Parametrical 3D Structural Co-modelling of Stringed Instruments

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PACS: 43.40.At

ABSTRACT

The paper attempts to give a contribution to structural and dynamic analyses of musical instruments, through co-modelling procedures based on numerical standard codes. Particular reference is made to stringed instruments. The availability on the market of powerful CAD and FE codes are usefully exploited, proposing a low cost co-modelling approach able to support the construction phases of high quality handcrafts products. The proposed co-modelling approach virtually emulates the construction phases followed by a lute maker during its job: numerical geometrical and modal models having different level of accuracy are generated step-by-step. Structural and dynamic simulated results can be compared to experimental procedures and suggests interactive modification of geometries or parameters.

INTRODUCTION

Acoustic performances of musical instruments are strictly related to their mechanical, structural, dynamic and vibratory responses. Numerical advanced procedures of mechanical, structural and dynamic analysis and diagnosis are often unknown to lute makers, developing their activity on own experience, based on practical knowledge and empirical tests.

Mechanical characteristics (e.g. Young's, shear and bulk moduli, Poisson's ratio, density and thickness) can be defined parametrically, for each elements or group of elements. Results concern the evaluation of natural frequencies and the modal shapes of the whole instrument or of parts of them. The interactive approach allows adjusting the wood characteristics or local thickness and simulating the dynamic result of this variation: but also to go back and modify the 3D geometry and simulate again. Main performances and features of the proposed co-modelling approach are practically described in the paper with reference to a specific violin.

MODELING APPROACH

The proposed approach is based on the use of different and interlaced design and structural codes. The modelling procedure follows the phases typically actuated by a violin maker during the craft construction. The purpose is the setup of a modelling procedure able to scientifically support the violin maker in its activity, allowing preventive simulations and structural analyses.

A piece of wood, even if selected for musical instruments construction, is characterized by defects and irregularities. Taking into account the techniques typically used to choose the woods devoted to soundboard and to back the modelling approach begins to define the relationships between deformations γ and stresses σ. With reference to Figure 1 the modelling assumes an ideal behaviour of the wood: in according to the references shown in Figure 1d) the Hooke law can be expressed as:

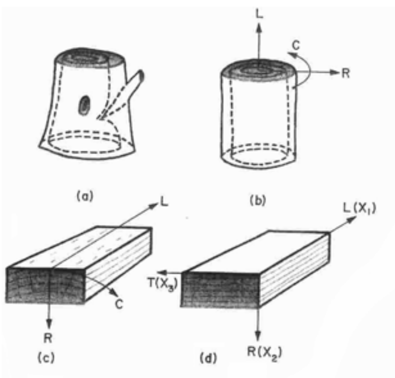


Figure 1. Wood element and references

$$\begin{bmatrix}\gamma_L\\ \gamma_R\\ \gamma_T\\ \gamma_{RT}\\ \gamma_{LT}\\ \gamma_{LR}\end{bmatrix}=\begin{bmatrix}\frac{1}{E_L} & \frac{-\nu_{RL}}{E_R} & \frac{-\nu_{TL}}{E_T} & 0 & 0 & 0\\ \frac{-\nu_{LR}}{E_L} & \frac{1}{E_R} & \frac{-\nu_{TR}}{E_T} & 0 & 0 & 0\\ \frac{-\nu_{LT}}{E_L} & \frac{-\nu_{RT}}{E_R} & \frac{1}{E_T} & 0 & 0 & 0\\ 0 & 0 & 0 & \frac{1}{G_{RT}} & 0 & 0\\ 0 & 0 & 0 & 0 & \frac{1}{G_{LT}} & 0\\ 0 & 0 & 0 & 0 & 0 & \frac{1}{G_{LR}}\end{bmatrix}\begin{bmatrix}\sigma_L\\ \sigma_R\\ \sigma_T\\ \sigma_{RT}\\ \sigma_{LT}\\ \sigma_{LR}\end{bmatrix}$$

Assuming orthotropic the material and considering the symmetry of the stiffness matrix Poisson's coefficients and Young moduli are related:

$$\frac{\nu_{RL}}{E_L} = \frac{\nu_{RL}}{E_R}$$

$$\frac{\nu_{LT}}{E_L} = \frac{\nu_{TL}}{E_T}$$

$$\frac{\nu_{RT}}{E_R} = \frac{\nu_{TR}}{E_T}$$

and consequently:

$$\gamma_L = \frac{1}{E_L}\left(\sigma_L - \nu_{LR}\,\sigma_R + \nu_{LT}\,\sigma_T\right)$$

$$\gamma_R = \frac{1}{E_R}\left(\sigma_R - \nu_{RL}\,\sigma_L + \nu_{RT}\,\sigma_T\right)$$

$$\gamma_T = \frac{1}{E_R}\left(\sigma_T - \nu_{TL}\,\sigma_L + \nu_{TR}\,\sigma_R\right)$$

Mechanical features can be deduced by experimental tensile tests or detecting the sound speed propagation V in a piece of wood having density ρ, in direction parallel and orthogonal to the fibres:

$$V_{//} = \sqrt{\frac{E_{//}}{\rho}}$$

$$V_{\perp} = \sqrt{\frac{E_{\perp}}{\rho}}$$

$$\frac{V_{//}}{V_{\perp}} = \sqrt{\frac{E_{//}}{E_{\perp}}}$$

These considerations are introductory to the implementation of a numerical modal analysis. As well known the reference equation describing the dynamic response is:

$$[\Psi]^T[K][\Psi][Z] - \omega^2[\Psi]^T[M][\Psi][Z] = \{0\}$$

being [Ψ] the shape vector, [Z] the amplitude, [M] the mass matrix, [K] the stiffness matrix and ω the natural frequency. Following the Rayleigh-Ritz approach the displacement vector ν (t) can be written as

$$\{v(t)\} = \{\psi\}z(t) = \{\psi\}Z_0\sin(\omega \cdot t)$$

being ψ the shape and e z (t) the generalized coordinate defining the amplitude. Defining

$$[K^*] = [\Psi]^T[K][\Psi]$$

$$[M^*] = [\Psi]^T[M][\Psi]$$

The reference equation can be rewritten as:

$$\left([K^*] - \omega^2[M^*]\right)\{\hat{Z}\} = 0$$

$\{\hat{Z}\}$ is the eigenvector corresponding to $\{Z\}$ and able to satisfy this equation. Numerical solution is implemented using Lanczos coordinates (derivatives of Ritz vectors).

In order to implement the numerical model geometrical data are necessary: in the present experience thickness of soundboard and back are experimentally acquired starting from components engraved by violin maker. A 3D scanner unit is used (Figure 2).

Figure 2. 3D scanner unit

The unit involves a video camera operating in conoscopic holography. It measures with high accuracy the distance between the sensor and the object. The electro optic probe operates at 100 mm from the body with acquisition speed of 1000 pixels/s. The measurement accuracy depends on the lens from 500 nm up to 50 µm for profiles from 0 to 140 mm. The physic principle is the conoscopic holography, consisting on the optic interference due to crystals bi-refracting illuminated by polarized light.

The result is the definition of cross and longitudinal sets of the violin (Figure 3).

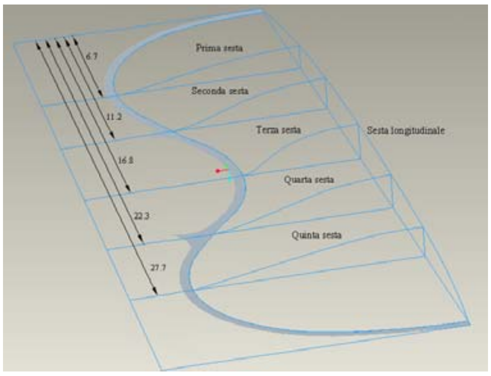


Figure 3. Cross and longitudinal sets

Starting from these geometrical references a parametric 3D model of the violin is implemented.

APPLICATION OF 3D CAD CODES

3D general purpose CAD codes (Pro/Engineer by PTC and AutoCAD by AutoDesk) are applied. Geometrical description of different parts of the violin corresponds to the various phases followed by lute makers during their craft construction, from the cut of external profiles of soundboard and back up to the realization of their final surfaces. The approach is organized on generation of surfaces sets, compatible to the description of corresponding finite shell elements: it makes possible the parametric description of variable thickness not only between elements but also within each element.

From the geometrical references previously acquired the basic sketch of soundboard and back profiles is defined (Figure 4).

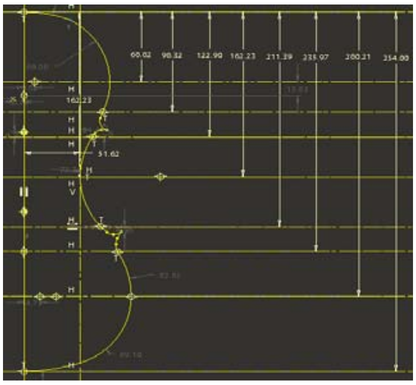
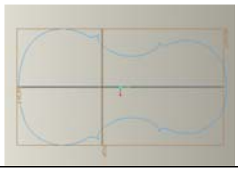
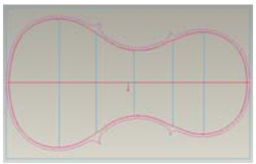
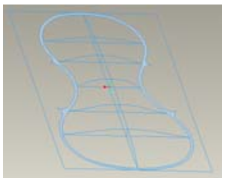
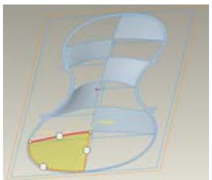
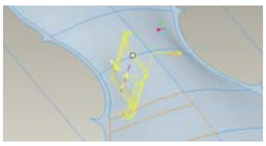
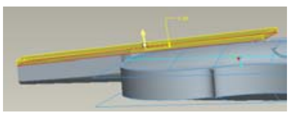
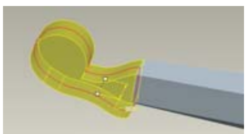


Figure 4. Basic sketch

A sequence of ten main steps is organized: each step corresponds to a significant construction phase. Results are collected in Table 1.

Table 1. Steps of modelling

1: Plant	2: External edge
	
3: Generation of sets	4: Curved surfaces generation
	
5: F holes generation	6: Merge of surfaces
	
7: Lateral ribs	8: Overall merge
	
9: Neck	10: Scroll
	

The final result of this procedure is the 3D representation reported in Figure 5.

Figure 5. 3D final drawing

STRUCTURAL ANALYSIS

The finite elements model is generated using a general purpose structural analysis code (ANSYS), importing the CAD geometry and applying elements, having membrane and flexional attributes.

Meshing requires suitable compromises. Discontinuities (e.g. f holes on the soundboard) are solved through original procedures able to automatically adjust the element size as function of the different local areas involved.

Good structural mesh distributions applied to soundboard and back are shown in Figure 6.

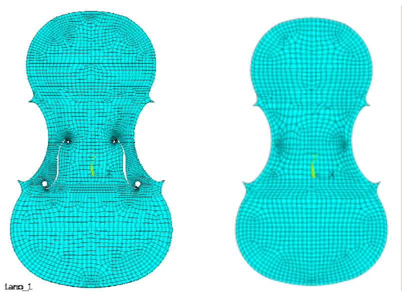


Figure 6. FE mesh of soundboard and back

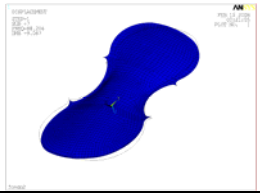
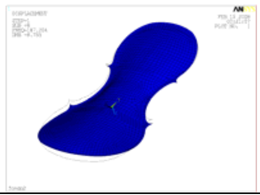
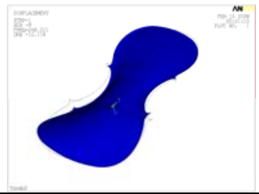
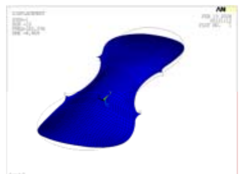
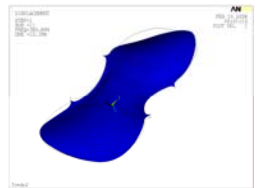
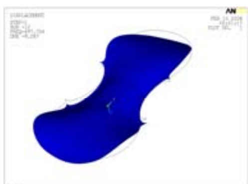
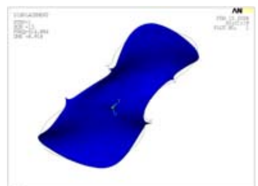
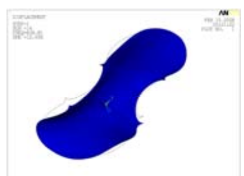
Applying the material properties collected in Table 2 the numerical modal analysis is developed.

Table 2. Properties of the material

Back	E_{xx}= 1.7 e10 Pa	E_{yy} = 1564 e6 Pa	E_{zz} = 731 e6 Pa
	G_{xy} = 1275 e6 Pa	G_{xz} = 187 e6 Pa	G_{yz} = 1173 e6 Pa
	ν_{xy} = 0.02	ν_{yz} = 0.4	ν_{xz} = 0.4
	ρ= 580 kg/m^3		
Soundboard	E_{xx} =1.3e10 Pa	E_{yy} = 890e6 Pa	Ezz = 649e6 Pa
	G_{xy} =1015e6 Pa	G_{yz} =715e6 Pa	G_{xz} = 416e6 Pa
	ν_{xy} = 0.02	ν_{yz} = 0.4	ν_{xz} = 0.4
	ρ= 400 kg/m^3		

Table 3 collects an example of result of the numerical modal analysis, reporting first eight mode shapes of the back under study.

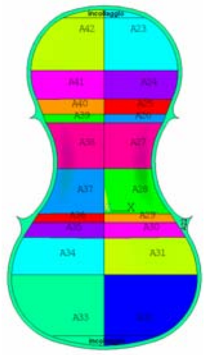
Table 3. Mode shapes

Mode	*f [Hz]*	*Type*	*Shape*
1	88.2	Torsional	
2	187.2	Flexural	
3	246.2	Flexural	
4	325.4	Flexural	
5	365.7	Flex-torsional	
6	497.7	Flexural	
7	514.9	Flex-torsional	
8	638.8	Flexural	

Next step concerns the definition of the variable thickness. Also for this step a direct measurement on prototype is made (Figure 6). Results are reported in Table 4.

Figure 6. Thickness measurement

Table 4. Thickness distribution (mm)

	BACK		*SOUNDBARD*	
 	Gluing	4	Gluing	3.8
	A42 A23	2.3	A31 A58	2.8
	A41 A24	2.8	A57 A32	3
	A40 A25	3.2	A56 A33	3
	A36 A26	3.2	A55 A34	3
	A38 A27	3.7	A54 A36	3.2
	A37 A28	4.4	A37 A51 A39 A49	3.2
	A36 A29	3.6	A42 A48	3.2
	A35 A30	3.8	A43 A46	3
	A34 A31	2.6	A44 A45	3.2
	A33 A32	2.4	A41 A40 A47 A50	2.6
	A22	4	A38 A52	3
	Gluing	3.8	A53 A55	3

The co-modelling approach described for soundboard and back is implemented for the other components, in order to achieve a complete modal analysis of the violin. Figure 7 shows an intermediate result corresponding to first mode shape of the body.

Including other components (neck, tailpiece,…) the procedure of numerical modal analysis is extended to the complete instrument (Figure 8). Table 5 collects the list of twenty mode shapes with the corresponding extracted frequencies.

The integrated model based on 3D CAD and FE sections is fully parametric: the used can modify geometrical, physical, or structural parameters and simulate the corresponding effect. In addition, the proposed co-modelling is implemented on violin geometry but it is completely general and can be applied to other families of stringed instruments.

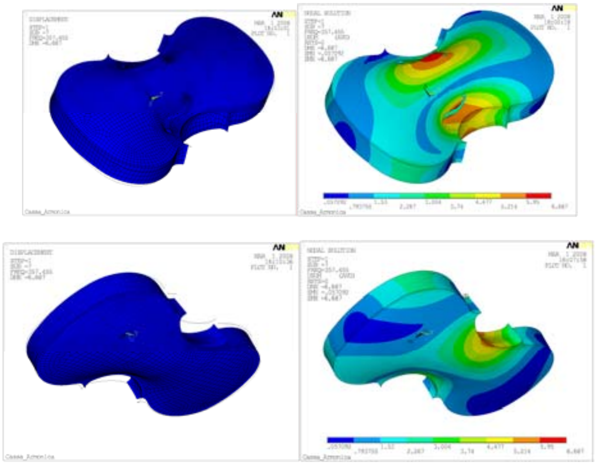


Figure 7. First mode shape of the body

Table 4. Mode shapes vs. frequencies (complete instrument)

Vibration mode	*Frequency [Hz]*
Mode 1	357.4
Mode 2	412.4
Mode 3	568.9
Mode 4	617.8
Mode 5	626.0
Mode 6	707.2
Mode 7	763.9
Mode 8	788.6
Mode 9	862.0
Mode 10	879.1
Mode 11	947.7
Mode 12	998.7
Mode 13	1032.4
Mode 14	1072.5
Mode 15	1111.9
Mode 16	1148.7
Mode 17	1206.
Mode 18	1267.5
Mode 19	1271.2
Mode 20	1342.2

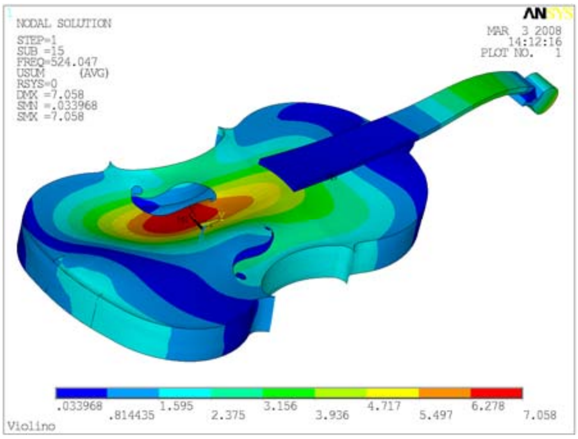


Figure 8. Modal analysis of the complete instrument

SENSITIVITY ANALYSIS

Co-modelling based on integrated CAD and FE packages is organized in parametric way: the user can modify physical parameters (Young's moduli, Poisson's coefficients, densities,…), geometrical characteristics (thickness, curvatures,…) or structural constraints and simulate the corresponding dynamic effect. Sensitivity analysis is a useful tool to supporting the violin maker: making decisions is not based on sensations or intuitions but is a consequence of methodical tests.

Hereafter some sensitivity tests are described. Different mechanical characteristics of spruce are available in literature: four different cases, selected in Table 5, are compared.

Table 5. Mechanical characteristics of spruce

Case	E_{xx} [MPa]	E_{yy} [MPa]	E_{zz} [MPa]	G_{xy} [MPa]	G_{yz} [MPa]	G_{xz} [MPa]
1	700	16230	400	780	630	40
2	910	13700	490	510	730	30
3	830	17000	650	870	640	40
4	920	18000	510	730	760	40

The corresponding frequency modification is shown in Figure 9.

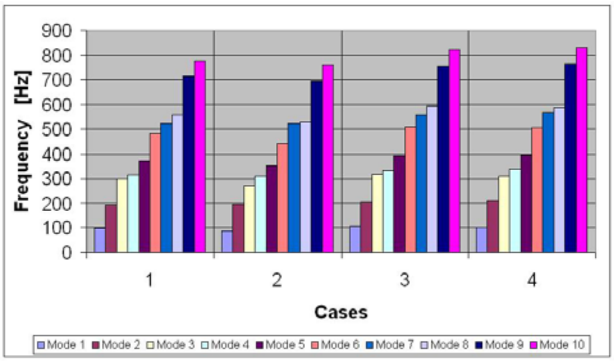


Figure 9. Frequency modification

Within the considered six mechanical characteristics E_{yy} seems to be the most significant: minimum is detected in the case 2 where Eyy assumes the minimum value. Among G moduli G_{xy} influences the frequencies. That is proved assuming E as constant and varying G as shown in Table 6: the corresponding results are collected in Figure 10.

Table 6. Sensitivity analysis on G

Case	E_{xx} [MPa]	E_{yy} [MPa]	E_{zz} [MPa]	G_{xy} [MPa]	G_{yz} [MPa]	G_{xz} [MPa]
1	700	16230	400	780	630	40
2	700	16230	400	510	730	30
3	700	16230	400	870	640	40
4	700	16230	400	730	760	40

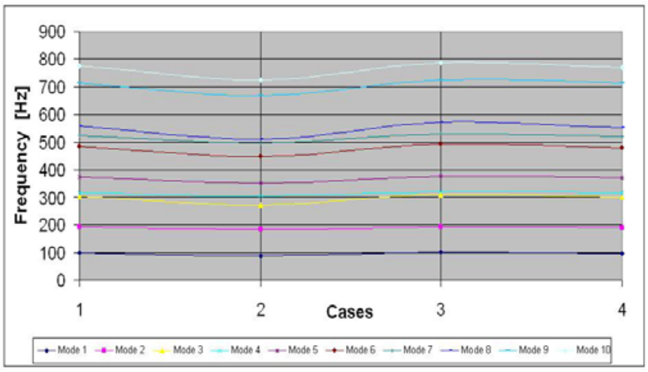


Figure 10. Effects of G variation

CONCLUDING REMARKS

An approach of co-modelling applied to structural and dynamic analysis of violins is implemented. 3D CAD and FE codes are integrated, solving problems related to automatic data transfer.

The structure of the instrument has been parameterized: to each significant area geometrical and physical attributes are assigned and con be easily modified by the user. Sensitivity analyses can be implemented, simulating effects on frequencies and mode shapes.

The integrated package is based on general purpose software, has general characteristics and can be applied to other families of stringed instruments.

Further developments are oriented to improve the parameterization procedure and to develop studies under forced vibrations.

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