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Blind source separation and localization in case of one reflection problem of one source signal

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ABSTRACT

The blind source separation and specification of location of source signal considering one reflected signal are conducted. The mathematical formulation is shown that not only the separation of signals but also the specification of location can be done completely, assuming that the observation signal is represented by addition of a direct signal and one reflected signal. First of all, the observation signals are transformed to Fourier domain, and the relationship between observation signals is introduced by dint of elimination of source signal. The relationship is represented by attenuation coefficients and time lags. We make use of the characteristics of time lags which are represented by the term of exponential in discrete Fourier domain. Then, by use of relationship between the real value of attenuation coefficients and the complex value of Fourier domain, the attenuation coefficients and time lags are completely specified. Then source signal and the location of it are specified. Moreover, the numerical test is conducted to confirm our method. Then, source signal and the location of it can be specified within the numerical error.

1 INTRODUCTION

The cocktail party effect is known as auditory ability to distinguish particular sound signal from other sounds like noise signals or reflected signals. The cause of cocktail party effect is tried to solve from various fields and various point of view. The blind source separation is used as one of the methods to solve the cocktail party effect. Authors proposed the methods for specification of the locations of source signals and the separation of the source signals using time-frequency information and time lags of source signals as a technique of blind source separation [1], [2]. However, in our method, the reflected signals can not be considered, so in case of actual problem, it is somewhat not practical.

In this paper, we propose the method which can separate the source signal from the observation signal and specify the location of the source signal completely in case of one reflection problem of one source signal.

2 FORMULATION

2.1 Assumption

Let $s(t)$ be one source signal data, assuming that the signal is originated from one point. Let $x_k(t)$ $(1 \le k \le M)$ be observation signal data, and subsuffix k denotes the observation point number, where we assume $M \ge 4$ to specify the location of source signal.

In this paper, it is assumed that the wall exists only in one direction, and there are direct signal and one reflected signal from the wall, which is transmitted from the source point to the observation point. Hereafter, j=1 indicates the direct signal and j=2 the reflected signal. Let us assume the following relation between $x_k(t)$ and $s(t)$ on the free sound field.

$$x_k(t) = \sum_{j=1}^{2} a_{kj} s\left(t - c_{kj}\right) \qquad (1 \le k \le M) \tag{1}$$

where a_{kj} represents real-valued attenuation coefficient and c_{kj} represents real-valued time lag between $x_k(t)$ and $s(t)$. In addition, $c_{k1} \le c_{k2}$ is obvious because the arrival time of direct signal is shorter than that of reflected signal. Here, only $x_k(t)$ is known data, and $s(t)$, a_{kj} and c_{kj} are all unknown data.

2.2 Specification of attenuation coefficients ratio

The Fourier information of eq.(1) is represented by

$$\hat{x}_k(\omega) = (a_{k1} e^{-i\omega c_{k1}} + a_{k2} e^{-i\omega c_{k2}})\hat{s}(\omega) \tag{2}$$

where the symbol ^ means Fourier domain. Here, we introduce $\hat{\hat{s}}(t)$ by

$$\hat{\hat{s}}(\omega) = a_{11} e^{-i\omega c_{11}} \hat{s}(\omega) \,. \tag{3}$$

By use of eq.(3), $x_1(t)$ and $x_2(t)$ respectively can be represented by

$$\hat{x}_1(\omega) = \left(1 + \frac{a_{12}}{a_{11}} e^{-i\omega(c_{12} - c_{11})}\right)\hat{\hat{s}}(\omega) \,, \tag{4}$$

$$\hat{x}_2(\omega) = \left(\frac{a_{21}}{a_{11}} e^{-i\omega(c_{21} - c_{11})} + \frac{a_{22}}{a_{11}} e^{-i\omega(c_{22} - c_{11})}\right)\hat{\hat{s}}(\omega) \,. \tag{5}$$

In addition, we introduce symbols a_v and C_v (v=1,2,3) by

$$a_1 = \frac{a_{12}}{a_{11}}\ ,\ a_2 = \frac{a_{21}}{a_{11}}\ ,\ a_3 = \frac{a_{22}}{a_{11}}\ , \tag{6}$$

$$C_1 = c_{12} - c_{11}\ , C_2 = c_{21} - c_{11}\ , C_3 = c_{22} - c_{11}\ . \tag{7}$$

By using eqs.(6) and (7), eqs.(4) and (5) respectively can be expressed by

$$\hat{x}_1(\omega) = (1 + a_1 e^{-i\omega C_1})\hat{\hat{s}}(\omega) \tag{8}$$

$$\hat{x}_2(\omega) = (a_2 e^{-i\omega C_2} + a_3 e^{-i\omega C_3})\hat{\hat{s}}(\omega)\ . \tag{9}$$

Here, complex valued quotient function $\hat{h}(\omega)$ is introduced and defined by

$$\hat{h}(\omega) = \frac{\hat{x}_2(\omega)}{\hat{x}_1(\omega)} = \frac{a_2 e^{-i\omega C_2} + a_3 e^{-i\omega C_3}}{1 + a_1 e^{-i\omega C_1}}\ . \tag{10}$$

In eq.(10) only $\hat{h}(\omega)$ is known data and a_v and C_v (v=1,2,3) are unknown values, subsuffix v=1,3 indicates the reflected signal and v=2 indicates the direct signal. So a_v and C_v can not be generally specified. But by calculating some candidate values a_v in the following conditions, we can limit the values of a_v.

Condition i) $e^{-i\omega C_v}(v=1,2,3)$ is limited to some candidate values at certain ω. Then, we can obtain some combinations of possible solutions a_1, a_2 and a_3.

Condition ii) In eq.(10), since $\hat{h}(\omega)$ and $e^{-i\omega C_v}$ are the complex values, most of a_v calculated for arbitrary C_v takes the complex value. But adequate a_v is the real value by assumption. So, the value of a_v obtained in i) is still limited.

From i) and ii), three equations constructed from a_v and $\hat{h}(\omega)$ are derived and we obtain a_v by solving those equations. After this, we explain above two conditions in detail.

The index part of $e^{-i\omega C_v}(v=1,2,3)$ in eq.(10) is rewritten as discrete data. Here, $\Delta\omega$ denotes the fundamental frequency, l an integer from 0 to $N/2$($N/2$: Nyquist Frequency), Δt the time interval of discrete data point, D_v the relative step lag which is an integer and N the total number of discrete data point which is the power of two. By substituting $\omega=\Delta\omega\times l$, $C_v=\Delta t\times D_v$, $D_v=O_v\times 2^{n_v}$ ($n_v \geq 0$) into the index part, the following equation can be obtained.

$$\begin{aligned} -i\omega C_v &= -i\cdot\Delta\omega\cdot l\cdot\Delta t\cdot D_v \\ &= -i\cdot\frac{2\pi}{N}\cdot l\cdot D_v \\ &= -i\frac{l\cdot O_v\cdot 2^{n_v+1}}{N}\pi \\ &= -i\theta_v\pi \quad \left(\theta_v = \frac{l\cdot O_v\cdot 2^{n_v+1}}{N}\right) \end{aligned} \tag{11}$$

where O_v denotes the part of the odd number of D_v, n_v an integer number greater than or equal to 0.

In eq.(11), we set $l=N/2^n$, then θ_v is represented by the following equation.

$$\theta_v = O_v\cdot 2^{n_v-n+1}\ . \tag{12}$$

Where n is an integer and greater than 0. In eq.(12), the θ_v takes odd number with $n_v=n-1$ and even number with $n_v \geq n-1$.

Here, we introduce symbol m as the following equation.

$$m = \mathrm{Min}\left(n_1, n_2, n_3\right) + 1\ . \tag{13}$$

If we set $n=m$, at least one of the θ_v becomes odd number. It means that when we set $l=N/2^m$ the value of $e^{-i\theta_v\pi}$ is determined as

$$e^{-i\theta_v\pi} = \pm 1 \ \ (\theta_v\pi = I\pi) \tag{14}$$

where I denotes an integer. Here, $e^{-i\theta_v\pi}$ is rewritten as the polar form and the unit circle. Then eq.(14) can be expressed by the unit circle shown as Figure 1.

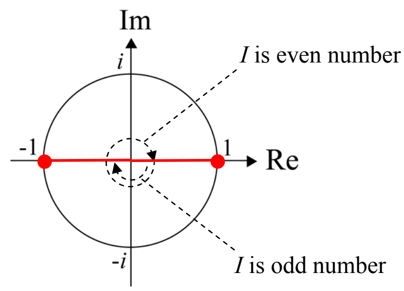

Figure 1 Illustration of the unit circle of $e^{-i\theta_v\pi}$ when $\theta_v=I\pi$.

In addition, when we set $l=N/2^{m+1}$ ($n=m+1$), one half of l in eq.(14), the value of $e^{-i\theta_v\pi}$ is determined as

$$e^{-i\theta_v\pi} = \begin{cases} \pm 1 & \left(\theta_v\pi = I\pi\right) \\ \pm i & \left(\theta_v\pi = \dfrac{\pi}{2} + I\pi\right) \end{cases} \tag{15}$$

When $e^{-i\theta_v\pi}$ takes $+1$ in eq.(14), then $e^{-i\theta_v\pi}$ takes either $+1$ or -1 in eq.(15). On the other hand, when $e^{-i\theta_v\pi}$ takes -1 in eq.(14), then $e^{-i\theta_v\pi}$ takes either $+i$ or $-i$ in eq.(15). Which value of $\pm i$ in eq.(15) $e^{-i\theta_v\pi}$ takes can be expressed by following figure.

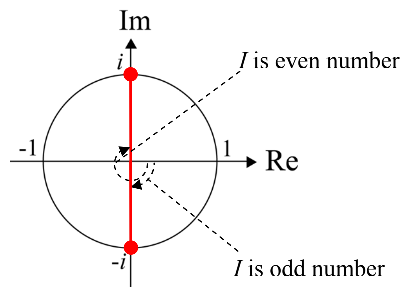

Figure 2 The value of $e^{-i\theta_v\pi}$ when $\theta_v=\pi/2+I\pi$.

Moreover, when we set $l=N/2^{m+2}$ ($n=m+2$), one half of l in eq.(15), the value of $e^{-i\theta_v\pi}$ is determined as

$$e^{-i\theta_v\pi}=\begin{cases}\pm 1 & (\theta_v\pi = I\pi)\\ \pm i & \left(\theta_v\pi=\dfrac{\pi}{2}+I\pi\right)\\ \pm\dfrac{1}{\sqrt{2}}\pm\dfrac{i}{\sqrt{2}} & \left(\theta_v\pi=\dfrac{3\pi}{4}+I\pi\right)\\ \pm\dfrac{1}{\sqrt{2}}\mp\dfrac{i}{\sqrt{2}} & \left(\theta_v\pi=\dfrac{\pi}{4}+I\pi\right)\end{cases} \tag{16}$$

The case of $e^{-i\theta_v\pi}=\pm 1$ or $e^{-i\theta_v\pi}=\pm i$ is explained in the same way as noted above. When $e^{-i\theta_v\pi}$ takes $+i$ in eq.(15), then $e^{-i\theta_v\pi}$ takes either $+1/\sqrt{2}+i/\sqrt{2}$ or $-1/\sqrt{2}-i/\sqrt{2}$ in eq.(16). On the other hand, when $e^{-i\theta_v\pi}$ takes $-i$ in eq.(15), then $e^{-i\theta_v\pi}$ takes either $+1/\sqrt{2}-i/\sqrt{2}$ or $-1/\sqrt{2}+i/\sqrt{2}$ in eq.(16). Which value of $\pm 1/\sqrt{2}\pm i/\sqrt{2}$ or $\pm 1/\sqrt{2}\mp i/\sqrt{2}$ in eq.(16) $e^{-i\theta_v\pi}$ takes can be expressed by following figures.

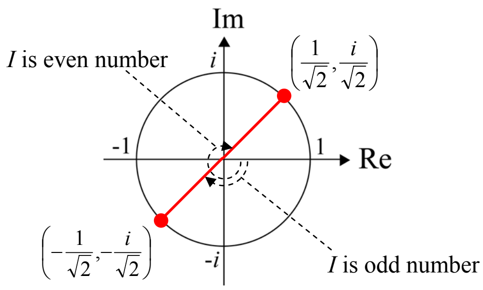

Figure 3 The value of $e^{-i\theta_v\pi}$ when θ_v=3π/4+$I\pi$.

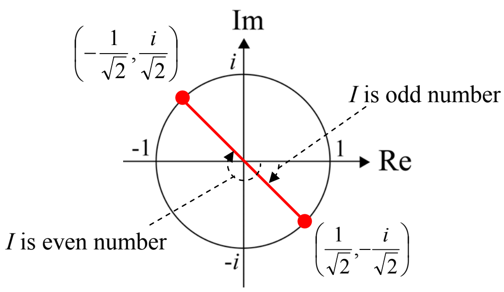

Figure 4 The value of $e^{-i\theta_v\pi}$ when θ_v=π/4+$I\pi$.

Since $e^{-i\theta_v\pi}$ takes the same value in 2π-period and l is set one half of the previous value of l, θ_v become also one half of the previous value of it and then $e^{-i\theta_v\pi}$ takes the different value in π-period. That is to say, there are always two possible values of $e^{-i\theta_v\pi}$ respectively, and those values depend on the previous value of $e^{-i\theta_v\pi}$.

Here, when we set l=$N/2^m$, eq.(10) can be represented by

$$\hat{h}(\omega)=\hat{h}\left(\Delta\omega\cdot\frac{N}{2^m}\right)=\frac{a_2p_2+a_3p_3}{1+a_1p_1}=\alpha_1 \tag{17}$$

where p_v (v=1,2,3) replaces $e^{-i\theta_v\pi}$ and takes either $+1$ or -1 and at least one of the p_v takes the value -1 because of eqs.(13),(14). Since a_v is the real values, α_1 is also the real value.

Next, we set l=$N/2^{m+1}$. Then eq.(10) can be represented by

$$\hat{h}\left(\Delta\omega\cdot\frac{N}{2^{m+1}}\right)=\frac{a_2q_2+a_3q_3}{1+a_1q_1}=\alpha_2 \tag{18}$$

where q_v also replaces $e^{-i\theta_v\pi}$ and takes one of the values $+1$, -1, $+i$ and $-i$ and at least one of the q_v is either $+i$ or $-i$ if p_v takes -1. The reason for these is that the arguement of the q_v is a half value of that of the p_v.

Next, we set l=$N/2^{m+2}$. Then eq.(10) can be represented by

$$\hat{h}\left(\Delta\omega\cdot\frac{N}{2^{m+2}}\right)=\frac{a_2r_2+a_3r_3}{1+a_1r_1}=\alpha_3 \tag{19}$$

where r_v also replaces $e^{-i\theta_v\pi}$ and the argument of r_v is a half value of that of q_v. At least the argument of r_v is a half value of that of q_v= i or q_v=$-i$.

Here, eqs.(17),(18),(19) are rewritten as follows.

$$\begin{aligned}&-p_1\alpha_1a_1+p_2a_2+p_3a_3=\alpha_1\cdot\\&-q_1\alpha_2a_1+q_2a_2+q_3a_3=\alpha_2\cdot\\&-r_1\alpha_3a_1+r_2a_2+r_3a_3=\alpha_3\cdot\end{aligned} \tag{20}$$

Eq.(20) can be written in matrix form.

$$\begin{bmatrix}-p_1\alpha_1 & p_2 & p_3\\ -q_1\alpha_2 & q_2 & q_3\\ -r_1\alpha_3 & r_2 & r_3\end{bmatrix}\begin{Bmatrix}a_1\\a_2\\a_3\end{Bmatrix}=\begin{Bmatrix}\alpha_1\\\alpha_2\\\alpha_3\end{Bmatrix}. \tag{21}$$

we rewrite eq.(21) as

$$\mathbf{Ba}=\boldsymbol{\alpha} \tag{22}$$

where

$$\mathbf{B}=\begin{bmatrix}-p_1\alpha_1 & p_2 & p_3\\ -q_1\alpha_2 & q_2 & q_2\\ -r_1\alpha_3 & r_2 & r_2\end{bmatrix},\ \mathbf{a}=\begin{Bmatrix}a_1\\a_2\\a_3\end{Bmatrix},\ \boldsymbol{\alpha}=\begin{Bmatrix}\alpha_1\\\alpha_2\\\alpha_3\end{Bmatrix}. \tag{23}$$

Here, the vector **α** is known by eqs.(17),(18),(19). The matrix **B** is known because p_v, q_v, r_v are known values although they are given from some candidate values by eqs.(17),(18),(19). Therefore the unknown vector is only the vector **a**. Then, the entries of the vector **a** are real values because a_v are defined as real values and the entries of the matrixs **B**, **α** are complex values. Therefore, using an arbitrary combination of p_v, q_v, r_v, most of the entries of the **a** are obtained as complex values, however using the adequate combination of p_v, q_v, r_v, the entries of the **a** are obtained as real values. But, the possibilities that the entries of **a** are obtained as real values by using an arbitrary combination of p_v, q_v, r_v still remain. For above process, some combinations of a_v taking real values are obtained.

2.3 Specification of the relative time lags

Eq.(10) is deformed and rewritten by eq.(11) as following equation.

$$e^{-i\frac{2\pi}{N}lD_1}=\frac{1}{\hat{h}(\Delta\omega l)}\left(\frac{a_2}{a_1}e^{-i\frac{2\pi}{N}lD_2}+\frac{a_3}{a_1}e^{-i\frac{2\pi}{N}lD_3}\right)-\frac{1}{a_1}. \tag{24}$$

Substituting some candidates a_v which are obtained at **2.2** into eq.(24) and calculating by varying D_2, D_3 with fixed l, we obtain the pair (D_2, D_3) with which the absolute value of the right side of eq.(24) takes 1(eq.(25)).

$$\left|\frac{1}{\hat{h}(\Delta\omega l)}\left(\frac{a_2}{a_1}e^{-i\frac{2\pi}{N}lD_2}+\frac{a_3}{a_1}e^{-i\frac{2\pi}{N}lD_3}\right)-\frac{1}{a_1}\right|=1. \tag{25}$$

If the inadequate values of a_v, D_2, D_3 are substituted into the left side of eq.(25), we don't have equality at eq.(24) and the left side of eq.(25) is not 1. When the left side of eq.(25) is 1, there is a possibility that the obtained pair (D_2, D_3) is adequate values. Substituting the pair (D_2, D_3) into eq.(24), we obtained D_1.

If the a_v, D_v obtained at **2.2** and **2.3** are adequate values, for all ω, we have equality at eq.(24). If we substitute inadequate a_v, D_v into eq.(24), for all ω, there is a small possibility that we have equality at eq.(24). Therefore we need to check if we have equality at eq.(24) with the obtained a_v, D_v for any ω. Then, we calculate relative time lags C_v as following equation by using D_v.

$$C_v = \Delta t \cdot D_v\,. \tag{26}$$

In eq.(10), we use $x_3(t)$, $x_4(t)$ instead of $x_2(t)$ and the same operations are repeated from **2.2** to **2.3**, the attenuation ratios a_v and relative time lags C_v which are necessary to specify and separate the source signal can be obtained.

2.4 Specification of the location of source signal

Using obtained relative time lags, the location of the source signal s can be specified. From eq.(26), relative distance d_v is calculated as following equation.

$$d_v = C_v \cdot V = \Delta t \cdot D_v \cdot V\,. \tag{27}$$

where V denotes the propagation velocity (≒340 [m/sec] in case of sound). The relative distance of direct signal is used in order to specify location of source signal. So, the relative distance d_v can be known for each k. In the case of the two-dimensional space (the three-dim), if three (four) relative distance of direct signal can be known, the location of source signal is specified. The details of the above process are shown in reference [1], [2].

2.5 Separation of source signal

We rewrite eq.(8) as follows.

$$\hat{\tilde{s}}(\omega) = \frac{\hat{x}_1(\omega)}{1+a_1 e^{-i\omega C_1}}\,. \tag{28}$$

Therefore, by substituting the a_1, C_1 obtained at **2.2**, **2.3** into eq.(28) and solving it for each ω, $\hat{\tilde{s}}(\omega)$ can be calculated. Finally, $\tilde{s}(t)$ can be calculated by Inverse Fourier Transform (F^{-1}).

$$\tilde{s}(t) = F^{-1}\left[\hat{\tilde{s}}(\omega)\right]$$

$$= F^{-1}\left[\frac{\hat{x}_1(\omega)}{1+a_1 e^{-i\omega C_1}}\right]. \tag{29}$$

The difference between $s(t)$ and $\tilde{s}(t)$ is the multiple of constant value a_{11} and the time lag of the constant value c_{11} in the time domain from eq.(3). In this paper, $\tilde{s}(t)$ can be calculated instead of $s(t)$.

3 NUMERICAL TEST

3.1 Problem setting

Numerical test can be conducted to confirm our method. In this example, the two-dimensional space is assumed. The locations of source point and observation points (M=4) are illustrated in Figure 5. The propagation velocity sets 340 [m/sec]. The source signal is shown in Figure 6. The source signal $s(t)$ is male voice in English. Sampling frequency is 44100[Hz], total duration time is 2.98[sec], and number of total step is 131072(=2^{17}). The observation data are constructed under the assumption of eq.(1) and shown in Figure 7. Figure 8 shows the close-ups of $s(t)$ and $x_1(t)$ from 0 to 0.5[sec]. The attenuation coefficients, the step lags, the attenuation ratios and the relative step lags are set in the following Tables.

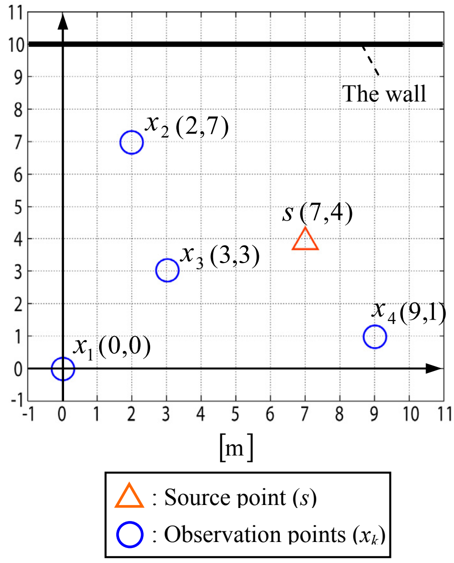

Figure 5 Locations of s and x_k.

Table 1 The setting values of the attenuation coefficients.

	Direct signal	Reflected signal
x_1	0.124	0.057
x_2	0.171	0.097
x_3	0.243	0.074
x_4	0.277	0.066

Table 2 The setting values of the step lags.

	Direct signal	Reflected signal
x_1	1046	2265
x_2	756	1335
x_3	535	1764
x_4	468	1963

Table 3 The setting values of the attenuation ratios.

	a_1	a_2	a_3
x_2/x_1	0.462	1.383	0.783
x_3/x_1	0.462	1.955	0.593
x_4/x_1	0.462	2.236	0.533

Table 4 The setting values of the relative step lags.

	D_1	D_2	D_3
x_2/x_1	1219	-290	289
x_3/x_1	1219	-511	718
x_4/x_1	1219	-578	917

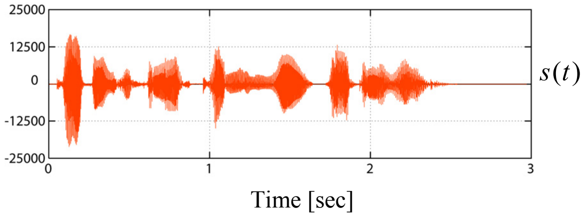

Figure 6 Source signal $s(t)$.

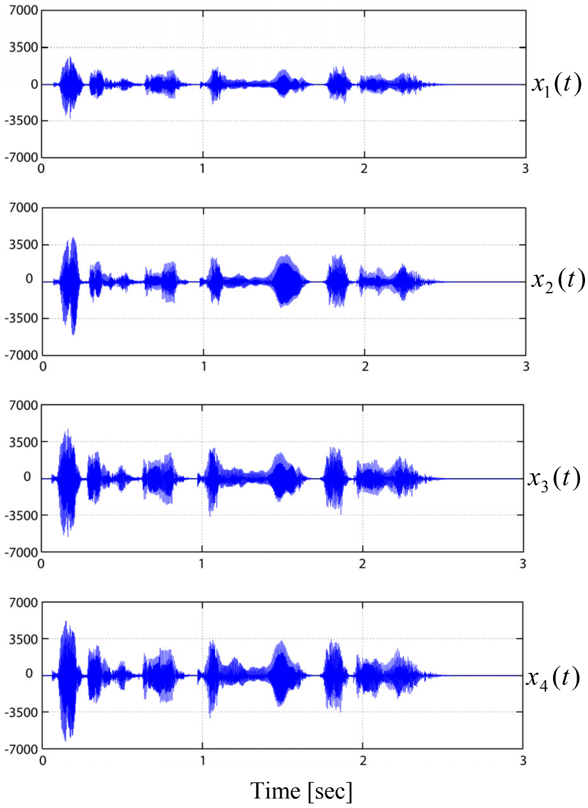

Figure 7 Observation signals $x_k(t)$.

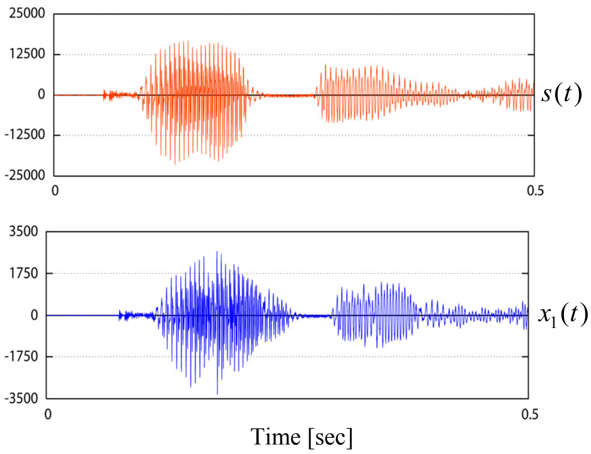

Figure 8 The close-ups of $s(t)$ and $x_1(t)$ from 0 to 0.5[sec]

3.2 Specification of the attenuation ratios and relative time lags

From now, the observation signals $x_k(t)$ and their locations are only known data. From **2.2** and **2.3**, the attenuation ratios a_v and the relative step lags D_v are obtained. Obtained those values are shown in Table 5 and Table 6. Compared with the setting values and obtained values, there are no errors in those Tables.

Table 5 Obtained attenuation ratios.

	a_1	a_2	a_3
x_2/x_1	0.462	1.383	0.783
x_3/x_1	0.462	1.955	0.593
x_4/x_1	0.462	2.236	0.533

Table 6 Obtained relative step lags.

	D_1	D_2	D_3
x_2/x_1	1219	-290	289
x_3/x_1	1219	-511	718
x_4/x_1	1219	-578	917

3.3 Specification of the location of source signals

From eq.(27) and **3.2**, the relative distance can be known. Now, there are four observation points so that three relative distances of direct signal can be found by the source point. Figure 9 is hyperbolic curves from the source point described in the relative distances of direct signal. In this case, a source point must exist on three curves. Therefore, the location of the source point is specified to compute the point of intersection. The result of identified the source point is shown in Figure 10. There is an error of about 0.45 mm.

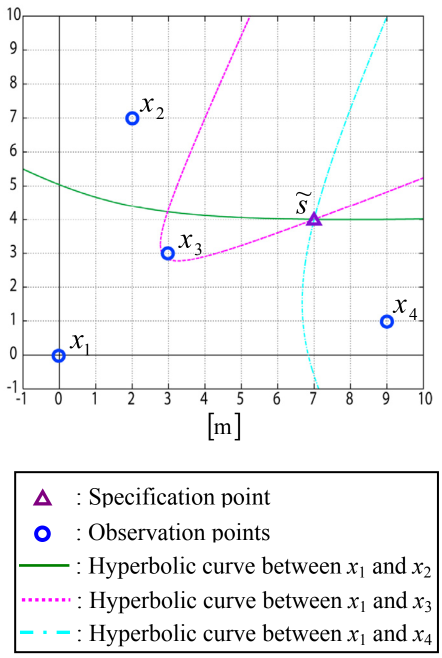

Figure 9 Hyperbolic curves to specify source point.

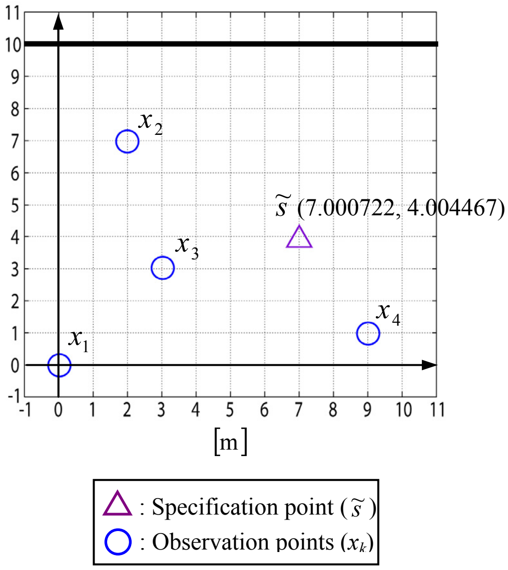

Figure 10 Result of specification of source point.

3.4 Separation of the source signal

The attenuation ratio a_1 and the relative time lag C_1 are obtained from **3.2** and eq.(26). Namely, each of unknown variable in eq.(28) except for $\hat{\tilde{s}}(\omega)$ has turned out to be known. So $\hat{\tilde{s}}(\omega)$ can be calculated to solve eq.(28) on every ω. Therefore $\tilde{s}(t)$ can be obtained by Inverse Fourier Transform of $\hat{\tilde{s}}(\omega)$ in eq.(29).

However, the calculated source signal $\tilde{s}(t)$ doesn't coincide with actual source $s(t)$ concerning with the size of amplitude. If we assume that the attenuation coefficient is in inverse proportion to the propagation distance, the attenuation a_{11} can be calculated as 0.1240. In a similar way, the step lag $c_{11}\times44100$ between the location of s specified at **3.3** and that of x_1 can be calculated as 1046.

By multiplying $\tilde{s}(t)$ and the a_{11}, shifting the step lag $c_{11}\times44100$, we finally get $\tilde{s}(t)$ which coincides with $s(t)$. Figure 11 shows calculated signal $\tilde{s}(t)$. Compared with actual source signal $s(t)$ (Figure 6), there are good coincidence and errors of less than 0.04%. The close-up of $\tilde{s}(t)$ from 0 to 0.5[sec] is shown in Figure 12.

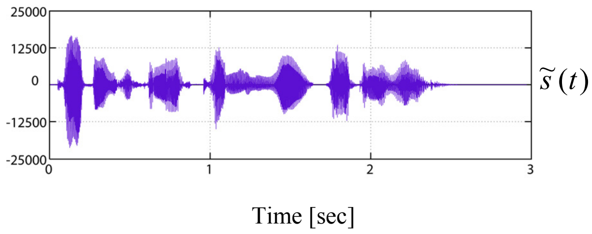

Figure 11 Calculated source signal $\tilde{s}(t)$.

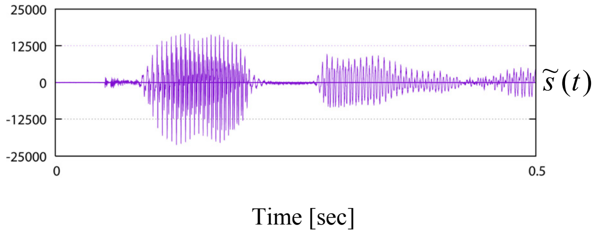

Figure 12 The close-up of $\tilde{s}(t)$ from 0 to 0.5[sec].

4 CONCLUSION

The method for the separation of source signal and the specification of location of source signal was formulated considering one reflected signal. Using this method, the separation and the specification of location can be conducted, under the assumption of relationship between source signal and observation signals. It is confirmed through the numerical test that the specification of location has been archieved almost completely and the separation has been done with high accuracy.

5 REFERENCES

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