

The effect of grazing-bias flow on the self sustained oscillations in a side branch

Andreas Holmberg (1), Mikael Karlsson (2)

(1) KTH Linné Flow, the Marcus Wallenberg Laboratory for Sound and Vibration Research, Stockholm, Sweden

(2) KTH Cicero, the Marcus Wallenberg Laboratory for Sound and Vibration Research, Stockholm, Sweden

PACS: 43.28.PY, 43.28.RA

ABSTRACT

The flow-acoustic interaction in a T-junction can result in both amplification and attenuation of incoming waves. The frequency ranges of amplification are mainly governed by the time scales of convection of hydrodynamic instabilities over the side branch opening. With bias flow, i.e. flow through the side branch the amplification can increase or completely vanish. In this paper this effect is studied for a T-junction of rectangular ducts, with a grazing Mach number of 0.1 and a varying bias inflow. When the bias inflow Mach number changes from 0 to 0.01 the amplification is increased severely. Further increasing the bias inflow has the effect of lower the amplification again towards zero at a bias Mach number of 0.05.

INTRODUCTION

Junctions and cavities are common elements in flow ducts such as automotive intake and exhaust systems, ventilation systems or pipelines. Both the active and passive aero-acoustic responses of such elements are strongly influenced by the mean flow configuration in the system. The fluid-acoustic interaction is in low Mach number applications often described as the continuous interaction of hydrodynamic instabilities with the acoustic field as they are convected across the aperture. The interaction can be constructive or deconstructive, that is, both attenuation and amplification of incident sound is possible. At low amplification rates the system is still linear; however if the amplification rate is too high, the interaction becomes nonlinear leading to a self-sustained oscillation. This can lead to intense noise and even mechanical failure. This can occur if the junction is coupled to a resonant system, e.g. an open duct termination or another junction.

Aurégan and Starobinski [1] showed how both the attenuation and the amplification potentiality of a junction can be obtained from a measured scattering matrix [2], which is one possible model of the passive acoustic properties of the junction. Often though these properties are expressed in terms of normalized acoustic impedances. In side branch orifices the general trend for a grazing flow configuration is an increase in the resistance and a decrease in the reactance with an increasing Mach number [3-10]. Analytical modelling of single orifices have been attempted [11, 8], but it have been shown [12, 8] that the correlation with experimental data is generally unsatisfactory. Regarding empirical models, Lee and Ih [9] have shown that the versatility of the models with respect to geometry and flow changes are limited.

Adding bias flow to the configuration significantly alters the acoustic response. In addition, Sun et al [10] observed in their experimental work on perforated plates that the direction of the bias flow is an important factor as well. Recent work treating mixed grazing/bias flow cases consists of e.g. Belfroid et al [13] who studied the flow induced sound of a T-junction numerically, and Karlsson and Åbom [14] where an analysis of potential sound attenuation and amplification, derived from measured scattering matrices of a Tjunction were presented. The results from the latter show that the amplification found without bias flow is significantly increased when a bias flow with a velocity of about 10% of the grazing flow velocity is led into the main branch. When the bias flow is further increased however, the amplification drops monotonically with the bias flow velocity. Also, due to an increased mass flow in the region of the junction, the Strouhal based solely upon the upstream grazing flow will increase when bias flow is added. This shift doesn't continue linearly however, and for sufficiently high bias flow Mach numbers the shift is invariant of the added flow velocity. In order to further investigate these observed phenomena, a T-junction of rectangular cross section is studied experimentally in this paper, using the method developed in [14].

THEORY

Limiting the frequency range to the plane wave region of a duct, it is possible to describe the acoustic field as a superposition of upstream and downstream propagating waves. Complex geometries are modelled by considering the relations between input and output waves.

Aeroacoustic Model

For linear time-invariant acoustic duct elements the N-port scattering matrix formulation is often chosen, which in addition to a source term relates incoming pressure waves to those reflect or transmitted. The outgoing waves are in this model given by [2]

$$\mathbf{p}_+ = \mathbf{S}\mathbf{p}_- + \mathbf{p}_+^s \quad (1)$$

where \mathbf{p}_+^s is the source vector, \mathbf{p}_+ and \mathbf{p}_- contains the outgoing and incoming complex pressure amplitudes of all ports, and \mathbf{S} is the scattering matrix which for a three-port system is given by [15]

$$\mathbf{S} = \begin{bmatrix} R_I & T_{II,I} & T_{III,I} \\ T_{I,II} & R_{II} & T_{III,II} \\ T_{I,III} & T_{II,III} & R_{III} \end{bmatrix} \quad (2)$$

R_m has the physical meaning of reflection of waves propagating towards the three-port in side branch m , and $T_{m,n}$ is the transmission of acoustic pressure from side branch m to side branch n , see Figure 1.

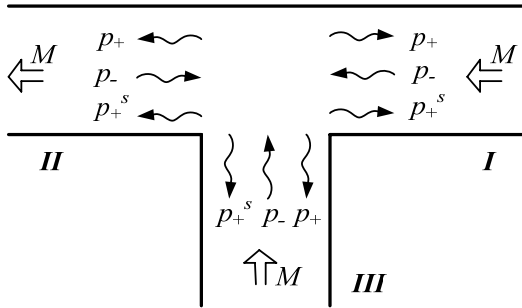


Figure 1. Definitions of coordinate systems, positive directions for the Mach numbers and numbering of side branches for the studied T-junction.

For random signals the source vector formulate in equation (1) is inadequate, instead the source cross spectrum matrix is used

$$\mathbf{G}^s = \mathbf{p}_+^s (\mathbf{p}_+^s)^H \quad (3)$$

where H refers to the Hermitian transpose. The diagonal elements of \mathbf{G}^s are then the auto spectra of the generated source pressures, while the rest of the elements consists of cross spectra. As argued in [14] the source vector only represents the generated sound which is independent of the acoustic field, and is thus not sufficient when flow-acoustic interaction effects are present in the three-port. It was postulated that the independent part and the flow-acoustic interaction

part of the sound generation can be separated, so that the latter can be included in the scattering matrix formulated via

$$\mathbf{p}_+ = (\mathbf{S}_0 + \mathbf{S}_{\text{mod}})\mathbf{p}_- + \mathbf{p}_{0+}^s \quad (4)$$

where the index 0 refers to the part which is independent of the acoustic field. Thus this formulation implies that the flow-acoustic interaction will be included in the scattering matrix. It is possible to study the attenuation/amplification effects of an incident wave in one of the branches by assuming anechoic terminations in the other branches. The power balance can then be derived as [14]

$$\frac{\langle W_{out}^I \rangle}{\langle W_{in}^I \rangle} = \frac{|R_I|^2 (1-M_I)^2}{(1+M_I)^2} + \frac{|T_{I,II}|^2 (1+M_{II})^2 A_{II}}{(1+M_I)^2 A_I} + \frac{|T_{I,III}|^2 (1-M_{III})^2 A_{III}}{(1+M_I)^2 A_I} \quad (5)$$

$$\frac{\langle W_{out}^{II} \rangle}{\langle W_{in}^{II} \rangle} = \frac{|R_{II}|^2 (1+M_{II})^2}{(1-M_{II})^2} + \frac{|T_{II,I}|^2 (1-M_I)^2 A_I}{(1-M_{II})^2 A_{II}} + \frac{|T_{II,III}|^2 (1-M_{III})^2 A_{III}}{(1-M_{II})^2 A_{II}} \quad (6)$$

$$\frac{\langle W_{out}^{III} \rangle}{\langle W_{in}^{III} \rangle} = \frac{|R_{III}|^2 (1-M_{III})^2}{(1+M_{III})^2} + \frac{|T_{III,I}|^2 (1-M_I)^2 A_I}{(1+M_{III})^2 A_{III}} + \frac{|T_{III,II}|^2 (1+M_{II})^2 A_{II}}{(1+M_{III})^2 A_{III}} \quad (7)$$

Here W refers to the acoustic power, and A is the duct cross section area. The ratios will be larger than unity for amplification and smaller than unity for attenuation of the incident power. This analysis is similar to the more generic approach suggested by Aurégan and Starobinski [1], and will be used in this work. The frequencies of aeroacoustic phenomena are often described using the dimensionless Strouhal number, which here is defined as

$$St = \frac{fd}{u} \quad (8)$$

where f is the frequency, d is the size of the side branch III opening in the grazing flow direction, and u is the convection velocity of hydrodynamic instabilities travelling across the opening. For ducts of circular cross section an amplification can occur at Strouhal numbers of approximately 0.4 when only grazing flow is present [14, 16-18]. Adding bias flow, it was found [18] that the frequency increased, however already at a bias Mach number of 0.05, the shift converged to an increase of around 25 %, invariant of the bias Mach number.

Measurement method

The analysis of the experimental data is based upon the ability to decompose the sound field in planar waves. For flow ducts the microphone methods consists of the two-microphone method [19-20], multi-microphone methods [21-22] or the full wave decomposition method [23]. In this paper, the multi-microphone method described in [21] is applied. Thus, at each side branch a microphone array is used to obtain input data put on the right hand side of the equation system

$$\begin{bmatrix} e^{-ik_+x_1} & e^{ik_-x_1} \\ e^{-ik_+x_2} & e^{ik_-x_2} \\ \vdots & \vdots \\ e^{-ik_+x_j} & e^{ik_-x_j} \end{bmatrix} \begin{bmatrix} p_+ \\ p_- \\ \vdots \\ p_j \end{bmatrix} = \begin{bmatrix} p_1 \\ p_2 \\ \vdots \\ p_j \end{bmatrix} \quad (8)$$

where k_+ and k_- are the wave numbers in the positive and negative directions respectively, x_j is the position and p_j is the complex pressure of microphone j . The sound field is obtained by having external loudspeakers mounted on the test rig, and the three port source part which is independent on the acoustic field is suppressed by averaging over the cross spectra between the microphone pressure and the loudspeaker signal. Now, to solve for \mathbf{S} measurements of three linear independent acoustic fields are required. Here the method of having external loudspeakers switched on at different side branches [24] is used in order to excite the fields. The scattering matrix is then solved from the equation

$$\mathbf{S} = \mathbf{P}_+ \mathbf{P}_-^{-1} \quad (9)$$

with

$$\mathbf{P}_+ = [\mathbf{p}_+^{L1} \quad \mathbf{p}_+^{L2} \quad \mathbf{p}_+^{L3}] \quad \mathbf{P}_- = [\mathbf{p}_-^{L1} \quad \mathbf{p}_-^{L2} \quad \mathbf{p}_-^{L3}] \quad (10)$$

Where the superscripts $L1$, $L2$ and $L3$ refers to vectors obtained at the three different loudspeaker excitations. The wave numbers in equation (8) are obtained from a model proposed by Dokumaci [25], which includes the effects of viscothermal damping

$$k_{\pm} = \frac{\omega}{c_0} \frac{K_0}{1 \pm K_0 M} \quad (11)$$

where ω is the angular frequency, c_0 the adiabatic speed of sound, M the averaged Mach number over the cross section and K_0 is given by

$$K_0 = 1 + \left((1-i)/s \right) \left(1 + (\gamma-1)/\sqrt{\text{Pr}} \right) / \sqrt{2} \quad (12)$$

with γ being the ratio of specific heats, Pr the Prandtl number and s the shear wave number

$$s = r \sqrt{\rho_0 \omega / \mu} \quad (13)$$

where ρ_0 is the ambient density, μ the dynamic viscosity and r is the duct radius. In this work, since the duct cross section is rectangular the radius is defined as that of a circle with the same circumference as the rectangular duct.

Measurement setup

The measurement rig consists of three rectangular ducts, each terminated by a lined expansion chamber, filled with mineral wool. The rig, which is depicted in Figure 2, was connected to the MWL wind tunnel.

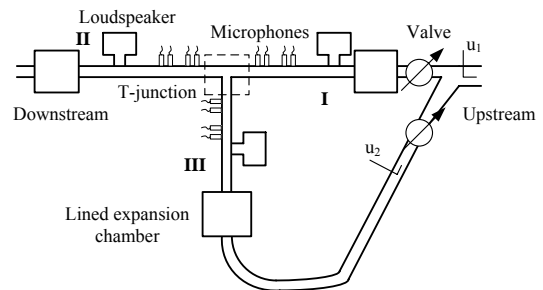


Figure 2. Schematic drawing of the test rig, showing positions of velocity measurements.

The rectangular ducts have the inner dimensions 25x120 mm, the walls made of 15 mm thick steel. The T-junction is constructed so that the length of the opening is 25 mm in the grazing flow direction. The velocity in each branch was tuned by the use of two valves, and monitored via prandtl-tubes in circular ducts, one put upstream of the lined expansion chamber of branch III but downstream of the valve, and one put upstream of the junction where the flow is separated into the main duct and into the side branch duct. The microphone array on each side consists of four microphones, placed in order for the wave decomposition to be valid from 100 Hz up to the cuton of the first non-plane mode in the test rig.

RESULTS AND DISCUSSION

In this paper, a stepped sine signal is used to map the aeroacoustic response of the T-junction in steps of 12.5 Hz, with a frequency resolution of 1.25 Hz. The measurement time for each loudspeaker and flow velocity case was about two hours, which from experience is short enough to ensure an approximately constant temperature. However to validate this, the temperature was measured in all three branches before and after the measurement, and the maximum increase in temperature was about 2 °C, thus increasing the speed of sound by less than 4 %.

The grazing flow Mach number is calculated by considering the difference in mass flow at the two velocity measurement points, indicated in Figure 2. Since both measurements are

conducted in circular ducts it is possible to use the 1/7th-power law [26] to predict the ratio between mean and maximum velocity, which for the Reynolds numbers found in this work are approximately 0.82. The grazing flow was kept constant at $M_g = 0.1$ (grazing Mach number), while the bias flow was varied in the range $M_b = 0.01n$, with $n = 0, 1 \dots 5$. First, the power balance is shown for the six cases as a function of Strouhal number based on M_g solely, see Figures 3-5.

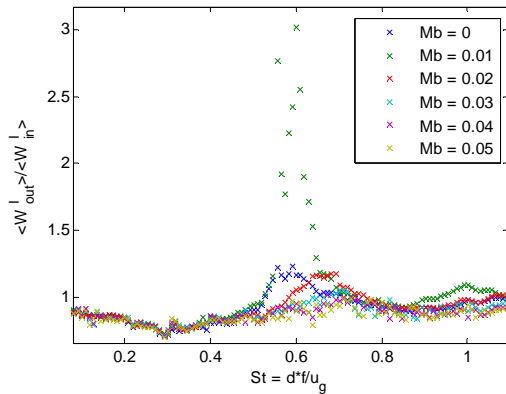


Figure 3. Power balance for waves approaching the T-junction from branch I.

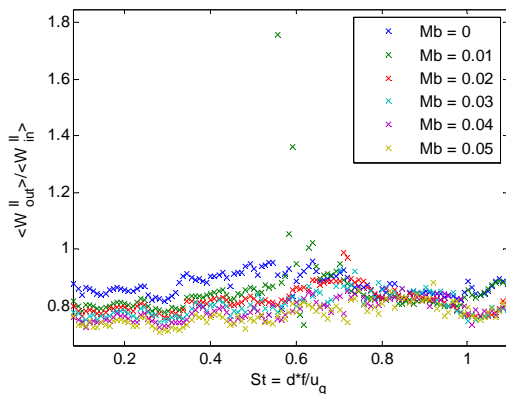


Figure 4. Power balance for waves approaching the T-junction from branch II.

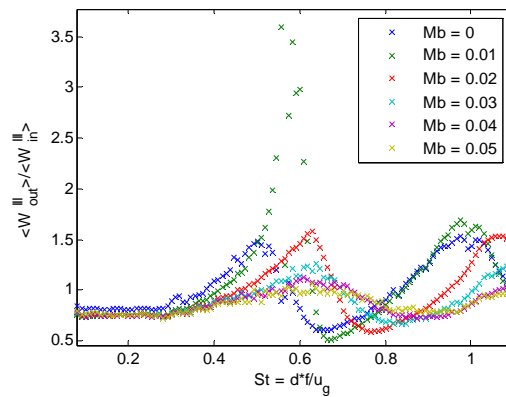


Figure 5. Power balance for waves approaching the T-junction from branch III.

From these figures several things are noted. First off, the power incident from branch I is slightly attenuated at a broad range of Strouhal numbers, although there is a range where the opposite is true. For waves coming from branch II the general behaviour is attenuation of acoustic power. For waves coming from the side branch (III) there are two distinct regions of power amplification, which for $M_b = 0$ is found at Strouhal number 0.5 and 1.0, based on the mean grazing flow velocity. All these observations agree fairly well with those made for circular ducts [14, 16-18]. Considering incidence from branch III the two amplification regions are shifted upwards in frequency as M_b is increased up to about 0.03. Perhaps most interesting is the fact that just by having $M_b = 0.01$ a huge increase in the first amplification region is obtained, an amplification which is found regardless of which branch the sound power originated from. This effect is probably non-linear, and it is thus not certain that the scattering matrix is an appropriate model to use for the power balance calculations.

It is interesting to compare the results of this rectangular duct T-junction to those from a circular one, subjected to a grazing flow of $M_g = 0.1$. The main difference of the two configurations is that for circular ducts, the duct opening is a function of both the axial and radial coordinates. To obtain a Strouhal number, Bruggeman et al [16] suggested using an effective length of $\pi r/2$ for the side branch opening. The results at $M_b = 0, 0.2, 0.3$ and 0.5 for the rectangular duct T-junction are plotted in Figures 6-8, along with results obtained in [18] for a circular T-junction with a diameter of 0.057 m.

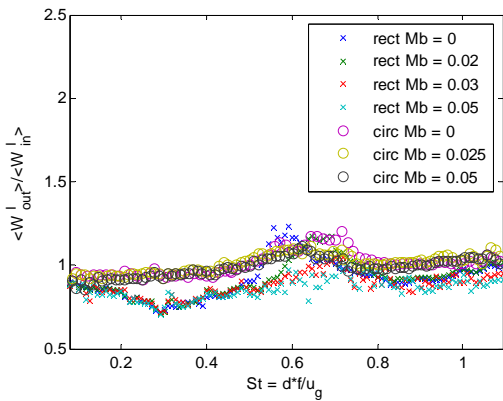


Figure 6. Power balance for waves approaching the T-junction from branch I, comparison of rectangular (x) and circular (o) ducts.

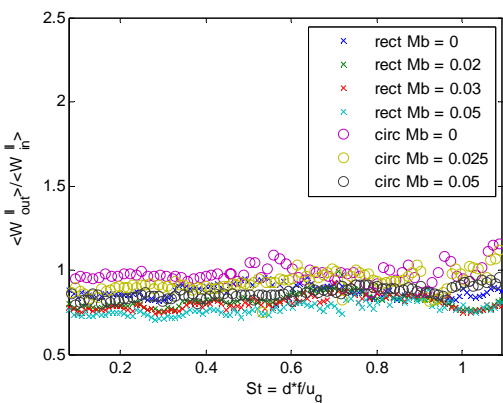


Figure 7. Power balance for waves approaching the T-junction from branch II, comparison of rectangular (x) and circular (o) ducts.

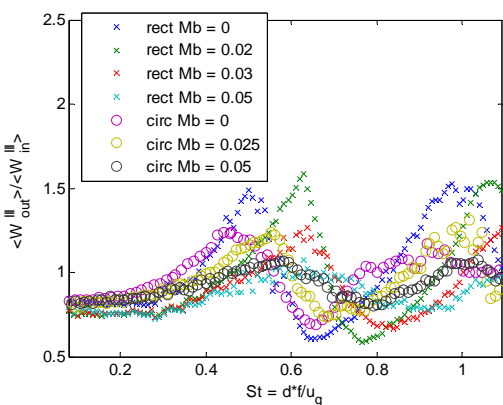


Figure 8. Power balance for waves approaching the T-junction from branch III, comparison of rectangular (x) and circular (o) ducts.

Overall it is seen that the flow-acoustic interaction is slightly more pronounced for the rectangular duct than for the circular. The regions of amplification agree fairly well, although for the circular duct they peak at about 10 % lower Strouhal number. A main conclusion is however that observations made for circular ducts seems to apply for rectangular ducts as well, but in addition the amplification will be very large if

a small bias flow is added. The system appears to be extremely sensitive to the amount of bias flow, since this effect is lost by increasing the bias Mach number by 0.01. What remains to be seen from future work is whether this narrow bias flow range either scales with, is approximately invariant of or restricted to certain ranges of the grazing flow velocity.

REFERENCES

- 1 Y. Aurégan and R. Starobinski, "Determination of acoustical dissipation/production potentiality from the acoustical transfer function of a multiport" *Acta Acustica* **85**, 788–792 (1999)
- 2 M. Åbom, "Measurement of the scattering-matrix of acoustical two-ports" *Mechanical Systems and Signal Processing* **5**, 89–104 (1991)
- 3 D. Ronneberger, "The acoustic impedance of holes in the wall of flow ducts" *J. Sound. Vib.* **24**, 133–150 (1972)
- 4 A. L. Goldman and R. L. Panton, "Measurement of acoustic impedance of an orifice under a turbulent boundary layer" *J. Acoust. Soc. Am.* **60**, 1397–1404 (1976)
- 5 J. W. Kooi and S. L. Sarin, "An experimental study of the acoustic impedance of Helmholtz resonator arrays under a turbulent boundary layer" *AIAA Paper* **81-1998**, (1981)
- 6 A. Cummings, "The effects of grazing turbulent pipe-flow on the impedance of an orifice" *Acta Acustica* **61**, 233–242 (1986)
- 7 N. S. Dickey and A. Selamet, "An experimental study of the impedance of perforated plates with grazing flow" *J. Acoust. Soc. Am.* **110**, 2360–2370 (2001)
- 8 X. Jing, X. Sun, J. Wu and K. Meng, "Effect of grazing flow on the acoustic impedance of an orifice", *AIAA Journal* **39**, 1478–1484 (2001)
- 9 S.-H. Lee and J.-G. Ih, "Empirical model of the acoustic impedance of a circular orifice in grazing mean flow" *J. Acoust. Soc. Am.* **114**, 98–113 (2003)
- 10 X. Sun, X. Jing, H. Zhang and Y. Shi, "Effect of grazing-bias flow interaction on acoustic impedance of perforated plates" *J. Sound. Vib.* **315**, 849–874 (2008)
- 11 M. S. Howe, *Acoustics of Fluid-Structure Interactions* (Cambridge University Press, Cambridge, 1998)
- 12 K. S. Peat, J.-G. Ih and S.-H. Lee, "The acoustic impedance of a circular orifice in grazing mean flow: comparison with theory" *J. Sound. Vib.* **114**, 3076–3086 (2003)
- 13 S. P. C. Belfroid, R. M. C. A. M. Peters, W. Schiferli and J. Buffing, "Flow induced pulsation caused by split flow in a T-branch connection" *ASME 6th FSI, AE & FIV+N Symposium*, Canada (2006)
- 14 M. Karlsson and M. Åbom, "Aeroacoustics of T-junctions – An experimental investigation" *J. Sound. Vib.* **329**, 1793–1808 (2010)
- 15 H. Bodén and M. Åbom, "Modelling of fluid machines as sources of sound in duct and pipe systems" *Acta Acustica*, **3**, 549–560 (1995)
- 16 J. C. Bruggeman, A. Hirschberg and M. E. H. van Dongen, "Self-sustained aero-acoustic pulsations in gas transport systems: experimental study of the influence of closed side branches" *J. Sound. Vib.* **150**, 371–393 (1991)
- 17 H. R. Graf and S. Ziada, "Excitation source of a side-branch shear layer" *J. Sound. Vib.* **329**, 2825–2842 (2010)
- 18 M. Karlsson, "The acoustic impedance of a confined circular side branch orifice subjected to grazing-bias flow" *16th AIAA/CEAS*, Sweden (2010)
- 19 W. E. Schmidt and J. P. Johnston, "Measurement of Acoustic Reflection from Obstructions in a Pipe with Flow" *NSF Report*, PD-20 (1975)

- 20 A. F. Seybert and D. F. Ross, "Experimental determination of acoustic properties using a two-microphone random-excitation technique" *J. Acoust. Soc. Am.* **61**, 1362–1370 (1997)
- 21 T. Fujimoro, S. Sato and H. Miura, "An automated measurement system of complex pressure reflection coefficients" *Internoise 84*, 1009–1014 (1989)
- 22 M. G. Jones and T. L. Parrott, "Evaluation of a multi-point method for determining acoustic impedance" *Mech. Sys. and Sign. Proc.* **3**, 15–35 (1989)
- 23 S. Allam and M. Åbom, "Investigation of damping and radiation using full plane wave decomposition in ducts" *J. Sound. Vib.* **292**, 519–534 (2006)
- 24 M. L. Munjal and A. G. Doige, "Theory of a two source-location method for direct experimental evaluation of the four-pole parameters of an aeroacoustic element" *J. Sound. Vib.* **141**, 323–333 (1990)
- 25 E. Dokumaci, "A note on transmission of sound in a wide pipe with mean flow and viscothermal attenuation" *J. Sound. Vib.* **208**, 653–655 (1997)
- 26 H. Slichting, *Boundary-Layer Theory*, (McGraw – Hill, USA, 1968)