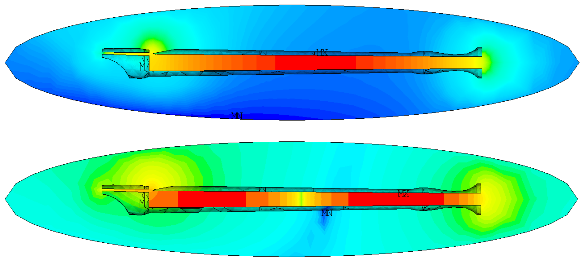

Figure 2: Eigenvectors of C5 (526 Hz). Upper figure: fundamental, lower figure: first overtone.

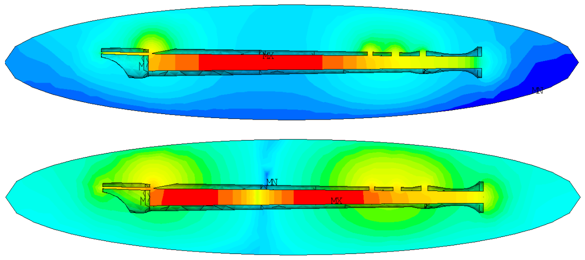

Figure 3: Eigenvectors of F5 (702 Hz). Upper figure: fundamental, lower figure: first overtone.

computational grid consists of 785.000 hexahedrons, with the smallest element size of approximated 0.05 mm at the front edge of the labium and the largest element size of nearly 5 cm at the outer boundary. This results into a computational effort of approximately three weeks on four CPUs for only one configuration (e. g. one fingering). The instrument walls are defined hard and full reflective, the outer boundary is defined reflectionless using a characteristic splitting scheme. At the inflow the total pressure is given with 100 Pa, resulting from accompanying measurements.

Coupling strategy

Two primary sound production mechanisms can be detected while playing the recorder: Vortex sound sources in the area between the exit of the wind channel and the labium and pressure forces acting on the labium surface. The vortex sound can be defined using the vortex sound theory by Howe (2002) with $q = -\rho(\nabla \cdot \boldsymbol{L})$; $\boldsymbol{L} = \boldsymbol{\omega} \times \boldsymbol{u}$, the surface forces are defined as $q = \nabla \cdot \boldsymbol{F}$ with the forcing term $\boldsymbol{F} = -p\boldsymbol{n}$. Both sound sources are dipole sources. Using the following equation

$$p^p = \frac{\mathrm{i}\omega\rho}{4\pi} D\cos(\theta)\left(-\frac{1}{r^2} + \frac{\mathrm{i}k}{r}\right) e^{\mathrm{i}(kr+\varphi)} \tag{5}$$

the dipole sources are implemented into the Helmholtz solver. As a preliminary study the sound production was approximated by two dipole sources at the upper and lower side of the labium. The resulting acoustic field is illustrated in Fig. 7.

Outlook

This paper presents first preliminary results based on a simplified sound source approximation. In further studies we simulate the sound production mechanism in detail by using a two-dimensional and high-order discontinuous Galerkin scheme (Richter et al. 2008, Richter and Stiller 2009). For this we estimate the source field around the labium for a wide range of different labium geometries and fingering. These results make a more realistic and parametrized sound source model possible, which can be used in the Helmholtz solver to perform the intended resonator optimization.

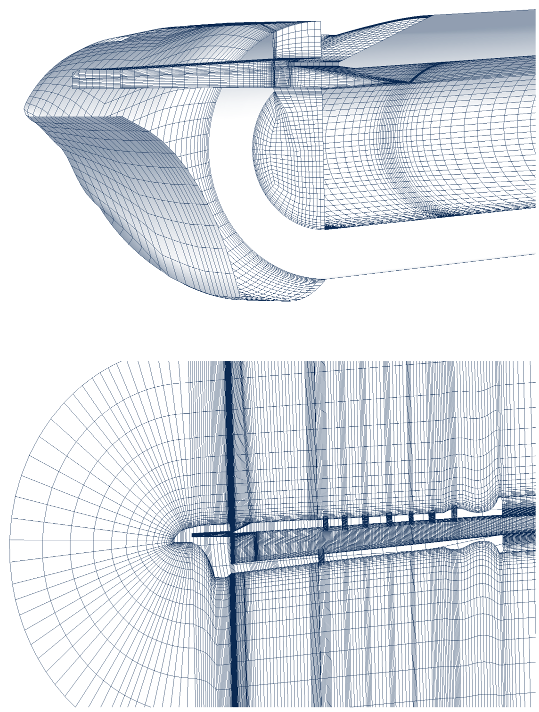

Figure 4: Simulation of the flow field of the soprano recorder. Computational grid. Upper figure: grid details, lower figure: whole domain.

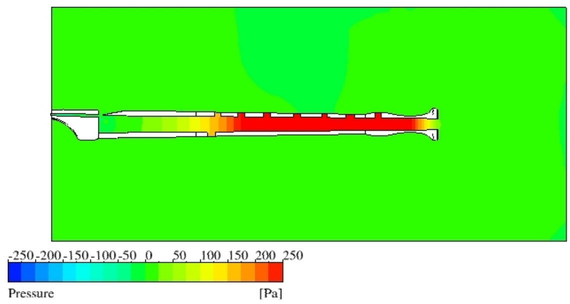

Figure 5: Pressure field, Note C5.

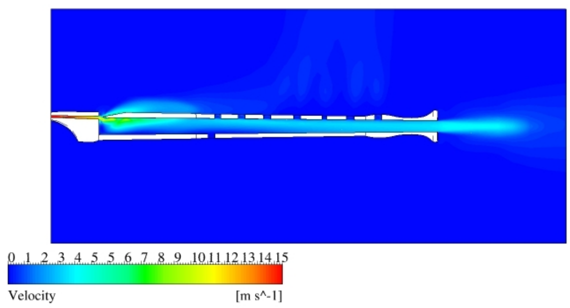

Figure 6: Velocity field, Note C5.

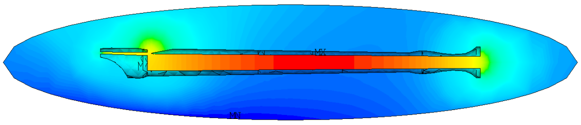

Figure 7: Sound production approximated with two dipole sources. Eigenvectors of C5 (526 Hz).

Acknowledgments

This work was supported by the German National Science Foundation (Deutsche Forschungsgemeinschaft, DFG) within the project *Numerical simulation of the sound spectrum and the sound radiation in and around a recorder (Numerische Simulation des Klangspektrums und der Schallausbreitung in und um eine Blockflöte)*. The authors also thank CFX Berlin Software GmbH for providing three-dimensional simulation results.

REFERENCES

R. J. Astley. Mapped spheroidal elements for unbounded wave problems. *Int. J. Num. Meth. Eng.*, 41:1235–1254, 1998.

M. S. Howe. *Theory of Vortex Sound*. Cambridge University Press, 2002.

A. Richter and J. Stiller. A two-dimensional DG-SEM approach to investigate resonance frequencies and sound radiation of musical woodwind instruments. In *ICOSAHOM 2009, Trondheim*, 2009.

A. Richter, J. Stiller, and R. Grundmann. Stabilized high-order discontinuous Galerkin methods for aeroacoustic investigations. In *ICCFD 2008, Seoul*, 2008.