Proceedings of 20th International Congress on Acoustics, ICA 2010

23–27 August 2010, Sydney, Australia

Performance Analysis of Dominant Mode Rejection Beamforming

Kathleen E. Wage (1) and John R. Buck (2)

(1) ECE Dept., George Mason University, Fairfax, VA, USA, kwage@gmu.edu
(2) ECE Dept., University of Massachusetts Dartmouth, N. Dartmouth, MA, USA, johnbuck@ieee.org

PACS: 43.60.Fg, 43.60.Jn, 43.60.Mn

ABSTRACT

Dominant Mode Rejection (DMR) adaptive beamforming replaces the covariance matrix for the Minimum Variance Distortionless Response (MVDR) beamformer with a modified sample covariance matrix (SCM). DMR modifies the SCM by first segmenting the eigenvalues into the signal (large eigenvalues) and noise (small eigenvalues) subspaces. The modified SCM uses the large signal eigenvalues but replaces the small noise eigenvalues with the average of these noise eigenvalues. The performance of the DMR beamformer in practical scenarios depends on the quality of the estimates of the rank of the signal subspace, as well as the quality of the estimated signal eigenvalues and associated eigenvectors. Therefore, an important challenge in practical applications of DMR is correctly estimating the rank of the signal subspace. Nadakuditi and Edelman recently developed an extension of the Akaike Information Criteria (AIC) for estimating the number of high dimensional signals from a relatively small number of observations exploiting results from infinite random matrix theory. The accuracy of the new Nadakuditi & Edelman AIC (N/E AIC) in estimating the dominant subspace rank was compared with the traditional AIC and Minimum Description Length (MDL) techniques. These simulations examined uniform linear arrays with one signal and varying numbers of array elements, snapshots and signal-to-noise ratios (SNRs). The N/E AIC performed better than the traditional AIC and MDL approaches in achieving a higher probability of correct rank estimation at a lower SNR in each case evaluated. Additionally, the N/E AIC performs well even in snapshot deficient cases where there are fewer snapshots than sensors. Both the standard AIC and MDL fail in snapshot deficient cases. The N/E AIC performance was also evaluated in simulations including a loud interfering source (+40 dB) and a relatively quiet source (-10 dB below the noise floor) observed by a uniform linear array with half-wavelength sampling over a range of array apertures and numbers of snapshots. The observed Signal to Interferer and Noise Ratio (SINR) for the standard DMR with N/E AIC suffered from a substantial degradation due to mismatch as the number of array elements grew. When the DMR algorithm was modified to incorporate the Cox/Pitre robust DMR method as well as the N/E AIC, the SINR closely tracked the performance of the omniscient beamformer with prior knowledge of the signal subspace rank.

INTRODUCTION

Adaptive beamformers allow for high spatial resolution for the detection and estimation of underwater acoustic sources. When the signal and noise statistics are perfectly known, the minimum variance distortionless response (MVDR) or Capon beamformer [1] [2, Sec. 6.2] is generally considered to be the optimal adaptive beamformer. In practice, the covariance matrix is not available, and must be replaced by a sample covariance matrix (SCM) estimated from the available array observations. As an estimate of the true covariance, the SCM almost invariably has some error relative to the true covariance matrix, and this error or mismatch can cause substantial degradation in the performance of adaptive beamformers. One proposed approach to managing this mismatch in the SCM is the dominant mode rejection (DMR) beamformer [3], [2, Sec. 6.8.3]. The DMR approach begins by performing an eigen-analysis of the SCM and separating the eigenvectors into a dominant signal subspace and a noise subspace based on the magnitude of the associated eigenvalues. All of the eigenvalues for the noise subspace in the SCM are replaced by their average value. The modified SCM is then constructed using the original signal eigenvalues and average noise eigenvalue, and this modified SCM is used in the expressions for the covariance matrix in the MVDR beamformer. The DMR can be interpreted as a modification of the MVDR by enforcing a constraint on the structure of the covariance matrix, specifically that all of the noise subspace dimensions have equal power. Like many adaptive beamformers, the performance of the DMR beamformer has been observed to depend on the ratio of the number of sensors in the array to the number of observations, or the sensor to snapshot ratio as it is often described. This paper explores in depth the factors that the DMR beamformer depends upon, and how these factors vary as the ratio of sensors to snapshots varies. In addition, we demonstrate that a recent result for estimating the number of large dimensional signals in relatively short time series [4] performs well in the critical task of estimating the rank of the signal subspace in DMR.

The rank estimation algorithm mentioned in the previous paragraph is one of several recent results from a topic variously known as infinite random matrix theory (IRMT) or stochastic eigen-analysis. Bai and Silverstein [5] provides a comprehensive overview of the important results in this area, while Nadakuditi [6] summarizes many of the key results as they pertain to adaptive beamforming and signal processing . IRMT provides results on the distributions of eigenvalues and eigenvectors of random matrices which hold in the limit as the matrix becomes infinitely large, but the ratio of rows to columns converges to a fixed constant. In array processing, these results provide asymptotic expressions for the SCM eigenvalues as the number of sensors and snapshots both become infinite, but do so in a manner that the ratio of sensors to snapshots is fixed. These expressions for the eigenvalue distributions form the basis for the rank estimator developed by Nadakuditi and Edelman [4].

The remainder of the paper is organized as follows. The next section presents the DMR beamformer in detail, the techniques used to make DMR robust to mismatch, and the factors determining the DMR beamformer performance. The following section describes the simulations used in this study to evaluate DMR performance in the presence of a strong interferer, then presents the results of those simulations. The final section discusses the results and summarizes the conclusions of the study.

DMR ADAPTIVE BEAMFORMER

The DMR adaptive beamformer [3] assumes that an N element hydrophone array receives a narrowband signal containing D planewaves that can be represented by a vector of complex phasors as

$$\mathbf{p} = \sum_{i=1}^{D} b_i \mathbf{v}_i + \mathbf{n}, \tag{1}$$

where $\mathbf{p}$ is the baseband complex phasors for the observed pressure, b_i is the amplitude of the ith planewave with associated replica vector $\mathbf{v}_i$, and $\mathbf{n}$ is the spatially white observation noise. Under this data model, the covariance matrix for the observed signal is

$$\mathbf{S} = \sum_{i=1}^{D} \sigma_{s_i}^2 \mathbf{v}_i \mathbf{v}_i^{\mathrm{H}} + \sigma_w^2 \mathbf{I}, \tag{2}$$

where $\sigma_{s_i}^2$ is the power of the ith source, and σ_w^2 is the noise power. The DMR beamformer assumes that the eigenvalues of $\mathbf{S}$ can be partitioned into a group of eigenvalues representing the strong signals to be nulled $\gamma_1, \ldots, \gamma_D$ and those due to the observation noise $\gamma_{D+1}, \ldots, \gamma_N$. Assuming that the eigenvalues of $\mathbf{S}$ are sorted in descending order, this can be written as

$$\mathbf{S} = \mathbf{E}\boldsymbol{\Gamma}\mathbf{E}^{\mathrm{H}} = \underbrace{\sum_{n=1}^{D} \gamma_n \mathbf{e}_n \mathbf{e}_n^{\mathrm{H}}}_{\text{large eigenvalues}} + \underbrace{\sum_{n=D+1}^{N} \gamma_n \mathbf{e}_n \mathbf{e}_n^{\mathrm{H}}}_{\text{small eigenvalues}}. \tag{3}$$

When the covariance matrix $\mathbf{S}$ is known, the Minimum Power Distortionless Response (MPDR) adaptive beamformer provides an unbiased estimate of the signal from a desired look direction θ_m while minimizing the output power of the beamformer [2, Sec. 6.2]. (Note: The MPDR is slightly different from the Minimum Variance Distortionless Response (MVDR) beamformer. The MPDR beamformer uses the covariance matrix for the combined signal and noise input, while the MVDR beamformer uses only the noise covariance. Some authors do not bother distinguishing the two. See [2, Sec. 6.2] for a discussion.) The MPDR beamformer weights for a steering vector $\mathbf{v}_m$ are

$$\mathbf{w}_{\mathrm{opt}} = \left(\mathbf{v}_m^{\mathrm{H}} \mathbf{S}^{-1} \mathbf{v}_m\right)^{-1} \mathbf{S}^{-1} \mathbf{v}_m \tag{4}$$

In practice, the true covariance matrix $\mathbf{S}$ is frequently not available, and must be estimated from a set of L observation vectors, or snapshots, $\mathbf{p}_1, \ldots, \mathbf{p}_L$. The result is a sample covariance matrix (SCM)

$$\widehat{\mathbf{S}} = \frac{1}{L} \sum_{n=1}^{L} \mathbf{p}_n \mathbf{p}_n^{\mathrm{H}} \tag{5}$$

$$= \sum_{n=1}^{L} \widehat{\gamma}_n \widehat{\mathbf{e}}_n \widehat{\mathbf{e}}_n^{\mathrm{H}}, \tag{6}$$

where $\widehat{\gamma}_n$ are the SCM eigenvalues and $\widehat{\mathbf{e}}_n$ are the associated SCM eigenvectors.

The DMR beamformer imposes a structural constraint on the SCM before substituting the SCM for the covariance matrix in Eq. (4). Specifically, DMR constrains the eigenvalues for the noise subspace to be constant as in Eq. (2). This constant noise eigenvalue is estimated by averaging the $N-D$ smallest SCM eigenvalues

$$\widehat{\sigma}_w^2 = \frac{1}{N-D} \sum_{n=D+1}^{N} \widehat{\gamma}_n. \tag{7}$$

Replacing $\gamma_{D+1}, \ldots, \gamma_N$ with $\widehat{\sigma}_w^2$ in Eq. (6) yields the modified SCM which the DMR beamformer then substitutes for $\mathbf{S}$ in Eq. (4)

$$\widetilde{\mathbf{S}} = \sum_{n=1}^{D_m} \widehat{\gamma}_n \widehat{\mathbf{e}}_n \widehat{\mathbf{e}}_n^{\mathrm{H}} + \sum_{n=D+1}^{N} \widehat{\sigma}_w^2 \widehat{\mathbf{e}}_n \widehat{\mathbf{e}}_n^{\mathrm{H}} \tag{8}$$

The resulting DMR beamformer weights are

$$\mathbf{w}_{\mathrm{DMR}}(\theta_m) = \frac{\mathbf{v}_m - \sum_{i=1}^{D} \beta_i (\widehat{\mathbf{e}}_i^{\mathrm{H}} \mathbf{v}_m) \widehat{\mathbf{e}}_i}{\mathbf{v}_m^{\mathrm{H}} \mathbf{v}_m - \sum_{i=1}^{D} \beta_i |\widehat{\mathbf{e}}_i^{\mathrm{H}} \mathbf{v}_m|^2} \tag{9}$$

$$\beta_i = \frac{\widehat{\gamma}_i - \widehat{\sigma}_w^2}{\widehat{\gamma}_i}, \tag{10}$$

where θ_m is the bearing corresponding to the steering vector $\mathbf{v}_m$.

Robust DMR

DMR is known to suffer from mismatch when a desired (weak) signal is included in the dominant subspace, i.e., when the signal replica $\mathbf{v}_s$ has a significant projection onto $\mathbf{e}_1, \ldots, \mathbf{e}_D$ [2, Sec. 6.8.3]. In this case, the array weights in Eq. (9) will null a significant portion of the signal energy when the beamformer chooses $\mathbf{v}_s$ as the steering vector. Several authors have proposed different techniques to make the DMR beamformer robust to this mismatch, including a white noise gain constraint, modifying β_i to have an exponential decay in $\widehat{\sigma}_w^2/\widehat{\gamma}_i$ [7], and introducing a weighting factor on β_i to disable the nulling for eigenvectors $\mathbf{e}_i$ which fall too close to the desired look direction [8]. This study implemented the latter approach proposed by Cox and Pitre [8], modifying the array weights according to

$$\mathbf{w}_{\mathrm{DMR}}(\theta_m) = \frac{\mathbf{v}_m - \sum_{i=1}^{D} \delta_i \beta_i (\widehat{\mathbf{e}}_i^{\mathrm{H}} \mathbf{v}_m) \widehat{\mathbf{e}}_i}{\mathbf{v}_m^{\mathrm{H}} \mathbf{v}_m - \sum_{i=1}^{D} \delta_i \beta_i |\widehat{\mathbf{e}}_i^{\mathrm{H}} \mathbf{v}_m|^2} \tag{11}$$

$$\delta_i = \begin{cases} 1 & |\widehat{\mathbf{e}}_i^{\mathrm{H}} \mathbf{v}_m|^2 \le \nu |\mathbf{v}_m^{\mathrm{H}} \mathbf{v}_m|^2, \\ 0 & |\widehat{\mathbf{e}}_i^{\mathrm{H}} \mathbf{v}_m|^2 > \nu |\mathbf{v}_m^{\mathrm{H}} \mathbf{v}_m|^2, \end{cases} \tag{12}$$

where $\nu > 0$ is a tuning parameter. This approach effectively disables the nulling for any eigenvector in the dominant subspace which has too large a projection onto the desired steering vector $\mathbf{v}_m$.

DMR Performance Issues

The performance of the DMR adaptive beamformer depends on several quantities estimated from the SCM. Probably the most crucial is the estimate of the rank of the dominant subspace $\widehat{D}$. Given the correct rank D, accurate estimates of the dominant eigenvalues and their associated eigenvectors are important for good DMR performance. Also, an accurate estimate of the average noise power in the noise subspace $\widehat{\sigma}_w^2$ is important for DMR performance. At present, no analytic results exist to predict or characterize the DMR beamformer's performance as a function the parameters mentioned above, or in terms of the number of snapshots or the ratio of sensors to snapshots. The section that follows describes simulations investigating the relative importance of all of these factors for the case of a weak signal and a single strong interferer.

SIMULATIONS

This section describes simulations of the performance of the DMR adaptive beamformer as a function of the number of sensors and snapshots. The first set of simulations compares three different algorithms for estimating the rank of the dominant subspace. The second set characterizes the DMR beamformer's ability to null a strong interferer using histograms of the beampattern in the interferer direction. The third set of simulations focuses on the overall performance, as defined by the signal-to-interference-and-noise ratio (SINR). The SINR results demonstrate the need for modifications of the standard DMR beamformer to control mismatch and illustrate the performance of the Cox & Pitre approach to controlling mismatch.

Subspace Rank Estimates

DMR beamforming requires knowledge of the dimension D of the dominant subspace. Standard estimators for subspace dimension include the Akaike Information Criterion (AIC) and the Minimum Description Length (MDL) approaches described by Wax and Kailath [9]. Both criteria include the log likelihood of the observed data, which reduces to a statistic that is the ratio of the geometric mean to the arithmetic mean for the smallest $N-d$ eigenvalues of the SCM:

$$t_d = \frac{\prod_{i=d+1}^{N} \widehat{\gamma}_i^{\frac{1}{(N-d)}}}{\frac{1}{N-d}\sum_{i=d+1}^{N} \widehat{\gamma}_i} \tag{13}$$

The AIC & MDL estimators are then defined as:

$$\widehat{D}_{\text{AIC}} = \arg\min_{d} \{-2(N-d)L\log(t_d) + 2d(2N-d)\} \tag{14}$$

$$\widehat{D}_{\text{MDL}} = \arg\min_{d} \left\{-(N-d)L\log(t_d) + \frac{d(2N-d)}{2}\log(L)\right\} \tag{15}$$

where $0 \le d < N$.

Nadakuditi and Edelman developed a new rank estimator using results from IRMT [4]. IRMT predicts the distribution of the sample covariance matrix eigenvalues in the noise-only case ($D = 0$). According to theory the distribution depends on the ratio of sensors (N) to snapshots (L): $c = N/L$. The prediction is valid in the limit as $N, L \longrightarrow \infty$ with c fixed. Using the predicted eigenvalue statistics in conjunction with the Akaike Information Criterion (AIC), Nadakuditi and Edelman (N/E) proposed the following estimator

$$\widehat{D}_{\text{N/E}} = \arg\min_{d} \left\{\frac{t_d^2}{2c^2} + 2(d+1)\right\} \tag{16}$$

$$t_d = N\left[(N-d)\frac{\sum_{i=d+1}^{N}\widehat{\gamma}_i^2}{\left(\sum_{i=d+1}^{N}\widehat{\gamma}_i\right)^2} - (1+c)\right], \tag{17}$$

where $0 \le d < \min(N, L)$.

Figure 1 compares the performance of the N/E AIC estimator with the standard AIC and MDL methods. The simulation environment contains a single planewave signal at broadside plus spatially white noise. The plots show the probability that the rank estimate is correct (*i.e.*, equal to one) as a function of the output SNR of the conventional beamformer. Probabilities were determined using 1000 Monte Carlo trials. Results are shown for three arrays of different lengths and three values of c for each array. The plots indicate that performance depends primarily on the ratio of sensors to snapshots (c). For the snapshot-rich cases ($c = 0.1$), the N/E AIC estimator achieves good results for output SNR's of 0 dB or above. As the figures show, the N/E AIC estimator performs as well as AIC and better than MDL for these snapshot-rich cases. As the number of snapshots decreases and c gets larger, the N/E AIC estimator requires higher SNR to achieve the same performance it did for smaller values of c. When the ratio of sensors to snapshots is 10 ($c = 10$), the N/E AIC estimator works well for the two large arrays ($N = 50, 100$), but fails for the 20-sensor array. Thus the N/E rank estimator is well-suited to large arrays operating in non-stationary environments. In contrast, the AIC and MDL methods fail completely for all array sizes for this snapshot deficient case.

Beampattern Histograms

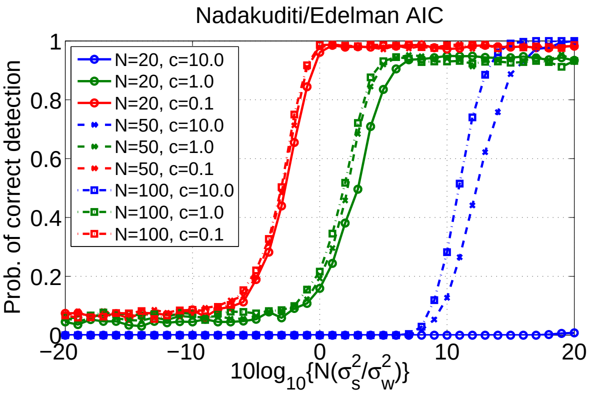
The goal of the DMR beamformer is to null strong interferers in order to facilitate the detection and measurement of weak sources. This section investigates how the null depth of a DMR beamformer varies as a function of snapshots for a 50-element array with half-wavelength spacing. The coordinate system for the simulation measures the arrival angle θ from the positive endfire direction, and source directions are defined by the directional cosine $u = \cos(\theta)$. The simulation environment contains spatially white background noise, a quiet source located at broadside ($u = 0$), and a loud interferer located at $u = 0.1$. The source and interferer have SNR's of -10 dB and +40 dB, respectively, relative to the white noise background. For the beampattern results described below, the DMR beamformers assume the rank of the interference subspace (D) to be equal to 1. Figure 2 compares the beampatterns for the conventional beamformer (uniform weighting), the DMR beamformer, and the optimal MPDR beamformer. The DMR and MPDR beamformers were designed with the true spatial covariance matrix. As Figure 2 indicates, the interferer lies at the peak of the second sidelobe of the conventional beamformer. The adaptive beamformers place a null at the location of the loud interferer. The null depth for the ideal MPDR beamformer is -132 dB and the null depth for the ideal DMR beamformer is -120 dB. These nulls are sufficiently deep to eliminate the influence of the strong interferer on the beamformed output in the look direction.

Ideally the DMR beamformer can effectively null out the strong interferer. In practice the depth of the null achievable by the DMR approach depends on how many snapshots are available to estimate the covariance matrix. Figure 3 shows histograms of the beampattern at the location of the +40 dB interferer for different ratios of sensors to snapshots. The histograms were computed using the results of 300 Monte Carlo trials. Figure 3 illustrates how the null deepens as c decreases, meaning that the DMR processor does a better job of blocking the loud interferer when it has more snapshots to estimate the spatial statistics. With $c = 0.1$ the average null depth is -78 dB. While the DMR beamformer designed using $10N$ snapshots substantially reduces the impact of the loud interferer, note that the null depth has not reached the optimal value of -120 dB achievable when the covariance matrix is known *a priori*.

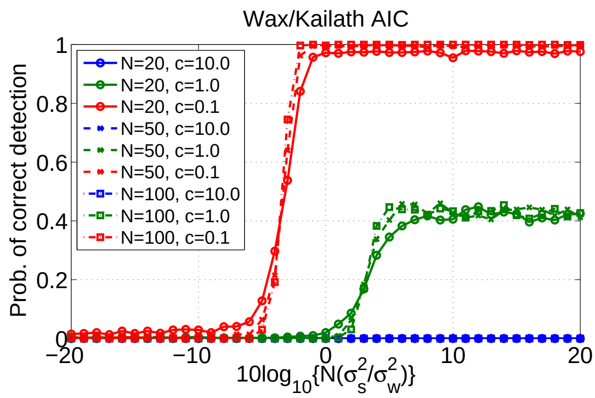
SINR Performance

SINR is a useful metric for characterizing the overall performance of adaptive beamforming algorithms. Given a particular realization $\widehat{\mathbf{w}}_{\text{DMR}}$ of the adaptive weight vector, the SINR at the output of the beamformer is defined as

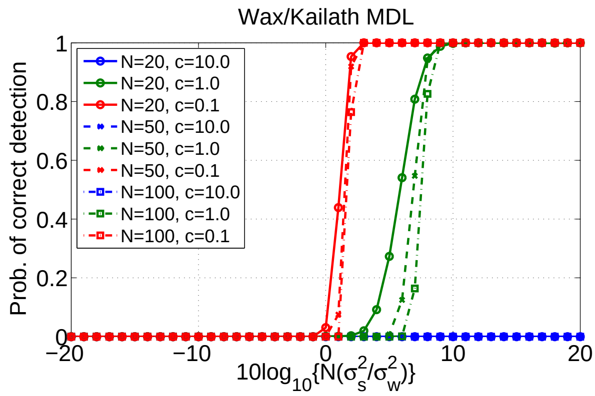
$$\text{SINR} = \frac{\sigma_s^2 \widehat{\mathbf{w}}_{\text{DMR}}^{\text{H}} \mathbf{v}_s \mathbf{v}_s^{\text{H}} \widehat{\mathbf{w}}_{\text{DMR}}}{\widehat{\mathbf{w}}_{\text{DMR}}^{\text{H}} \mathbf{S}_{\text{i+n}} \widehat{\mathbf{w}}_{\text{DMR}}}, \tag{18}$$



(a) Nadakuditi/Edelman AIC [4]



(b) Wax and Kailath AIC [9]



(c) Wax and Kailath MDL [9]

Figure 1: Comparison of the probability of subspace rank detection for the three estimators as a function of output SNR. Colors represent the ratio of sensors per snapshot, and line types represent number of sensors.

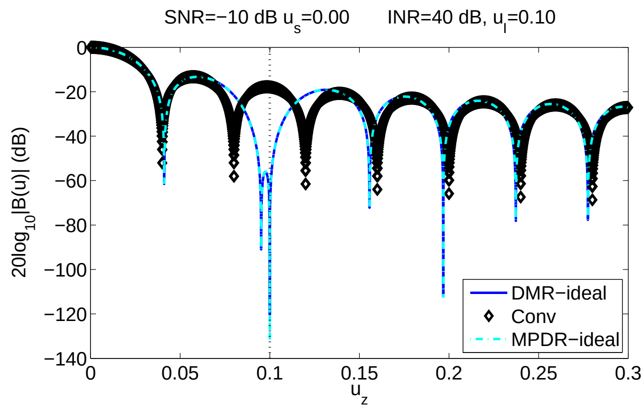


Figure 2: Beampatterns for the conventional beamformer, the ideal MPDR beamformer, and the ideal DMR beamformer (known covariance) for a 50-element linear array. The environment contains a -10 dB source at broadside, a +40 dB interferer at $u = 0.1$, and spatially white noise.

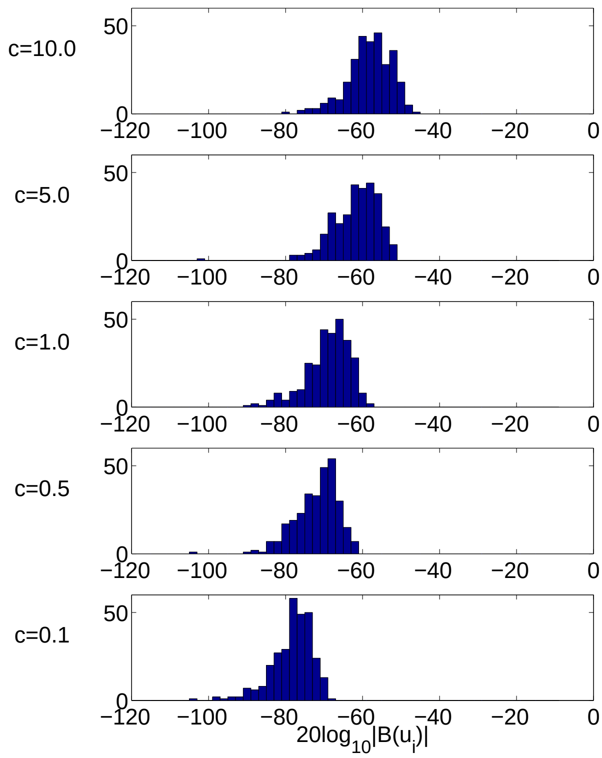


Figure 3: Histogram of null depth in interferer direction when look direction is broadside. Each panel shows a different ratio of sensors per snapshot with the number of sensors fixed at $N = 50$.

where $\mathbf{S}_{\mathrm{i+n}}$ is the interference plus noise covariance matrix. The SINR depends on the adaptive weight vector (a random quantity), thus it is a random variable whose statistics can be quantified. For the results presented below the known interference plus noise covariance matrix $\mathbf{S}_{\mathrm{i+n}}$ for the simulation was used in the SINR calculation since it produces smoother results [2].

Figure 4 characterizes the performance of the DMR beamformer using the average SINR. The plot shows how SINR varies as a function of the number of snapshots (L). Each data point represents 300 Monte Carlo trials. As L increases the performance of the DMR beamformer gradually approaches that of the optimal MPDR beamformer designed using clairvoyant statistics.

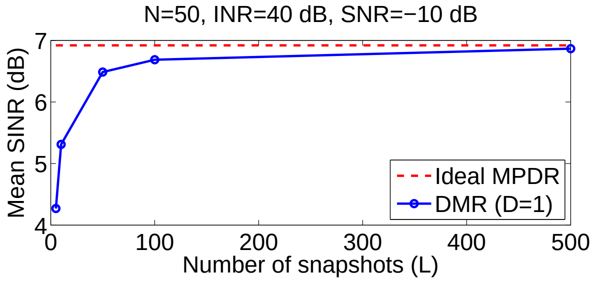


Figure 4: Average SINR as a function of the number of snapshots for simulation example with 50-element array. Plot shows how the SINR of the DMR beamformer approaches the SINR achievable with the ideal MPDR beamformer designed with a known covariance matrix.

The normalized SINR metric η defined below indicates how close the DMR beamformer comes to the optimal MPDR result:

$$\eta = \frac{\overline{\mathrm{SINR}_{\mathrm{DMR}}}}{\mathrm{SINR}_{\mathrm{MPDR-ideal}}}. \tag{19}$$

Figure 5 shows the normalized SINR for the standard DMR beamformer as a function of array length. The solid lines show the performance for different values of c when the rank of the dominant subspace is assumed equal to one. Excluding a transient for low numbers of sensors, the performance is flat, meaning that η is essentially determined by the ratio $c = N/L$ and is not dependent on N. The dashed lines show the corresponding results for the case when the rank of the dominant subspace is estimated from the snapshot data using the N/E AIC algorithm. In contrast to the known-rank case, these results show a substantial degradation in performance as the number of sensors increases. As N increases, the N/E AIC algorithm is more likely to estimate a rank of two for the dominant subspace. The second eigenvector in the subspace will be very close to the replica of the desired signal (though not exactly the same due to mismatch). The DMR processor has a well-known problem when the signal is included in the dominant subspace, since the processor attempts to null the modes in this subspace.

Obviously this type of performance degradation is undesirable. Mismatch control techniques, such as the one proposed by Cox/Pitre, can be used to mitigate this problem. Figure 6 shows that the Cox/Pitre robust method with $\nu = 0.5$ in Eq. (12) yields the same normalized SINR as for the known-rank case.

DISCUSSION AND CONCLUSIONS

This paper presented simulations studying the performance of the dominant mode rejection (DMR) adaptive beamformer for a uniform linear array observing narrowband signals that contain a weak source signal in the presence of a strong interferer and

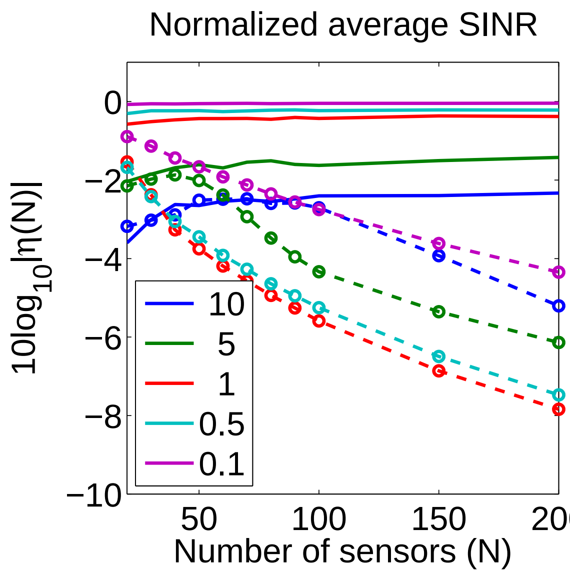


Figure 5: Comparison of DMR output SINR normalized by the ideal MPDR SINR as a function of the number of array sensors N. The plot compares the performance of DMR assuming $D = 1$ (solid lines) with DMR estimating dominant subspace rank using the N/E AIC (circle/dashed lines). Colors represent different ratios of sensors per snapshot.

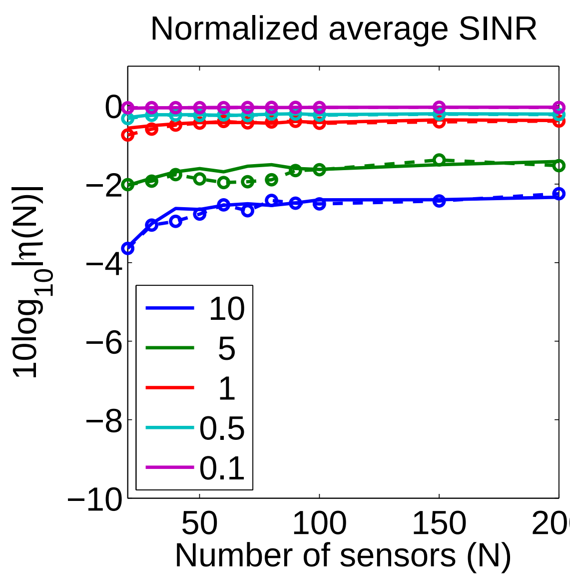


Figure 6: Comparison of DMR output SINR normalized by the ideal MPDR SINR as a function of the number of array sensors N. The plot compares the performance of DMR assuming $D = 1$ (solid lines) with DMR estimating dominant subspace rank using the N/E AIC and Cox/Pitre mismatch control [8] (circle/dashed lines). Colors represent different ratios of sensors per snapshot.

white noise. The simulations demonstrate that the rank estimator in Nadakuditi and Edelman [4] performs better than the classic MDL or AIC estimators for the signal subspace rank proposed by Wax and Kailath [9], especially for scenarios with fewer snapshots per sensor. The simulations also demonstrate that the depth of the null for a strong interferer also depends on this ratio of sensors to snapshots, and that even for a snapshot-rich scenario the null depth falls well short of the theoretical null depth for an omniscient processor that knows the covariance matrix. The simulations results also demonstrated that the sensitivity of the N/E AIC estimator to weak signals means that the DMR implementation for large arrays requires adequate mismatch control to prevent performance degradation. As the array grows, the N/E AIC is actually too successful at detecting the presence of the weak signal and begins including an eigenvector in the dominant subspace that is too close to the steering vector for the weak signal. Using the N/E AIC puts a higher premium on some form of mismatch control such as Cox and Pitre [8] to prevent this undesired nulling of the desired signal. Incorporating this mismatch control allowed the N/E AIC DMR to achieve comparable SINR performance to a DMR algorithm told *a priori* that signal subspace rank was $D = 1$.

All of the simulations found that the two major parameters determining DMR performance were the input SNR and the ratio of sensors to snapshots (c). This latter factor is consistent with infinite random matrix theory results that the density of the eigenvalues of sample covariance matrices are parametrized by this same ratio of sensors to snapshots. In light of this, the success of the N/E AIC rank estimator is not surprising, since it is the only of the three signal subspace rank estimators that incorporates c into its estimator statistics.

ACKNOWLEDGMENTS

This work was supported by the US Office of Naval Research Code 321US through grants N00014-09-1-0114 and N00014-09-1-0167. The authors wish to thank Doug Abraham, Arthur Baggeroer, Harry Cox, Keith Davidson, Raj Rao Nadakuditi, and John Tague for helpful discussions.

REFERENCES

[1] J. Capon. High-resolution frequency-wavenumber spectrum analysis. *Proceedings of the IEEE*, 57(8):1408–1418, August 1969.

[2] Harry L. Van Trees. *Optimum Array Processing*. John Wiley and Sons, New York, NY, 2002.

[3] Douglas A. Abraham and Norm L. Owsley. Beamforming with dominant mode rejection. In *Proc. IEEE Oceans*, pages 470–475, 1990.

[4] Rajesh Rao Nadakuditi and Alan Edelman. Sample eigenvalue based detection of high-dimensional signals in white noise using relatively few samples. *IEEE Trans. Signal Processing*, 56(7):2625–2638, July 2008.

[5] Zhidong Bai and Jack W. Silverstein. *Spectral Analysis of Large Dimensional Random Matrices*. Springer, New York, second edition, 2010.

[6] Rajesh Rao Nadakuditi. *Applied Stochastic Eigenanalysis*. PhD thesis, Massachusetts Institute of Technology/Woods Hole Oceanographic Institution, 2007.

[7] N. Li, J. Tang, Y. Peng, and X. Wang. An improved dominant mode rejection adaptive beamforming algorithm. In *CIE Int'l Conf. on Radar*, pages 1–4, October 2006.

[8] Henry Cox and Richard Pitre. Robust DMR and Multi-Rate Adaptive Beamforming. In *Proceedings of the 32nd Asilomar Conference on Signals, Systems, and Computers*, pages 920–924, 1998.

[9] Mati Wax and Thomas Kailath. Detection of signals by information theoretic criteria. *IEEE Trans. on Acoustics, Speech, and Signal Processing*, ASSP-33(2):387–392, April 1985.