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Parameters estimation of point moving source with time-frequency transformation

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ABSTRACT

A new parameter estimation method for a point moving harmonic source with unknown moving velocity and frequency is presented in this paper. The time-frequency representation of the source signal is taking the place of traditional time correlation estimation methods. For a harmonic source moving at constant velocity, the received signal which is amplitude and frequency modulated has no spatial correlation between microphone pairs in time or frequency domain which make the estimation problem become complicated. Besides, it is observed that the Doppler shifted frequency of the signal correlate well between spatial distributed microphone signals. Moreover, the second time derivative of the Doppler shifted frequency gives the reference time for the estimation of the source parameters in time domain. In this paper, the algorithm for estimation based on time-frequency transformation is presented. The adaptability of short-time Fourier transform (STFT), filtered short-time Fourier transform (FSTFT) and Polynomial Phase Estimation (PPE) method are also illustrated. In addition, the performance of the analysers on difference source velocity, frequency and signal to noise ratio in computer simulations is presented. The results demonstrate the validity of the proposed method to give a rigid estimation in constant speed moving source. This paper concludes with further investigation and discussion about the proposed method.

INTRODUCTION

The estimation of moving sound sources parameters through measurement is a significant process in acoustics. It is obvious that direct measurement of sound pressure from a moving source which is amplitude and frequency modulated of the original signal, signal information is unavailable to extract if no prior knowledge of the source. Pre-processing of the measured data is required for the parameter estimation. A commonly used method, sweeping focus method employed by Barsikow and King [1], Barsikow [2] and To and Yung [3] to eliminate the Doppler effect by adjusting the direction of focus to the source with a directional microphone array at the same speed of the source which assumed to be known. Besides, instantaneous frequency based methods has been used for moving source localization by Ferguson and Quinn [4,5] with Wigner-Ville transformation. The instantaneous frequency change also done by Poisson et al [6] with bilinear time-frequency transformation which simplifies the frequency modulation by linear approximation.

Three parameters are estimated including source velocity, frequency and power (amplitude). The harmonic source is travelling through a line trajectory which parallel to the array with constant speed. Under this situation, the received frequency and pressure remain a linear relationship at particular time instant, and only depend on the speed and the closest source microphone distance. The general way is to make use of the highest sound pressure time as the reference time. However, if the source velocity is unknown, the estimated frequency and power would be biased. Thus, the estimation of source velocity and representative time reference for the estimation problem are necessary.

This paper is organized as follows. Section 2 gives the review of the harmonic moving source model. The new reference time for the estimation based on maximum rate of change of received source frequency is presented in section 3. Three time-frequencies based instantaneous frequency estimators were reviewed including, short-time Fourier transforms (STFT), filtered short-time Fourier transforms (FSTFT), and Polynomials phase estimation (PPE) in section 4. Simulation results with various source speed, frequency and signal to noise ratio (SNR) are discussed in section 5.

SIGNAL MODEL

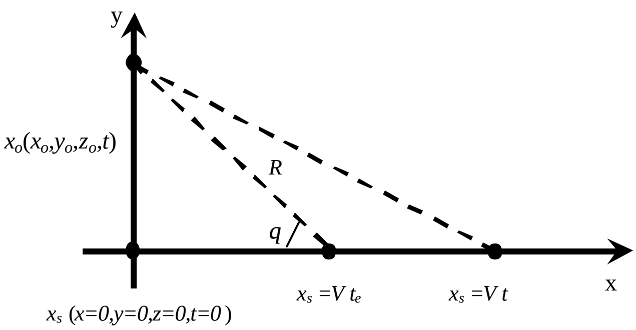

Figure 1. Point Harmonic Source Model

As shown in Fig 1, the distance R between the point moving source and the point of observation x_0 in Cartesian coordinate is,

$$R^2 = \left[x_0 - x_s\left(t - \frac{R}{c}\right)\right]^2 + \left[y_0 - y_s\left(t - \frac{R}{c}\right)\right]^2 + \left[z_0 - z_s\left(t - \frac{R}{c}\right)\right]^2 \quad (1)$$

For the source with strength q, moving along straight in x-axis at speed V smaller than speed of sound c, Morse [7] suggested the solution based on Lorentz transformation for the sound pressure p with respect to the observation point:

$$p(R,t) = \frac{q'(t - R/c)}{4pR(1 - M\cos q)^2} + \frac{(\cos q - M)Vq(t - R/c)}{4pR^2(1 - M\cos q)^3} \quad (2)$$

, where $M = V/c$ is the Mach number.

At time $t = 0$ and $y = z = 0$, equation (1) for R become

$$R = \frac{M(x_o - Vt) + R_1}{1 - M^2} \quad (3)$$

where $R_1 = \sqrt{(x_o - Vt)^2 + (1 - M^2)r}$, with $z > 0$ and $r = (y_o{}^2 + z_o{}^2)$. The angle θ is the direction of sound radiation at the instant of its emission t_e. This angle is related to V and R:

$$R\cos q - MR = x_o - Vt \quad (4)$$

For a point harmonic source with angular frequency w_0, $q = Ae^{jw_0 t}$. The sound pressure in equation (2) becomes

$$p(R,t) = \frac{jwRAe^{jw_0(t - R/c)}}{4p[(1 - M^2)R - M(x - Vt)]^2} + \frac{(x - Vt)VAe^{jw_0(t - R/c)}}{4p[(1 - M^2)R - M(x - Vt)]^3} \quad (5)$$

The received signal have the phase $f = w_0(t - R/c)$, the instantaneous frequency is defined as the first time derivative of the phase,

$$w = \frac{df}{dt} = \frac{w_0}{1 - M\cos q} \quad (6)$$

Normalization

Normalization Scale

Length scale: z_0, perpendicular distance of array center from the source path

Velocity scale: V_0, source velocity

Time scale: $t_0 = z_0 / V_0$

Pressure scale: $P_0 = r_0 c_0{}^2$

Frequency scale: $f_0 = \frac{V_0}{z_0}$

Normalized quantity

$x' = x / z_0$, $y' = y / z_0$, $z' = z / z_0$

$V' = V / V$, $M = V'/c'$, $t' = t\frac{V_0}{z_0}$,

$f' = f\frac{z_0}{V_0} = ft_0 = St$ (Strouhal Number)

PROPOSED ESTIMATION METHOD

The problem of moving source parameters estimation is complicated since the model depends on spatial variation in both source and receiver side. The highly non-linear characteristic of its received sound pressure and frequency enhance the difficulties. Most of the approaches for source power estimation use the maximum received pressure with time difference method to determine the source velocity and location, however the priority of the method is the source frequency at stationary,

From equation (6), the second time derivative of received Doppler frequency w is,

$$\frac{\partial^2 w}{\partial t^2} = \frac{w_0 M\left((M\cos q - 1)\left(\frac{\partial^2 \cos q}{\partial t^2}\right) - 2M\left(\frac{\partial \cos q}{\partial t}\right)^2\right)}{(M\cos q - 1)^3} \quad (7)$$

For $\frac{\partial^2 w}{\partial t^2} = 0$ which is at the maximum rate of change of source frequency

$$(M\cos q - 1)\left(\frac{\partial^2 \cos q}{\partial t^2}\right) = -2M\left(-\frac{V}{R} + \frac{V(x - Vt)}{R^2}\frac{\partial R}{\partial t}\right)^2 \quad (8)$$

With an analytical solution at $t_m = x / v$, which is dependence only on time invariance factor, microphone position and source velocity.

The source velocity can be estimated by time difference method,

$$\hat{V} = \frac{x_i - x_j}{\hat{t}_{mi} - \hat{t}_{mj}} \quad (9)$$

with microphone pair ij with corresponding time of maximum rate of change of frequency .

The source frequency w_0 and the amplitude A can be found by substituting the estimated velocity $\hat{V}$ into corresponding Doppler frequency and pressure at time equal at $t = t_m$ for each microphone, equation (5) and (6) becomes

$$w_0 = w_{t=t_m}\left(1 - \hat{M}^2\right) \quad (10)$$

and

$$\hat{A} = \frac{\left|p_{t=t_m}\right|\sqrt{(1-\hat{M}^2)}}{w_{t=t_m}}\sqrt{r} \quad (11)$$

The least square error solution for source frequency and amplitude would be obtained through the microphone array signals.

Instantaneous frequency (IF) estimation [8,9]

The key parameter, maximum rate of change of source frequency, can be estimated by obtaining the instantaneous frequency of the source through various time-frequency transform methods. This section introduces four of them including the Short-time Fourier transforms (STFT), Filtered short-time Fourier transforms (FSTFT) method and Polynomials phase estimation (PPE).

For a harmonic source with frequency f and phase $f(t)$, the analytic signal model with amplitude a is

$$x(t) = ae^{jf(t)} \quad (12)$$

, its IF is defined as the first time derivative of instantaneous phase,

$$f(t) = \frac{1}{2p}\frac{df(t)}{dt} \quad (13)$$

The IF also has another definition based on time-frequency distribution (TFD) by locating the weighted average of the frequency which exist at particular time,

$$f(t) = \frac{\int_{-\infty}^{\infty} f\hat{P}(t, f)df}{\int_{-\infty}^{\infty} \hat{P}(t, f)df} \quad (14)$$

where $\hat{P}(t, f)$ is the estimated TFD of the signal.

Short-time Fourier transforms (STFT)

A classical representation of signal frequency is based the frequency spectrum which is the magnitude squared of the Fourier transform for stationary signal. For frequency domain of signal in equation (12) is,

$$x(f) = \int_{-\infty}^{\infty} x(t)e^{-j2pft}dt \quad (15)$$

The infinite integral of time in equation (15) neglects the variation of signal frequency in time domain. The STFT encounter the problem by segmenting the time signals into short time divisions and computes the Fourier transform for each division. The segmentation can be done by multiple the time domain data with a time weighting function with zeros beyond the segment width. The STFT of the harmonic signal is define as follow,

$$\hat{y}(t, f) = \int_{-\infty}^{\infty} W(t - t)x(t)e^{-j2pft}\,dt \quad (16)$$

where $W(t)$ is the time domain window with segment duration $2t$ and cantered on time t and the corresponding signal spectrogram which is magnitude squared STFT

$$\hat{P}_{STFT}(t, f) = \left|\hat{y}(t, f)\hat{y}^*(t, f)\right| \quad (17)$$

The estimated IF with STFT spectrogram can be found by substituting $\hat{P}_{STFT}(t, f)$ into equation (14),

$$\hat{f}_{STFT}(t) = \frac{\int_{-\infty}^{\infty} f\hat{P}_{STFT}(t, f)df}{\int_{-\infty}^{\infty} \hat{P}_{STFT}(t, f)df} \quad (18)$$

Filtered short-time Fourier transforms (FSTFT)

Since the center of gravity of the frequency domain is used as estimator of IF, the ratio of the IF and the sampling frequency would influence the result of estimation. Assuming background white noise present in the short time Fourier spectrum, the estimated IF would biased as ,

$$\hat{f}_{STFT}(t) \leq f_{STFT}(t) \text{ if } \frac{f_{STFT}(t)}{Fs/2} > 0.5 \quad (19)$$

and

$$\hat{f}_{STFT}(t) \geq f_{STFT}(t) \text{ if } \frac{f_{STFT}(t)}{Fs/2} < 0.5 \quad (20)$$

Time-frequency filter approach is used to minimize the effect of shifting frequency due to background noise. By applying a band pass filter adaptively for each time segmented spectrum which centered in the peak frequency and compute for the new IF as,

$$\hat{f}_{FSTFT}(t) = \frac{\int_{-\infty}^{\infty} f\hat{P}_{STFT}(t, f)H(t, f_0)df}{\int_{-\infty}^{\infty} \hat{P}_{STFT}(t, f)H(t, f_0)df} \quad (21)$$

where $H(t, f_0)$ is a time-frequency band pass filter with band width B and centered at frequency f_0 as ,

$$H(t, f_0) = \begin{cases} 1 & f_0 \pm B/2 \\ 0 & else \end{cases} \quad (22)$$

which eliminate the unwanted frequency content outside $f_0 \pm B/2$.

Polynomials phase estimation (PPE)

The phase of signal in (11) can also be approximated by a polynomial with pth order,

$$f(t) = a_0 + a_1 t + a_2 t^2 + \cdots + a_p t^p = \sum_{n=0}^{p} a_n t^n \quad (23)$$

whether the phase of the signal can be estimated by Hilbert transform.

With reference to equation (13), the instantaneous frequency can be finding by numerical differentiation with time,

$$\hat{f}_{PPE}(t) = \frac{1}{2p}\frac{df(t)}{dt} = \frac{1}{2p}\sum_{n=1}^{p} a_n t^{n-1} \quad (24)$$

The order of approximation is subjective to the signal's order. For linear frequency sweeping signal, P = 2 already gives appropriate results. For this study, P = 3 is used for analysing the point moving source.

NUMERICAL SIMULATION AND RESULTS

Computer simulations with software MATLAB were performed to illustrate the availability of the method above. Point harmonic source model was used and all time-frequency transformation methods above were applied for the estimation problem. A point harmonic source travelling with uniform speed with Mach number set to 0.1 to 0.3. The source with unit amplitude was located at 1 length unit above the reference level and with shortest distance at 1 length from the receiver array. The normalized simulating frequency (f_s) was set to 1000Hz and the normalized source resting frequency f was set from 50Hz to 450Hz which satisfy the sampling theorem with $f \leq 0.5 f_s$. For Mach number 0.2 and 0.3, the ceiling of source frequency was limit at 400Hz and 350Hz. It was assumed that the signal were received under homogeneous condition by a 11 uniformly spaced linear microphone array along x-axis with separation 0.1 unit length while the 6th element located at the zero of the coordinate system. The simulation time was set from -5 to 5 normalized time unit. The signals were embedded with background Gaussian white noise while the signal to noise ratio (SNR) was set from 10 to 40dB with reference to the peak level of the received sound pressure which is uncorrelated between microphones. For each case, 1000 simulations were performed to validate statistical performance of those methods.

Figs 2 to 7 show the standard deviation and the mean square error of the estimated velocity, frequency and amplitude with difference estimation methods. All methods give an improving accuracy with higher SNR. Note there is significiance enhancement in effectiveness of those methods around SNR 20dB. At SNR 20 to 40 dB ranges, the mean estimation error for velocity, frequency and amplitude are 11%, 4% and 0.06dB respectively, while standard deviation are 0.39, 10.62 and 0.04. All methods demonstrated stability with low variation and acceptable mean estimation error at SNR above 20dB, which give confidence for time-frequency based method to tackle moving source problems.

Improvement is expected for STFT estimation replaced with FSTFT. In the velocity estimating stage, FSTFT obtain promotion in terms of reducing the variance of the results, however FSTFT arise similar percentage error with conventional method. It means that although the constant band width band pass filter success in rejecting background noise as to eliminate the effect of shifting of estimating frequency, it gives insignificant improvement in estimating the peak of the shifting frequency rate and the referencing time for further estimation. Since the center frequency of the constant band width filter is obtain based on the peak of the sliced spectrum, the accuracy of the estimated IF would depends on the resolution of the discrete Fourier transform. It also leads to problem while high concentration of the peak frequency appears in the filtered spectrum, the continuity of the IF would lose and lower the performance in estimating the IF change. The imperfect of improvement in frequency rate estimation

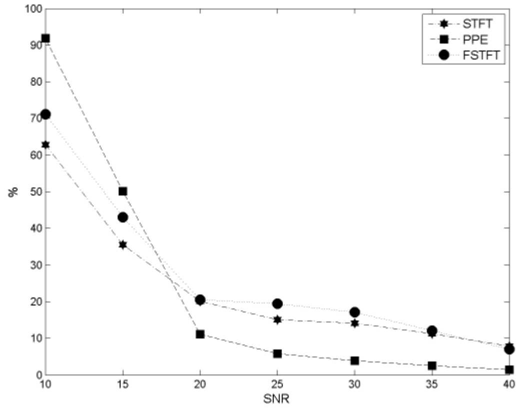

Figure 2.Source velocity mean estimation error

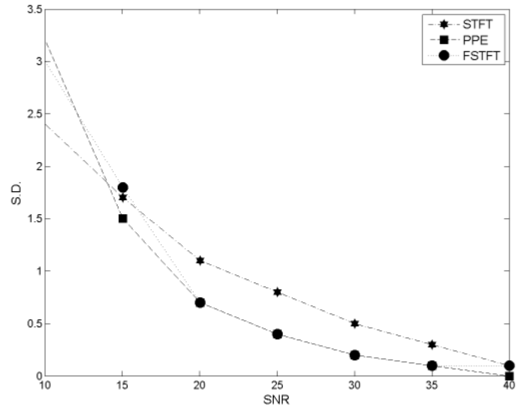

Figure 3.Source velocity standard deviation

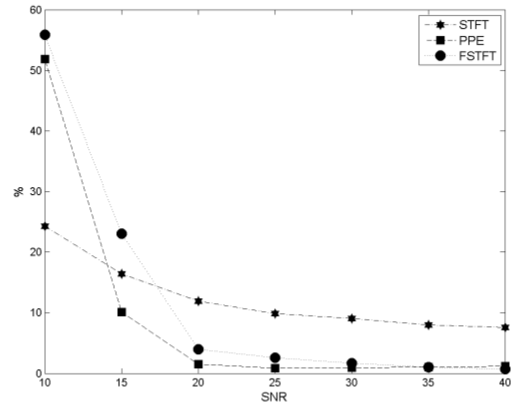

Figure 4.Source frequency mean estimation error

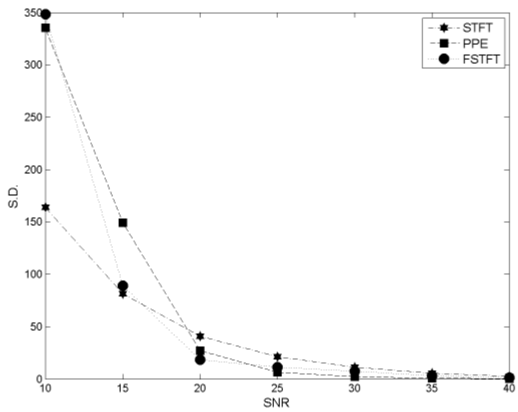

Figure 5.Source frequency standard deviation

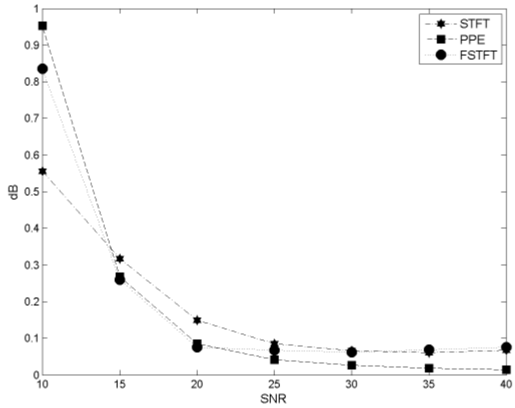

Figure 6.Source amplitude mean estimation error

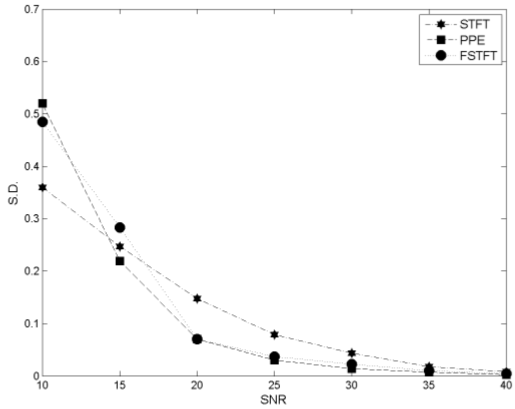

Figure 7.Source amplitude standard deviation

makes FSTFT not acting as best substitute of conventional STFT method.

Apart from getting information that how the different time-frequency perform in the estimation problem, it is also important to investigate the influences of source parameters in affecting the results. Since the PPE method gave the best estimation among those three, the following paragraph is based on the result analysed through PPE.

Table 1. Performance of PPE at difference Mach number

Parameter	Mach	Error	S.D.
Velocity	**0.1**	27%	1.06
	0.2	23%	0.85
	0.3	22%	0.77
Frequency	**0.1**	3%	24.4
	0.2	8%	67.8
	0.3	18%	131
Amplitude	**0.1**	0.17dB	0.06
	0.2	0.20dB	0.12
	0.3	0.23dB	0.18

Table 1 show the standard deviation and the mean square error for estimating the source parameters in different speed. It follows that better velocity estimation can be obtained under higher speed situation. Since we adopt to use the frequency gradient as the base parameter to estimate the source velocity, and the gradient is a function of square of the source speed, thus higher speed cause larger dynamic range and more dominant gradient to get source information. Moreover, as the solution is based on time difference in microphone pairs and thus value of estimated source velocity is not affected while errors in instantaneous frequency estimation. Therefore it is obvious to see that more exact estimation can be made at higher speed. However, higher speed minimizes the accuracy of the estimation for source frequency and amplitude. Since the estimation in higher speed would induce larger frequency and amplitude gradient, the error would be magnified if there is mis-locating in the time of maximum rate of Doppler shift.

Table 2. Performance of PPE at difference source frequency

Parameter	Frequency	Error	S.D.
Velocity	Low	22%	1.05
	Medium	21%	0.72
	High	37%	1.13
Frequency	Low	10%	39.3
	Medium	7%	70.7
	High	9%	95.9
Amplitude	Low	0.23dB	0.13
	Medium	0.17dB	0.10
	High	0.24dB	0.13

Table 2 show the standard deviation and the mean square error for estimating the source parameters in different source frequency. The medium frequency range (200 – 300Hz) give more desirable result than those in low and high frequency range. It is observable that in general, low and high source frequency gives a similar performance.

CONCLUSION

This paper presented a parameter estimation method for harmonic point moving source under constant speed situation using time-frequency transformation of signals from array of microphones. The suggested methodology shows success even without the knowledge of source velocity and frequency. The algorithm adopted to estimate source parameters based on the rate of source Doppler frequency shift which were investigated by three time-frequency methods. Numerical simulations were performed under various conditions in order to investigate the availability of the proposed method. Performances of the methods were discussed based on estimator's error and standard deviations. The results showed that PPE method gives the most reliable outcome. Finally, further investigations on the improvement of above method and algorithms development for other sound progogration condition are recommended.

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