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Efficient handling of parameter uncertainties in coupled fluid-structure finite element computations

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ABSTRACT

Various methods based on probability theories, fuzzy sets, and other approaches exist to take uncertain parameters in numerical computations into consideration. However, a major drawback of the methods that are most universal and relevant to real-life applications, like Monte Carlo simulation or the fuzzy transformation method, is their need to repeatedly evaluate a basic numerical model, which is continuously manipulated regarding its uncertain input parameters. Keeping in mind the traditionally high computational cost of vibro-acoustic finite element models, the calculation of hundreds or even thousands of model evaluations sets a practical limit to the applicability of these methods.
As a way out, a novel approach called iterative method for multiple evaluations (IMME) has been formulated, which is a structured methodology to efficiently perform an uncertain finite element analysis based on multiple evaluation runs. Its general philosophy is the fragmentation of the vibro-acoustic system into different structural and fluid substructures with few or no uncertainties. To evaluate the overall system, the different substructures are reconnected by component mode synthesis and an iterative coupling of fluid and structural parts.
The high advantage of the IMME for uncertain vibro-acoustic analysis regarding computation time is exemplified for representative investigations of different components of an aircraft cabin. Depending especially on the system size, the number of evaluation runs, and the number of examined load cases, a decrease of the overall computation time down to only a few percent of the direct-coupled solution is possible.

INTRODUCTION

The vibro-acoustic optimisation of technical components gains more and more in importance, as customers are increasingly sensible regarding the impact of noise and vibration on their subjective well-being. Especially within vehicle engineering, considerable efforts are made to design and to optimise the vibro-acoustic performance, and the significance of numerical investigations and predictions in this area is continuously increasing.

Unfortunately, the vibro-acoustic characteristics of complex technical system are observed to vary in a wide spectrum of several dB for supposedly identical products. Among others, production tolerances and varying environmental and operating conditions have a considerable impact on the vibro-acoustic behaviour, which is essentially based on resonant effects. In practice, huge deviations between supposedly 'identical' products from series production are observed. Between different passenger cars of the same model with equal configuration, deviations up to 15-20 dB can be observed [1][2][3][4]. Similar effects apply, for instance, also for trains, airplanes, or ships. The main reason for this immense scatter in the vibro-acoustic properties is that already small differences, e.g., in dimensions or joining forces, can have a significant effect on the resonances of a system. Within the variational range of the parameters of a subsystem, resonances with other subsystems may occur or, on the contrary, be avoided.

Additionally, corresponding numerical models often suffer from a lack of knowledge regarding single model parameters. On the one hand, this may be caused by generally limited information about a parameter, for instance regarding structural damping. On the other hand, parameters may not finally be specified, e.g. in early design stages. Nevertheless, substantiated statements between different design alternatives have to be made.

Independent of the actual reasons for parameter uncertainties, a reliable numerical model should be able to reproduce their influences and allow for an estimation and evaluation of the vibro-acoustic variations. For this purpose, different approaches have been established in the past.

UNCERTAINTY ANALYSIS IN NUMERICAL SIMULATION

Generally, most existing methods for the treatment of parameter uncertainties in numerical simulation can be subdivided by the uncertainty triangle in three major groups (Figure 1). Intermediate methods exist that combine features of two groups at the same time, like the α-cut strategy for solving fuzzy set problems with an interval arithmetic approach.

All methods of the three groups enable for the consideration of parameter uncertainties. Yet, their characteristics and the requirements on the input data are quite heterogeneous, so that the choice of an appropriate method strongly depends on the actual problem and its type of parameter uncertainties.

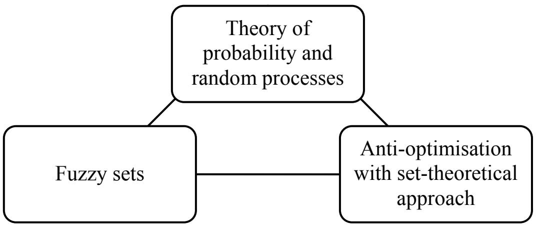

Figure 1. Uncertainty triangle according to [5]

Independent of the classification, methods that break down the uncertain problem on single deterministic evaluations are most relevant for practical applications, like the probabilistic Monte Carlo simulation (MCS) [6] or the fuzzy transformation method [7]. A common attribute of these methods is the modification of a basic model according to the uncertain input parameters by a special systematic. As a result, a set of single deterministic models is generated, which can be solved separately. After evaluation of the corresponding set of deterministic solutions, the uncertain output of the overall problem is generated.

As a direct consequence of this approach, existing models and standard software packages can be used, which are in most cases not able to process other than deterministic data. Still, practical problems with more than one uncertain parameter result often in a huge number of multiple evaluation runs of several hundred or even thousand times. This may become a severe limitation for an uncertainty analysis, if simulation models are used, which are costly regarding computation time and resources, like coupled finite element (FE) models of vibro-acoustic systems.

EFFICIENT SOLUTION OF UNCERTAIN FINITE ELEMENT SYSTEMS

Complex structures are often composed of different components, which are joint together to the overall system after separate production. Together with the fluid domain, a vibro-acoustic system consists of partly independent substructures, which influence the characteristics of the system by their interaction (note that both structural and fluid domains will be denoted as ‘substructures’ in the following to assure consistent terminology). Whenever a system can be regarded as a sum of different substructures, it is likely that also some of the parameter uncertainties concentrate on substructure level.

As an example, a coupled vibro-acoustic FE system consisting of a plate and two beams with attached fluid cavity is illustrated in Figure 2. A specific feature of the depicted system is the absorbing boundary condition on the five free faces of the fluid cavity that are not connected to the plate. With this configuration of the FE model, free-field conditions should be approximated, which enable for investigations of transmission loss and radiation efficiency analogous to typical experimental setups. A detailed description of the application of such models for vibro-acoustic investigation can be found in [8].

The general philosophy to obtain an efficient solution of the uncertain vibro-acoustic system by methods based on multiple evaluation runs can be described by the following steps:

1. Definition of as many substructures as possible that are not affected by any uncertainties at all (e.g. the fluid cavity in Figure 2).

2. Identification of uncertainties, which apply only to a limited part of the overall model and isolation in corresponding substructures (e.g. the plate in Figure 2).

3. If possible, classification of similar recurring components with identical uncertainties and definition as substructures (e.g. the beams in Figure 2).

4. Separate reduction or solution of the different substructures.

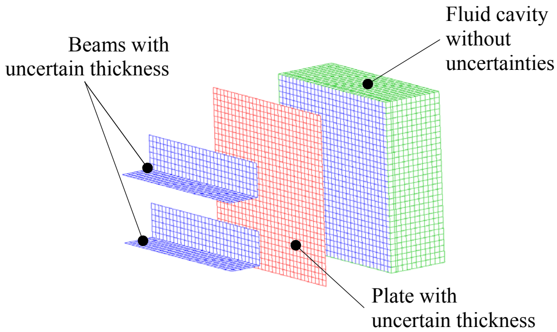

Figure 2. Vibro-acoustic FE system of a plate with two beams and connected fluid cavity with absorbing boundary condition (exploded view)

Common methods for the reduction and solution of FE substructures are typically based on modal approaches. In particular, methods related to component mode synthesis (CMS) are widely accepted for the efficient solution of uncertain FE systems [9], which are also capable of considering fluid domains as separate substructures [10]. However, major restrictions of modal approaches exist in connection with unbounded or highly damped fluid domains [11][12][13]. Compared to reverberant cavities, the resulting acoustic field of such fluid domains will have an entirely different shape and cannot be well described by normal modes based on cavity resonances and standing wave phenomena.

ITERATIVE COUPLING OF FLUID AND STRUCTURE

To be able to separately consider unbounded fluid domains or cavities with a considerable amount of absorption, an iterative coupling strategy is used instead. By doing so, the coupled fluid-structure system in the frequency domain for time-harmonic excitation

$$\begin{bmatrix} -\omega^2\mathbf{M}_{str} + \mathrm{i}\omega\mathbf{C}_{str} + \mathbf{K}_{str} & -\mathbf{S}^T \\ -\omega^2\rho_0\mathbf{S} & \omega^2\mathbf{M}_{fl} + \mathrm{i}\omega\mathbf{C}_{fl} + \mathbf{K}_{fl} \end{bmatrix} \begin{bmatrix} \mathbf{D} \\ \mathbf{P} \end{bmatrix} = \begin{bmatrix} \mathbf{R}_{str,ext} \\ -\mathrm{i}\omega\mathbf{R}_{fl,ext} \end{bmatrix} \quad (1)$$

M	mass matrix	**P**	fluid pressure
C	damping matrix	ω	angular frequency
K	stiffness matrix	ρ_0	mass density of fluid
S	coupling matrix	*str*	structure
R	force	*fl*	fluid
D	structural displacement	*ext*	external

is divided in a sequential solution of fluid and structural domain regarding pressure and displacement, respectively, according to the iteration scheme depicted in Figure 3. Thereby, the effect of the fluid pressure on the structural domain is considered by an additional external force $\mathbf{R}_{fl-str}$, while the effect of the structural vibration on the fluid is accounted for by $\mathbf{R}_{str-fl}$.

The coupled solution for fluid pressure and structural displacement is obtained, if equilibrium of all quantities is reached. For practical application, a convergence criterion is evaluated and the iteration is stopped, if a certain accuracy of the solution is reached.

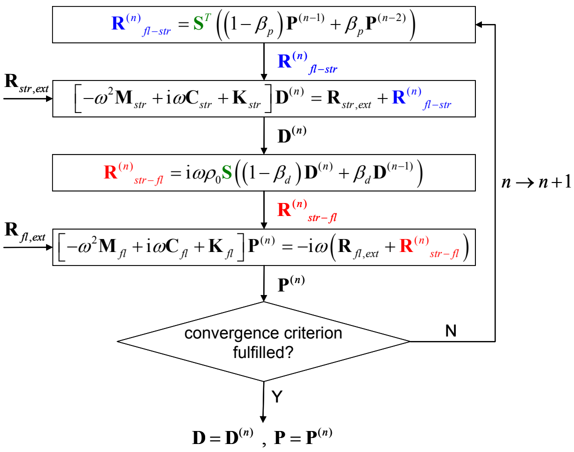

Figure 3. Iterative solution of the coupled fluid-structure-system with iteration number n, relaxation factor for fluid pressure β_p , and relaxation factor for structural displacement β_d according to [8]

The iterative coupling of fluid and structure is a widely accepted approach for investigations in the time domain [14][15]. However, stability of the iterative solution scheme is a general challenge. Depending on the characteristics of the coupled problem, the parameters of the iteration must be thoroughly adjusted to obtain a stable solution.

Nevertheless, a successful application to problems in the frequency domain is not known to the authors. Iteratively coupled system may be so sensible at single frequencies, that instability may not be controlled any more. This becomes most obvious, if the direct-coupled solution of fluid pressure and structural displacement is taken as initial value for the iteration. In theory, pressure and displacement should not change, if the iteration loop is repeatedly executed without relaxation, as equilibrium is already reached. In practice, however, the system may become instable at single frequencies after a certain number of executions, so that pressure and displacement rise towards infinity. This sudden instability of the equilibrium solution occurs only due to the bounded numerical accuracy of computer processors and software routines. As a consequence, an iterative solution at these instable frequencies will not be possible at all. For a further discussion of possible iteration instability in the frequency domain, it is referred to [8].

ITERATIVE METHOD FOR MULTIPLE EVALUATIONS (IMME)

Nonetheless, the separate treatment of the fluid and the subsequent iterative coupling to the structure offers a high potential to speed up the multiple evaluation runs needed for an uncertain FE analysis in the frequency domain. Once the fluid domain is solved, e.g., by CHOLESKY or LU decomposition, a stable iteration yields a much faster solution of the coupled quantities for many cases than the direct-coupled approach [8]. In order to reduced the effort of an uncertain analysis regarding computation time and use of resources, it seems therefore desirable to solve as many frequencies as possible by the iterative approach.

At this point it should be recalled that, in general, two different main reasons motivate the use of an iterative coupling for a fluid-structure FE system: On the one hand, the coupled system may simply be to large to be solved as a whole. On the other hand, the iterative approach may be favourable regarding solution time and computer resources, although a direct-coupled solution would be possible. Uncertain problems that are investigated with methods based on the multiple evaluation of a model, for instance MCS or the transformation method, do normally not belong to the first group. Typically, the used models are somewhat limited in size, so that a single evaluation run of the coupled problem can be performed by direct coupling. In consequence, a direct solution is possible for single frequencies, at which the iterative approach yields no result due to instability.

Based on these thoughts, the *Iterative Method for Multiple Evaluation* (IMME) has been formulated (see Figure 4). Similar to the component mode transformation method given in [16], the IMME conducts in a first step a segmentation of the structural domain by CMS, if this is meaningful with regard to overall computation time. Additionally, reverberant fluid cavities may also be treated by CMS, if their dynamic behaviour can satisfactorily be described by a modal approach. All other fluid domains, however, are solved by CHOLESKY/LU decomposition as a preparation for an effcient iterative coupling.

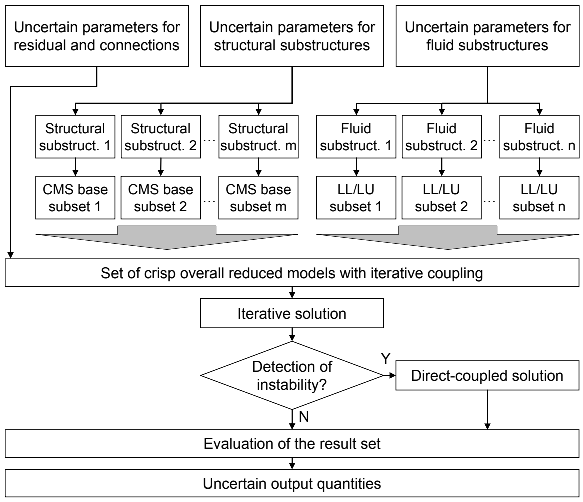

Figure 4. General methodology of the *Iterative Method for Multiple Evaluations* (IMME) for the solution of coupled fluid-structure systems with parameter uncertainties according to [8]

After assembling the set of crisp overall reduced models, the corresponding result set is generated by the iterative coupling scheme described before. During iteration, stability of the solution is monitored. As soon as instability is detected, the iteration is interrupted and the direct-coupled solution is calculated for the instable frequency instead.

The IMME is suitable for both structured and unstructured sampling methods, which require a slightly different approach regarding the general methodology for an efficient multiple evaluation. Structured sampling methods, like the fuzzy transformation method, obtain the different input samples from a systematic permutation of the uncertain parameters. For this kind of methods, the set of crisp overall models is generated straight-forward by the IMME and the predefined parameter combinations given by the used method for the uncertainty analysis.

Unstructured sampling methods like MCS, however, are characterised by a statistical sampling, so that the configuration of the different input samples is not known in advance. Although the CMS bases and matrix decompositions for the different substructures cannot be derived from a fixed permutation, the application of CMS or CHOLESKY/LU decomposition may still be advantageous regarding computational cost, if substructures that contain no uncertainties at all can be isolated. Furthermore, the iterative coupling is still possible also for unstructured sampling methods. For a detailed investigation of possible benefits and drawbacks regarding different operating conditions and configurations, it is referred to [8].

COMPARISON OF COMPUTATION TIME

To illustrate the high potential of the IMME to reduced the computational effort for uncertain FE analysis, three different vibro-acoustic models for components of an aircraft cabin have been investigated (Figure 5). All three components show a considerable amount of uncertainty in some of their material parameters and are investigated with the fuzzy transformation method. An overview of the basic properties of the three models and the corresponding uncertainty problem is given in Table 1. All models have been solved at 55 frequencies. Further details regarding the model set-up, the incorporated uncertain parameters, and the resulting uncertain transmission losses are found in [8].

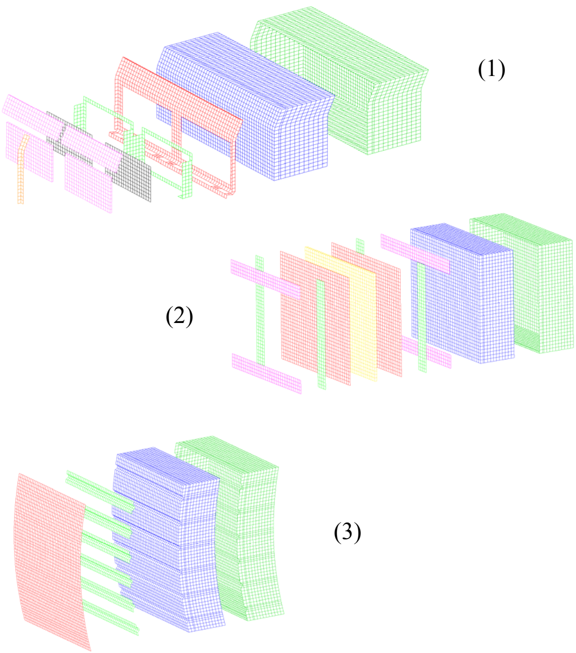

Figure 5. Vibro-acoustic FE models for three different components of an aircraft cabin: (1) dado panel, aluminium and plastics with foam insulation (2) ceiling panel, honeycomb core with GRP facing skins (3) fuselage structure, CFRP skin and CFRP stringers

Table 1. Basic properties of the uncertain FE models

	dado panel	ceiling panel	fuselage structure
structure d.o.f.	13932	15048	15006
structural modes	450	150	200
fluid d.o.f.	14469	16302	31570
uncertain parameters	4	4	3
fuzzy α-levels	4	4	5
model evaluations	354	354	225

Compared to the conventional approach, computation time of the uncertain FE problem could be reduced to values of only 3.4% to 7.8% of the direct-coupled solution by using the iterative scheme of the IMME (Figure 6). It can be noted that the portion of computation time for the modal transformation of the structural domain takes a considerable amount of the overall computation time, while the portion for the LU decomposition of the fluid is comparatively low. As the uncertain parameters of the three components concentrate on the structural domain, a separate modal transformation is needed for every model evaluation. For the deterministic fluid cavity, however, the LU decomposition has to be performed only once and can be re-used for all model evaluations. It should be pointed out that the computation time for modal transformation of the structure is identical both for the direct-coupled and the iterative approach, while the LU decomposition of the fluid is only required for the iteration.

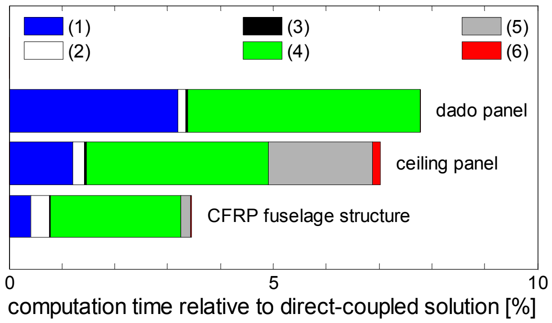

Figure 6. Reduction of computation time using the IMME for transmission loss calculations of typical aircraft cabin components with parameter uncertainties
Portions of time: (1) modal transformation of structure (2) LU decomposition of fluid (3) I/O (4) iterative solution (5) direct solution of instable frequencies (6) instable iterations

For all three components, the biggest portion of computation time arises for the iterative solution of the stable frequencies. Thereby, the number of iterations that are needed per frequency is in average comparatively low between 4.89 and 9.57, which is due to the weak coupling of fluid and structure caused by the absorbing fluid boundary conditions (Table 2). At single frequencies, however, this number can significantly deviate.

Table 2. Number of iterations per frequency and percentage of instable frequencies

	dado panel	ceiling panel	fuselage structure
average number of iterations	4.89	9.57	5.49
minimum number of iterations	3	3	2
maximum number of iterations	51	100	98
percentage of instable frequencies	-	2.01%	0.18%

Both for the ceiling panel and the fuselage structure, a certain amount of all calculated frequencies becomes instable (Table 2). Although the percentage of instable frequencies is very low, their direct-coupled solution causes a considerable amount of computation time, which illustrates the efficiency of the iterative approach (Figure 6). The additional computation time of the useless iteration loops until instability is detected, however, is negligible.

For all three FE models, no additional reduction by CMS has been used for the structural domain, as this only affects the computation time for the modal transformation. The cost both for the direct-coupled and the iterative solution, however, would remain the same, as the number of structural modes and thus the dimension of the resulting system of equations do not change. Therefore, it has to be decided from case to case, if CMS gives a real additional value in terms of computation time. For this decision, the effort for generating the CMS model and the possibly negative effect on result accuracy due to the approximate character of the component modes must be taken into account as well.

It should be accentuated that for all three components, the same performance of the IMME could have been achieved by using an unstructured sampling method like MCS instead of the structured fuzzy transformation method. This is due to the deterministic fluid properties, so that the fluid has only to be solved once independent of the applied sampling method.

CONCLUSIONS AND OUTLOOK

With the IMME, a highly-efficient method for the uncertainty analysis of coupled fluid-structure FE systems has been proposed. Common approaches for the solution of uncertain problems, like MCS or the transformation method, are based on the multiple evaluation of single deterministic models. Due to the iterative coupling of structure and fluid domain, a separate reduction and solution of the two domains becomes possible, which is especially beneficial, if the fluid domain does not show any uncertainties at all. It has been demonstrated for three different components with parameter uncertainties of the aircraft cabin, that reductions in overall computation time down to only a few percent of the direct-coupled approach can be realised.

The IMME is not restricted to problems in connection with uncertainty analysis. It is generally applicable for all tasks in FE vibro-acoustics that are based on multiple evaluations of a basic model, for instance optimisation problems. Additionally, coupled problems of, e.g., boundary and finite elements can be solved in the same manner.

Furthermore, an efficient application to related moderately coupled problems, like electro-magnetic radiation into an unbounded domain, seems to be promising.

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