Proceedings of 20th International Congress on Acoustics, ICA 2010

23-27 August 2010, Sydney, Australia

Characterization of non-exponential sound energy decays in multiple coupled volumes

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PACS: 43.55 Br.; 43.60 Uv.

ABSTRACT

Recent studies on acoustically coupled volumes, worship spaces, and in concert hall acoustics, have prompted an increasing interest in analyzing sound energy decays consisting of more than one decay slope, so-called non-exponential decays. It has been considered very challenging to estimate parameters associated with double-slope decay characteristics, even more challenging when the coupled-volume systems contain more than two decay processes. To meet the need of characterizing energy decays of multiple decay processes, this paper reports investigations using both acoustical scale-models and numerical models of three coupled volumes. Characterization is based on Bayesian probabilistic inference. Acoustic scale models, and diffusion-equation based models are used to evaluate the estimation strategy, and to validate the results. The analysis method is then applied to geometric-acoustics models of concert halls with more complex geometries. The analysis method within Bayesian framework is capable of determining more than two decay slopes and estimating the corresponding decay parameters.

INTRODUCTION

In single-volume spaces, steady-state sound energy decays are often of single-slope nature, given that diffusion and absorption on interior surfaces is well distributed, otherwise double-slope decay characteristics can occur. In coupled spaces, on the other hand, single-slope decays can also be found given that sound energy exchanges across coupling apertures between spaces may not be significant, or multiple-slope decays, beyond double-slope decays can also occur. In the room-acoustics literature, determinations of double-slope energy decays have often been reported, both from practicing acousticians and research scientists[1-4]. So far, detection and characterisation of more than two-slopped sound decay decays have been considered challenging.

This paper discusses strategies for characterisation of multiple-slope energy decays within Bayesian framework. In order to demonstrate the strategies and capabilities of multiple-slope decay analysis, scale models of three coupled rooms featuring clearly different natural reverberation times are used for the experimental studies [5,6]. A diffusion equation model [7-9] is also applied to three-coupled rooms to achieve numerically modeled results. The investigations using both experimental and numerical models validate the baseline of the Bayesian algorithms. The validated Bayesian methods for determining the number of slopes in measured and/or numerically modeled data, followed by pertinent decay parameter estimations are of practical significance, as this paper also applies the Bayesian methods to the room-acoustics simulations of a 'conceptual' concert hall configurations recently published in Ref.[10].

BAYESIAN METHODS

Energy decay model

Steady-state sound energy decays can be derived from acoustically measured or numerically modeled room impulse responses using Schroeder backwards integration. A parametric model H_M with M exponential decay terms can model the Schroeder decay functions [9]

$$H_M = A_0(t_{ULI} - t) + \sum_{i=1}^{M} A_i e^{-\frac{13.8}{T_i}t}, \; i = 1,\ldots,M \,. \qquad (1)$$

This model consists of M exponential decay terms, and one linear decay term $A_0(t_{ULI} - t)$, with t_{ULI} being the upper limit of Schroeder integration. A_0 is associated with background noise in the room impulse response under investigation. A_i, T_i are decay parameters, termed *linear coefficient* and *decay time* of i th decay slope, respectively. From these decay parameters, it is possible to estimate level differences ΔL_i (the logarithmic ratio of the individual linear parameters A_1, A_2 in the double-slope case, for example), decomposed decay lines

$$10\log_{10} A_i - 10\log_{10} \cdot \frac{13.8}{T_i} t\,, \quad i = 1,\ldots,M\,, \qquad (2)$$

and turning points.[9]

Model-based Bayesian inference

Decay parameter estimations within Bayesian framework employ Bayes rule, given the data vector $\mathbf{D}$ of K data points and model H_i as follows

$$p(\mathbf{A},\mathbf{T}\,|\,\mathbf{D},H_i) = \frac{p(\mathbf{A},\mathbf{T}\,|\,H_i)L(\mathbf{A},\mathbf{T}\,|\,\mathbf{D},H_i)}{p(\mathbf{D}\,|\,H_i)}, \quad (3)$$

where $L(\mathbf{A},\mathbf{T}\,|\,\mathbf{D},H_i)$ is the likelihood function [11]

$$L(\mathbf{A},\mathbf{T}\,|\,\mathbf{D},H_i) = (2\pi)^{-K/2}\,\Gamma\!\left(\frac{K}{2}\right)\frac{Q^{-K/2}}{2}, \quad (4)$$

with $Q = (\mathbf{D}-\mathbf{H})^{Tr}(\mathbf{D}-\mathbf{H})/2$ and model vector $\mathbf{H}$ also contains K model points determined by $2M+1$ parameters without marginalization. $\Gamma(\cdot)$ is Gamma function. Probability $p(\mathbf{D}\,|\,H_i)$ is termed *Bayesian evidence* (evidence, in short), which is of central importance of determining which model H_i is preferred by the data at the level of selecting models.

Bayes' rule in eq.(3) represents how one's prior knowledge on the decay parameters is modified in the presence of the data via the likelihood function. At the decay parameter estimation level, the evidence can be considered as a constant. If no prior knowledge on decay parameters is available, prior probability $p(\mathbf{A},\mathbf{T}\,|\,H_i)$ should be assigned constant, bounded with relatively large parameter ranges. Over these parameter spaces, the posterior probability can then be expressed as

$$p(\mathbf{A},\mathbf{T}\,|\,\mathbf{D},H_i) \propto L(\mathbf{A},\mathbf{T}\,|\,\mathbf{D},H_i)\,. \quad (5)$$

Figure 1 illustrates three representative marginal posterior probability distributions of experimentally measured results over two-dimensional parameter space. In the decay parameter space, there exists an extreme (maximum), the vicinity of which approximates a (multi-dimensional) Gaussian distribution. This is due to the fact that in this room-acoustics application, the number of data points K is often on order of hundreds and thousands (the central limit theorem).

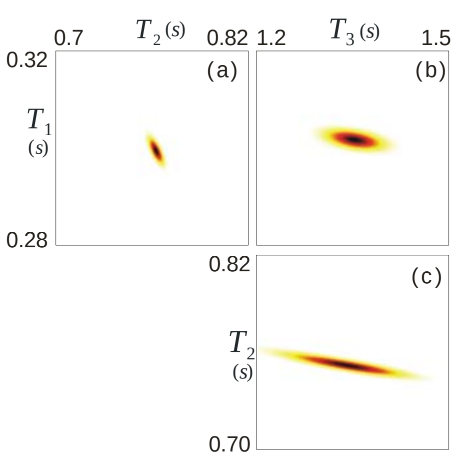

Figure 1. Marginal posterior probability distributions (MPPD) over two-dimensional decay parameter space from experimentally measured data in a scale model of three coupled rooms. (a) MPPD over T_1 and T_2. (b) MPPD over T_1 and T_3. (c) MPPD over T_2 and T_3.

When the posterior probability distributions or equivalently the likelihood distributions within a vicinity around the global peak approximate Gaussian distributions as shown in Fig.1, and there is a sufficiently large number of data points K involved in calculating likelihood distributions of M-slope decays, Bayesian information criteria (BIC) asymptotically approximates logarithmic Bayesian evidence [12]

$$\ln\hat{Z} \approx \ln L(\hat{\mathbf{A}},\hat{\mathbf{T}}\,|\,\mathbf{D},H_i) - (\ln K)(2M+1)/2\,, \quad (6)$$

where $\ln L(\hat{\mathbf{A}},\hat{\mathbf{T}}\,|\,\mathbf{D},H_i)$ represents the natural logarithm of global peak estimate of the likelihood distribution from eq.(3) or eq.(5), so-called maximum likelihood (or *a posterior*) estimate. This paper applies Bayesian information criteria (BIC) in form of eq.(6) to rank the decay models by choosing the largest $\ln\hat{Z}$ associated with model H_i, while uses eq.(5) via eq.(4) to conduct decay parameter estimation.

EXPERIMENTAL & NUMERICAL VALIDATION

Scale model of three coupled rooms

In order to validate the Bayesian algorithm being capable of characterizing decay functions with multiple-slopes beyond double-slope decays, an eighth scale model of three coupled spaces was used for obtaining experimental results back in 2001[5]. This paper reports about the experiments in 2001[5] and applies the Bayesian analysis method documented in 2004[6] and more recently in Ref. [11] using the fully parameterized model in eq.(1).

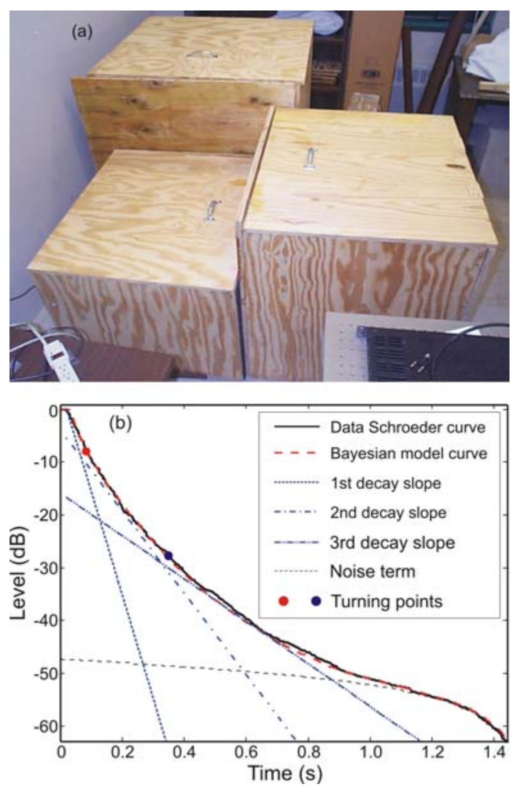

Figure 2. Scale model of three coupled rooms [5] and Bayesian analysis results achieved around 2001. (a) Photo of the scale model of three coupled rooms. (b) Comparison between the measured Schroeder decay curve and the Bayesian model curve along with decomposition of three slope lines.

Almost all interior surfaces of the modeled rooms feature sufficient scattering for frequency bands between 1kHz and 2 kHz (oct.), so that diffuse sound fields within these frequency ranges in individual rooms are expected. Given in real size, the main room (room 1) is of V_1=154 m^3 in volume, with $V_2 \approx 1.5\,V_1$, $V_3 \approx 1.5\,V_2$. Table I lists the natural reverberation times when each room is decoupled, which are measured in four different locations. When the main room is coupled both to room 2 and room 3 via an opening aperture of 4 m^2 to each room, the room impulse responses are measured in the main room. Measured room impulse responses are (oct.) bandpass filtered, followed by Schroeder integration.

Table I. Natural reverberation times (RT) of decoupled rooms and the Bayesian decay time estimations at 1kHz.

	Room 1	Room 2	Room 3
Natrual RT (s)	0.45-0.52	0.79-1.03	1.27-1.39
Decay time (s)	0.30	0.77	1.36

Table I lists three decay times from the Bayesian decay parameter estimation from one data set. Table II lists (natural) logarithmic Bayesian information criteria for double-slope, triple-slope, and four-slope models, respectively. BIC provides unambiguous evidence in preferring the triple-slope model in this measured data set.

Table II. Bayesian information criteria (BIC) evaluated from scale models of three-coupled rooms for 2-slope, 3-slope, and 4-slope model, respectively.

	2-slopes	**3-slopes**	4-slopes
BIC	3307.97	**3580.68**	3549.99

Diffusion equation model of three coupled rooms

Recently, diffusion equation modeling has gained acceptance in room-acoustics for modeling different types of spaces, including coupled spaces. Assuming sound particles travel along straight lines at sound speed c and follow a certain statistical process in the room(s) under investigation with diffusely reflecting walls, the sound-energy density $w(\mathbf{r},t)$ as a function of location $\mathbf{r}$ and time t is then governed by the diffusion equation [8]

$$\frac{\partial w(\mathbf{r},t)}{\partial t} - D\nabla^2 w(\mathbf{r},t) + cm\, w(\mathbf{r},t) = q(\mathbf{r},t)\,,\ \in V\,, \quad (7)$$

subject to a boundary condition on the interior surface S

$$D\frac{\partial w(\mathbf{r},t)}{\partial n} + \frac{c\ \alpha}{2(2-\alpha)}\, w(\mathbf{r},t) = 0\,, \quad (8)$$

where D is so-called diffusion coefficient, c is speed of sound, m is air absorption coefficient, and $q(\mathbf{r},t)$ is a source term. α is absorption coefficient. The diffusion equation model inherently assumes diffusely reflecting interior surfaces in the modeled spaces, so the degree of sound field diffuseness is expected to be high. This diffusion equation model is applied in this work to a three coupled rooms with similar geometries and acoustical configurations to the three-rooms as shown in Fig. 2(a). Table III lists absorption coefficients assigned to each room and their natural reverberation times.

Table III. Absorption coefficients and achieved natural reverberation times (RT) along with the Bayesian decay time estimation from the diffusion equation model of three rooms.

	Room 1	Room 2	Room 3
Absorption coef.	0.28	0.20	0.17
Natrual RT (s)	0.45	0.85	1.3
Decay time (s)	0.388	0.833	1.21

Table IV. Bayesian information criteria (BIC) evaluated for 2-slope, 3-slope, and 4-slope model, respectively, from the modeled result by the diffusion equation model.

	2-slopes	**3-slopes**	4-slopes
BIC	5417.42	**6912.40**	3454.05

Table IV lists Bayesian information criteria estimated from the diffusion equation modeled result for double-slope, triple-slope and four-slope models, respectively. The Bayesian information criteria yield unambiguous evidence in supporting the triple-slope model. Given the selected triple-slope model, Table III also lists the Bayesian decay parameter estimation from the diffusion equation modeled result.

APPLICATION TO GEOMETRICAL-ACOUSTICS MODEL

After validating the capability of the Bayesian algorithm of characterizing multiple-slope decays beyond single-slope and double-slope decays, using purposely designed acoustical scale models and the numerical diffusion equation models, the Bayesian algorithm is applied to characterization of sound energy decays derived from a geometrical-acoustics-based model. A recent paper [10] attempted to study more realistic geometries in modeling a conceptual concert hall with cou-

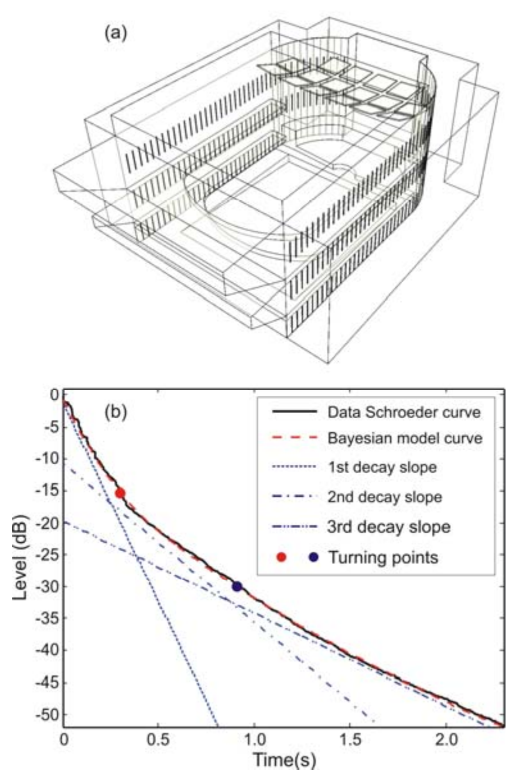

Figure 3. Geometrical-acoustics model of a virtual 'concert hall' with coupled reverberation chambers. (a) Model geometry. (b) Comparison between the CATT-acoustics^{TM} modeled Schroeder decay curve at 1 kHz (oct.) and the Bayesian model curve along with decomposition of three slope lines, and two turning points.

pled reverberation chambers. The recent paper [10], however, did not characterize the decay characteristics properly by employing two straight-line models on preselected level ranges along the energy decay curves.

In order to provide scientifically rigorous analyses, the present work applies Bayesian model selection and decay parameter estimation to the modeled results from a geometrical-acoustics-based model. Taking the same geometrical parameters and the same acoustical parameters, a CATT-acoustics™ model is created. Different aperture sizes and different absorption parameters are adjusted for a number of strategic receiver positions. Similar room impulse responses to those published in Ref.[10] have been obtained. Figure 3 illustrates a representative decay function for an aperture size 0.5% [10].

As listed in Table V, the Bayesian information criteria (BIC) indicate that triple-slope decay model has unambiguously the largest BIC value, the data prefer an explanation that three decay slopes are present. Table VI lists the relevant decay parameters. Of special note is that the second turning point is present within a level range between -25dB and -35dB [see Fig. 3(b)], exact the level range for estimating the late decay time (LDT) as proposed in Ref.[10]. The method as published in Ref.[10] seems to mispresent these data, leading inevitably to oversight of triple-slope decays. Analyses have shown that the aperture size 0.5% and 1.0% [10] for three different absorption configurations exhibits triple-slope decay characteristics in the 1 kHz octave band. Using two straight-line models (L_{10}/ LDT) in Ref.[10] automatically restricts expectations to either single-slope or double-slope decays, while "*the Bayesian analyses did not indicate presence of the double-slope decays* [10]", rather triple slope decays.

Table V. Bayesian information criteria (BIC) of the data set from the geometrical-acoustics model as shown in Fig.3(a), for 2-slope, 3-slope, and 4-slope model, respectively.

	2-slopes	**3-slopes**	4-slopes
BIC	6298.78	**6424.45**	6323.31

Using the estimated parameters, Fig.3(b) illustrates the Bayesian model curve in comparison with the Schroeder decay data. The decay slope line decompositions and the turning points are also illustrated for ease of comparison.

Table VI. Bayesian decay parameters estimated from the geometrical-acoustics model as shown in Fig.3 (a), given that the triple-slope model is preferred by the data.

Parameter	Value
A_1 (dB)	-1.28
$\boldsymbol{T_1}$ **(s)**	**0.96**
A_2 (dB)	-10.65
$\boldsymbol{T_2}$ **(s)**	**2.40**
A_3 (dB)	-19.70
$\boldsymbol{T_3}$ **(s)**	**4.13**

CONCLUDING REMARKS

The results from Bayesian analyses as illustrated in Figs. 1-3 and Table I-VI demonstrate that the decay parameters and their inter-relationship among the three decay terms in case of the triple-slope decays change from data-to-data. In a most recently published paper [10], the use of a ratio of two decay quantities based on two straight lines in the preselected, fixed level ranges, prior to any precise analysis, to quantify the decay characteristics is scientifically dubious. Furthermore, the use of two straight line models inevitably restricts one's expectation to either single-slope or double-slope decays, leading to oversight of triple-slope decays and beyond.

The sound energy decay analysis presents an excellent practical application of two levels of inference; the model selection and the parameter estimation. Bayesian framework provides scientifically rigorous analysis methods to cope with the challenging tasks. Upon available data one should first rank a set of finite completing models. The Bayesian model selection implicitly implements Ockham's razor by penalizing overfitting of overparameterized models. In case of two competing models for explaining the same data, it prefers a simpler one, yet with more generalized predictive power. After determining the correct decay model, the Bayesian formulation yields, at the same time, the decay parameter estimates, providing detailed descriptions of the decay characteristics as required for understanding the coupled-volume systems encountered in practice.

ACKOWLEDGMENT

This work is dediated to Prof. Dr. Manfred Robert Schroeder (1926 - 2009). Thanks are due to Donghua Li and Prof. Paul Goggans, who contributed to the collection of experimental data from scaled-down models of three coupled rooms as part of Li's Master's degree thesis, which was supervised by one of the present authors (N.X.) and Prof. Goggans. We are also grateful to Dr. Leo Beranek, Dr. Jason Summers and Tomislav Jasa for many insightful discussions.

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