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A method to calculate the acoustic intensity near an open end of a flanged round pipe

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ABSTRACT

The behaviour of the acoustic intensity field near an open end of a round pipe is complex and poorly understood. In this paper we propose an efficient method to study the acoustic intensity in a pipe with the Neumann boundary conditions on its walls. We assume that the sound field in this pipe is excited by a plane wave that is incident from the far field on its open end. We also assume that this end of the pipe is flanged in a rigid baffle. We present the total sound field in the pipe as a superposition of propagating and evanescent modes. We use the Huygens-Fresnel principle to formulate the radiation conditions at the open end of the pipe. We adopt the orthogonality condition for normal modes to derive the equation for the modal coefficients in the reflected sound field. We compute accurately the singular integrals which appear in the equation for the modal coefficients using the Telles numerical integration scheme. Finally, we validate the proposed model against experimental data obtained for an 80mm PVC pipe.

INTRODUCTION

Background

The problem of radiation and reflection of acoustic pressure in the circular pipe, for both unflanged and flanged pipes, has been extensively studied over the recent years. Nomura et al. solved the problem of acoustic radiation from a flanged circular pipe using the normal mode decomposition method [3], Felsen et al. studied the same problem using the ry method [4]. Commercial finite element packages exist that can calculate efficiently the acoustic pressure in the vicinity of the open pipe end. However, there has been little or no interest in the acoustic intensity field in the vicinity of the pipe end. This problem relates to the accuracy of acoustic instrumentation that is used to inspect quality of air-filled pipes as the intensity field near the open pipe end is complex and fluctuates rapidly. As a result, the quality of the measured data is very sensitive to the position of the intensity probe near the open pipe end and it is affected by the reflections from the open end which are not obvious but can interfere with required data. In this paper we are presenting a method to calculate the acoustic intensity in a vicinity of an open end of the pipe and comparing it with the experimental results, with theoretical model, using reflection coefficients for the open end of a pipe derived in ref. [1] and with the finite element model designed in Comsol software.

METHODOLOGY

Theoretical model

We consider a situation where a plane wave is incident on the open end of the pipe from the far field as shown in Figure 1.

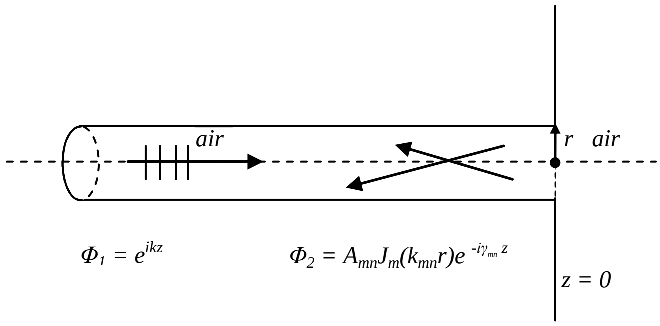

Figure 1. Schematic drawing of the flanged pipe model

The acoustic filed in the pipe is represented with a set of normal modes that are scattered by the open end of the pipe:

$$\Phi_1 = e^{ikz} + \sum_{m,n} A_{mn} cos(m\theta) J_m(k_{mn}r) e^{-i\gamma_{mn}z}, \qquad (1)$$

where k_{mn} are the zeros of the first derivative of the Bessel function and $\gamma_{mn} = \sqrt{k^2 - k_{mn}^2}$ are the wavenumbers of the normal modes propagating in the pipe.

Sound field outside the pipe is given by Huygens-Fresnel equation

$$\Phi_2 = \frac{1}{4\pi} \iint_S \frac{\partial \Phi_1}{\partial n} \frac{e^{ikR}}{R} dS \qquad (2)$$

The velocity continuity condition is used at the open end of the pipe, $\Phi_1|_{z=0} = \Phi_2|_{z=0}$ together with boundary and orthogonality conditions to derive unknown modal reflection coefficients in eq. (1), A_{mn}. Singular integrals, which appear in the solution, are accurately calculated using the numerical integration scheme proposed by Telles [2]

$$\int_0^b \frac{f(y)}{y} dy \cong \sum_{i=1}^{n} f(b\xi_i) w_i + f(0) \ln(b). \quad (3)$$

Once the modal reflection coefficients are known, it is possible to determine the acoustic potential, Φ_1, inside the pipe. The spatial derivatives of the acoustic potential can then be taken analytically to find the acoustic velocity and the acoustic intensity as a product of the acoustic pressure and the acoustic velocity.

Model validation

Theoretical results were validated against experimental data obtained using a laboratory setup. This setup consisted of a 2 m long metal pipe that had an 80mm internal diameter. The pipe was terminated with a PVC flange at one end as shown in Figure 2.

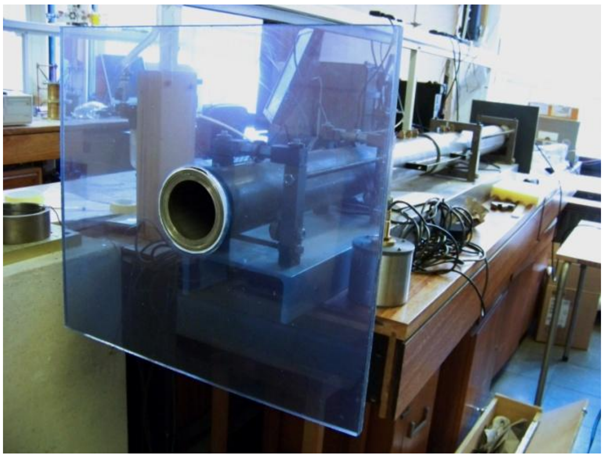

Figure 2. Experimental setup

A sound wave was produced by a loudspeaker at the pipe end opposite to the flanged end of the pipe. The acoustic signal was registered in the vicinity of an open end of the pipe using a Brüel and Kjær intensity probe Type 3520 (See Figure 3) with two microphones spaced 12mm apart. The signals were passed to a PULSE module through a pair of matched microphone channels. Standard PULSE software was used to calculate the sound pressure, velocity and the intensity.

Figure 3. Intensity probe Type 3520

The experimental data was also used to compare the prediction by an alternative theoretical model [1]. In this model the acoustic field in the pipe is represented as a superposition of an incident plane wave and its reflection from the pipe end

$$p(z) = e^{ikz} + Re^{-ikz} \quad (4a)$$

$$u(z) = ike^{ikz} - ikRe^{-ikz} \quad (4b)$$

$$I(z) = ike^{2ikz} - ikR^2 e^{-2ikz} \quad (4c)$$

where p, u and I are the acoustic pressure, velocity and intensity, respectively, and R is the reflection coefficient from the pipe end given by exp. (2) in ref. [1].

A model of the flanged pipe (see Figure 2) was also created using a FEM package Comsol assuming a 2-D axial symmetry of the problem. The flange was covered with a hemisphere of perfectly matching layer to prevent reflections and the size of a flange was chosen to be both greater than the wavelength and sensible for computations as shown in Figure 4.

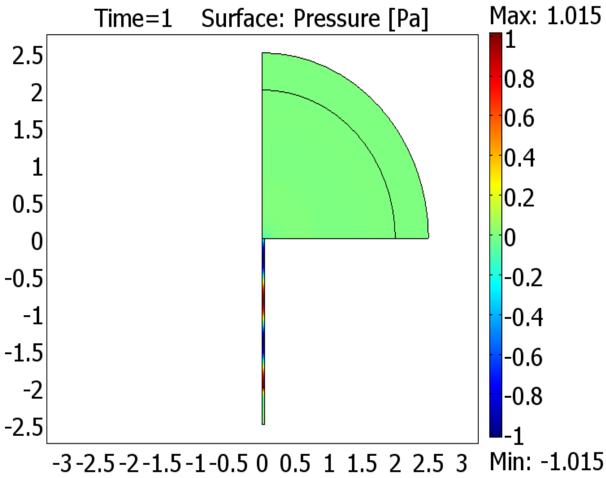

Figure 4. Flanged pipe model in Comsol. The left and bottom scales demonstrate model's dimensions in space. The right scale shows the pressure distribution in the model.

The length of the pipe was set to 2 m and the diameter of flange was 4 m. The model was divided into 1 cm triangular elements and solved using a UMFPACK solver.

RESULTS

Pressure

The theoretical pressure data were obtained by coding the equations in the proposed theoretical model and running the code in Matlab. These predictions were compared against the experimental data, predictions obtained via expressions (4) with the reflection coefficient calculated according to the formula presented in ref. [1] and, finally, against the results of Comsol FEM.

All the predictions and data were obtained at two frequencies: 300 and 600 Hz. In the proposed theoretical model the modal analysis was limited to the first 12 propagating modes, which was considered sufficient to achieve a balance between the accuracy and computation time. Figures 5-8 present the predicted and measured acoustic pressures and velocities.

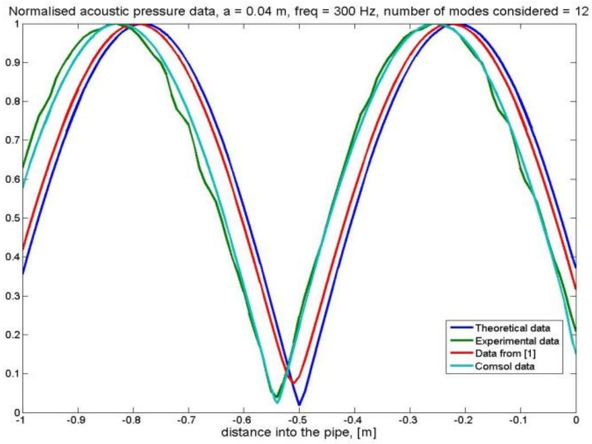

Figure 5. The normalised acoustic pressure as a function of the distance in the pipe (300 Hz).

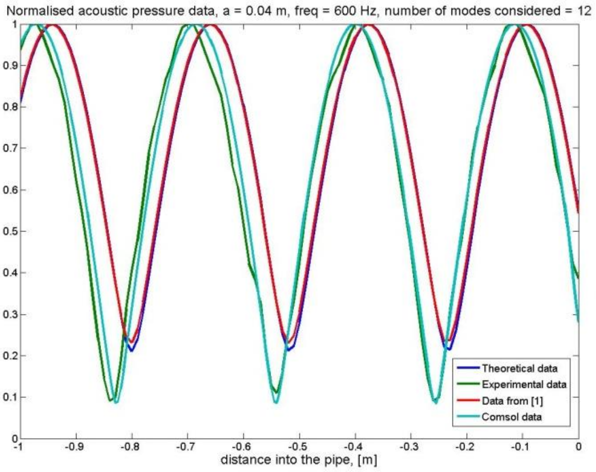

Figure 6. The normalised acoustic pressure as a function of the distance in the pipe (600 Hz).

From the graphs it can be seen that the data and predictions do not fully agree. The predictions by the proposed theoretical model are the closest to the predictions by the model detailed in ref. [1]. The cause for the disagreement between the theoretical predictions and the measured data is the absence of the far-end reflection as it was assumed that acoustic wave was incident on the flanged end from the infinity.

The other two sets of results, the experimental data and the predictions by Comsol, are in very close agreement with each other. These results are shifted approximately 5 cm further into the pipe when compared to the two theoretical predictions. Obviously, a semi-infinite pipe and infinite flange cannot be reproduced in the laboratory, so the sound wave is reflected from the both ends of the pipe during the measurement. This leads to the creation of standing waves in the pipe and the actual positions of the maxima and minima in these waves can differ from those predicted by an idealistic model.

Velocity

In addition to the acoustic pressure, the acoustic velocity in the pipe was measured and calculated. The measured and calculated results were compared as shown in Figures 7 and 8.

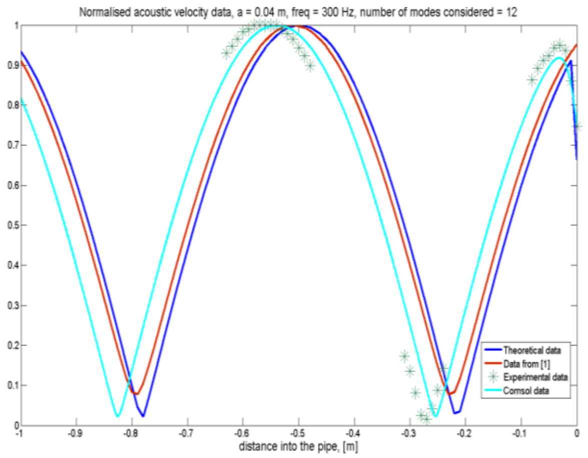

Figure 7. The normalised acoustic velocity as a function of the distance in the pipe (300 Hz).

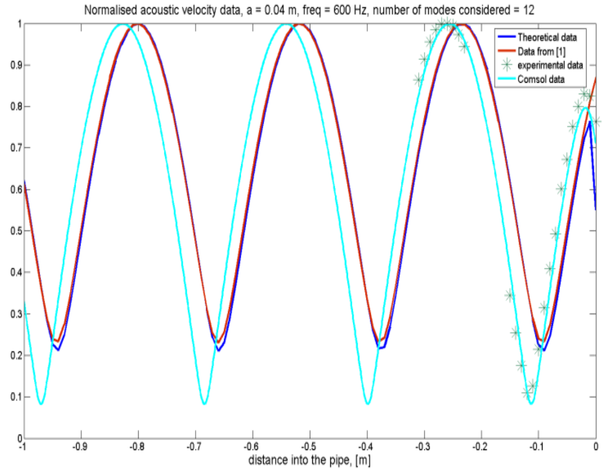

Figure 8. The normalised acoustic velocity as a function of the distance in the pipe (600 Hz).

This comparison suggests that the velocity predicted by the method detailed in ref. [1] is almost identical to that predicted by the proposed theoretical method except the vicinity of the flanged end of the pipe. These two theoretical models are based on the similar assumption and a good agreement between the two is expected. In the vicinity of the flanged end the prediction of the proposed theoretical model are close to the measured data. The model suggested in ref. [1] does not explain the behaviour of the experimental data near the pipe end where the discrepancy between the two theoretical methods is obvious from the comparison in Figures 7 and 8.

The acoustic velocity calculated by Comsol matches well the experimental data. This result is closer to the theoretical predictions near the pipe end. The result predicted by Comsol can be affected by the performance of the perfectly matching layers that were placed on both ends of the pipe and by the finite size of the mesh.

Intensity

The acoustic intensity was also measured. However, the quality of the measured intensity data is inferior to that of the pressure and velocity data presented in the previous sections. Figures 9 and 10 show a comparison between the measured and predicted acoustic intensities.

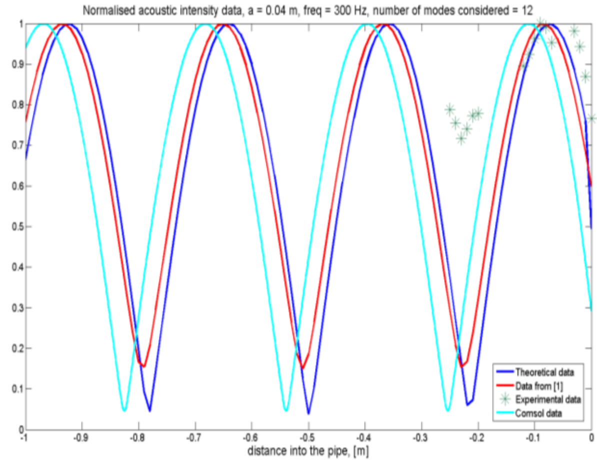

Figure 9. The normalised acoustic intensity as a function of the distance in the pipe (300 Hz).

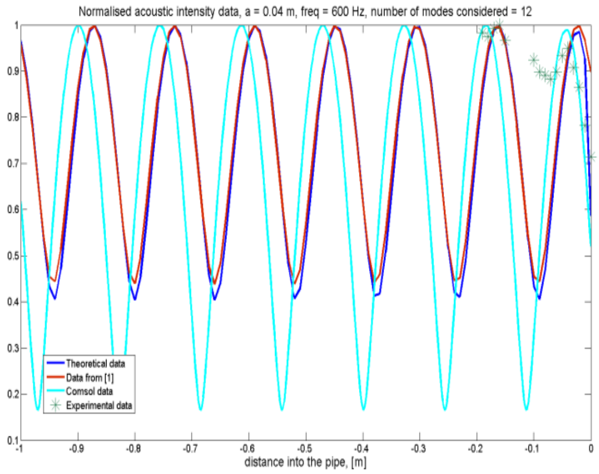

Figure 10. The normalised acoustic intensity as a function of the distance in the pipe (600 Hz).

There is a good agreement between the results predicted by the two theoretical models except near the pipe end (see Figures 9 and 10). The predicted intensities for 300 Hz are slightly shifted with respect to each other. This can be associated with the shift in the effective acoustic end of the pipe [5] assumed in expressions (4).

The agreement between the measured intensity data and the predictions by the proposed theoretical model is close in the immediate vicinity of the pipe end. Outside this region, the predicted intensity follows the trend in the measured data but does not match properly its rather complex behaviour. During the measurement, it was noticed that the intensity readings were unstable, which could be a result of: (i) microphone pair mismatch; (ii) inaccuracies in the intensity probe positioning; (iii) limited signal to noise ratio.

CONCLUSIONS

In this paper the acoustic pressure, velocity and intensity fields near an open end of a semi-infinite flanged pipe have been examined. A theoretical method has been proposed and validated against another theoretical method, FEM model and measured data. Theoretical predictions agree well in the case of the pressure, velocity and intensity except the immediate vicinity of the pipe end. The predictions by Comsol FEM model explain closely the behaviour of the acoustic pressure and velocity in the pipe, but do not fully explain the complex behaviour of the acoustic intensity. The predictions by the proposed theoretical model follow closely the the measured intensity data near the vicinity of the pipe end, but do not match the data outside this region. This could happen by a number of reasons: (i) the presence of microphone holes in the pipe as shown in Figure 11; (ii) imperfections in the measurement technique and equipment; (iii) inability of the model to represent the acoustic field in a pipe of a finite length. The effect of the loudspeaker on the acoustic field cannot be ignored as well. The boundary conditions at the loudspeaker are not perfectly absorbing or perfectly reflecting. As a result, the reflection coefficient from the loudspeaker is frequency-dependent and cannot be easily modelled and compensated for.

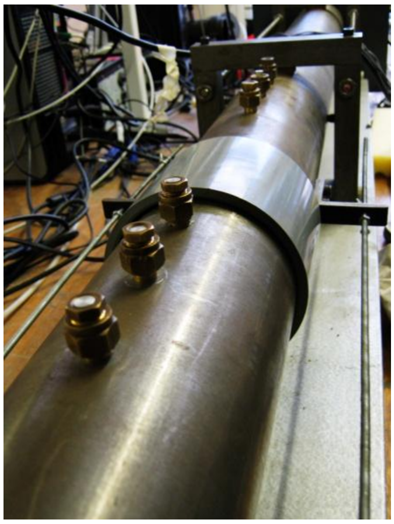

Figure 11. Microphone holes.

The predictions and measured data confirm that the intensity field in the pipe is rather complex and that the intensity changes rapidly near the flanged end of the pipe. The quality of the experimental data for the acoustic intensity has been limited. Therefore, more accurate experimental data for the intensity field and a better experimental setup are needed to validate the proposed theoretical model. This can be achieved through the improved experimental conditions and better quality measurement procedure.

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