Brake Vibration and Noise - A Review and Discussion

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ABSTRACT

Brake vibration & noise is the friction induced phenomenon which has been studied by many researchers in a variety of ways due to its importance and complexity since the early 20th century. Recently rapid progress has been made in both theoretical and experimental studies, especially on the modeling and analytical methodology of solution to the practical problems. Present paper besides a general review of the literature concerned tries to make emphasize on the common points of different investigation works. The literature is including our own works since 1984 in Tsinghua University, but many of them are none published in English. For its presentation it takes some pages. The analysis made is from point of view for suppression of brake vibration & noise.

INTRODUCTION

Brake vibration & noise problem almost has existed since brake product comes out, as a friction induced phenomenon. With the development and light weight design of brake product, its mechanism is becoming complicated and the problem is becoming more serious. Since 20th century, many investigators have studied this problem. Even though, due to its immense complexity, none comprehensive understanding has been reached and none general means for quiet brake design in the product developing stage have been approached. Recently big progress have been made in both experimental and theoretical modeling analysis, especially in the development of analytical methodology of suppressing brake vibration and noise occurrence for industrial use. Both experimental and analytical investigations are equally important. They promote each other in the research development. With rapid progressive development of computational science, the computational and simulational means can make big advantage in investigation and industry product development. So to develop powerful analytical and simulational means is necessary. As reviewing the experimental and analytical investigations, very similar results can be found. For example, comparison between Felsk's experimental work on drum brake squeal[1] and the work of Zhu and Guan[2,3], similar results are achieved by stiffening the back plate assembly, low frequency squealing of the two drum brakes can be eliminated respectively; another example is the comparison between Baba's work on disc brake squeal[4] and the work of Guan and Jiang[5,6], they obtained similar results by modifying the mounting bracket of the disc brake, and low frequency squealing of the two disc brakes can be eliminated. These are evidences supporting that there are some regularities exist, and the analysis method is practical in principle even if it is not completed at this moment. These methods should be advanced in further investigation and practice.

Recently, transient analysis method in time domain is rising. It can present different nonlinear characteristics of the system and with time dependent loads. It can give the amplitude of unstable vibration. But the shortcomings of transient analysis are its formidable computing time and difficulty to identify the nonlinear function of the varying parameters. For the linear complex eigen analysis method, the problem of over-prediction exists, which makes it difficult to judge the problem accurately when the product is in virtual development stage.The problem should be investigated in cooperation by two ways, i.e. complex eigenvalue analysis and transient simulation which can complement to each other.

A series of work done by Tsinghua University since 1984 and non-published in English are given in present paper. It will take some pages with figure illustrations.

THE FEATURE OF BRAKE VIBRATION & NOISE AND INFLUENCING FACTORS

THE FREQUENCY CHARACTERISTICS

In the very early stage of investigation, people have focused on brake frequency characteristics. Brake vibration and noise are classified by its frequency range mostly[7]. The brake vibration & noise frequency falls in frequency range from several Hz to 16 kHz. Depending on the frequency of bake vibration & noise, they are divided into low frequency vibration (under 300 Hz) and high frequency (3000~15000 Hz)[8]. But different classifications are made by investigators [9~12].

The vibration & noise of a certain brake has a specific peak at certain frequency, but not a pure tone for most cases (See Figure 1, 2)[13]. Such kind of feature supported by other studies (See Figure 3, 4)[2,5].

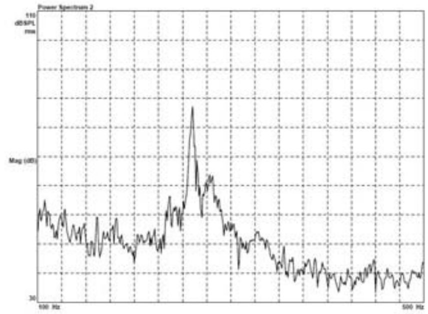


Figure 1. Interior sound pressure spectrum – sustained groan event[13]

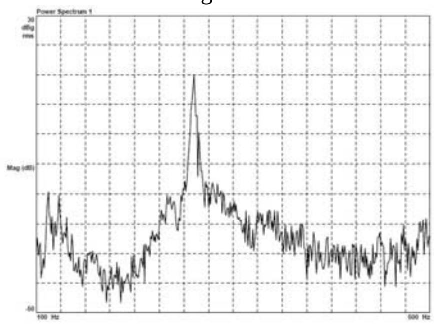


Figure 2. Lateral calliper acceleration spectrum – sustained groan event[13]

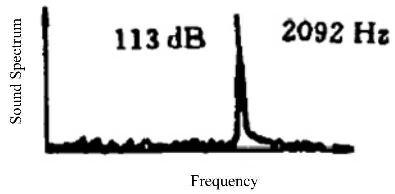


Figure 3. Sound spectrum of a drum brake squeal[2]

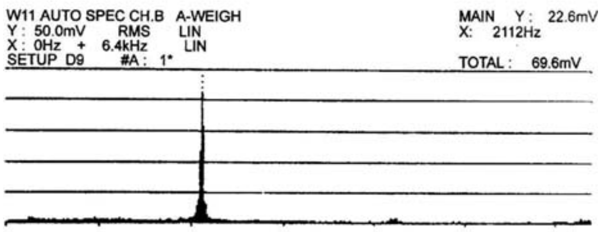


Figure 4. Sound spectrum of a car disc brake squeal[5]

THE INFLUENCING FACTORS AND RANDOMNESS OF BRAKE VIBRATION & NOISE OCCURRENCE

The influencing factors can be classified into two categories: one is can be known relative easily by its influence regularity through theoretical analysis. They are friction coefficient, component structural dynamics and material modulus and etc. They are making influence on brake vibration &noise occurrence directly. It will be presented later; another group of influencing factors, such as temperature, humidity and operation conditions (braking speed pad pressure, etc). They are probably the functions of frictional coefficient and lining stiffness which make influence indirectly but through frictional force.

Antii [7] gives two pictures (Figure 5, 6) of percentage occurrence of brake squeal obtained at PBR Automotive Pty Ltd using a Rubore drag type noise dynamometer and an AK noise matrix for various brake pressures and temperatures respectively, based on which it is summarized that there is no simple relationship between the percentage occurrence and pressure (Figure 5). Similarly, the influence of frequency of the brake squeal is quite complex (Figure 6).

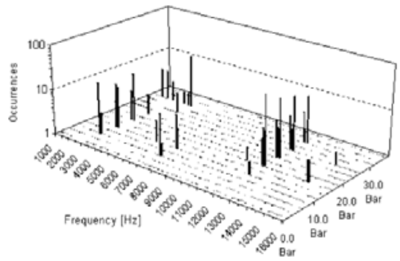


Figure 5. Variation of occurrences of brake squeal with frequency andbrake pad pressure [7]

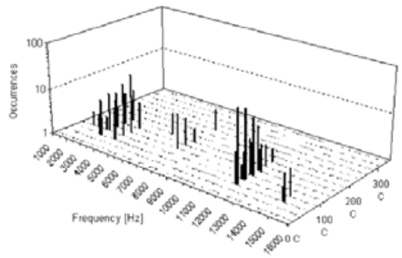


Figure 6. Variation of occurrences of brake squeal with frequency and temperature [7]

And brake working history especially the thermal history of brake makes influence on brake vibration & noise occurrence also. [13,14] pointed that some brake burnishing is usually necessary before groan will occur.

These influence factors lead the brake vibration & noise occurrence to have strong random characteristics. The logical analysis about it should be that most of them make influence on brake vibration & noise occurrence no directly, but through changing the characteristics of the factors, which make influence directly such as frictional and elastic characteristics of frictional materials. And the functions between them are immense complicated[15~18].

THE INVESTIGATION ON MECHANISM OF BRAKE VIBRATION & NOISE OCCURRENCE

FRICTIONAL MODEL

At the very early stage of investigation researchers focused on frictional characteristics of friction pairs.

A classical typical example is the friction coefficient opposes the relative motion speed, or negative slope between friction coefficient and relative motion speed between slider pair. But Hunt et al. [19] performed experiments on steady sliding of machine tools and concluded that velocity dependence of friction alone could not account for all observed effects in practical problems.

For the stick-slip vibration phenomenon the dynamic coefficient of friction is less than its static one. But these kind of explanation cannot be satisfied for understanding so complicated phenomenon of brake vibration and noise occurrence. An experimental investigation on low frequency drum brake vibration (judder) shows (see Figure 7) [14] (1988) that the stick-slip is occurred only at the end of the brake vibration process. This phenomenon explained by theoretical analysis also. The critical condition for stick-slip occurrence is $\dot{X}_{\max} \geq V_0$, where $\dot{X}$ is the relative speed between tangential speed of shoe and drum; V_0 is the drum circumferential speed. So stick-slip in most cases probably is not the triggering factor of brake vibration, but inversely the big enough brake vibration amplitude gives a condition the stick-slip can be occurred at the end of braking process, when the V_0 is becoming significant small.

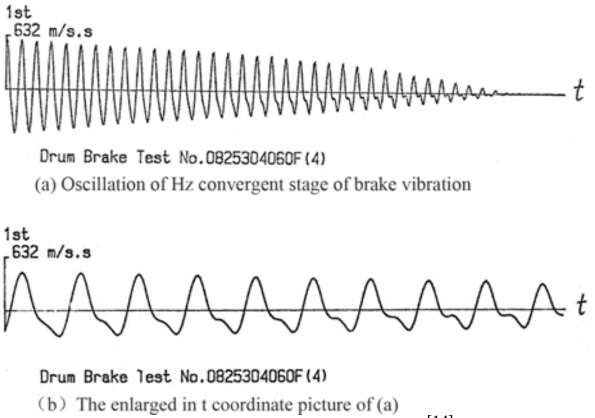


Figure 7. Stick-slip oscillation[14]

THE SCHEMATIC MODEL ON MECHANISM OF BRAKE VIBRATION & NOISE

The early well know example is Spurr's model (1961) (see Figure 8) [20].

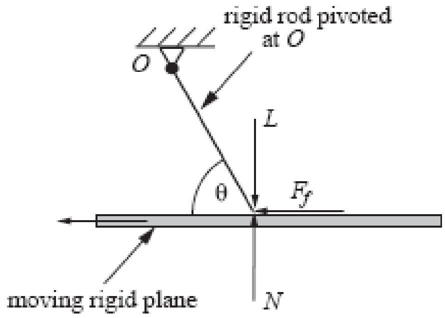


Figure 8. A rigid, massless rod, which is pivoted at O (whose position is fixed) and loaded by an external force L at its free end, contacting a rigid moving plane. At the point contact between the plane and the rod, a friction force Ff is present.[20]

The importance of this model is it includes systems designed parameter (θ), except frictional coefficient. It enlightens investigators as to structural design factors.

Above presented models belongs to structural non-closed coupling model.

A structural frictional closed-loop coupling model is given by [14] (1988) to the mechanism of brake vibration and noise occurrence (see Figure 9).

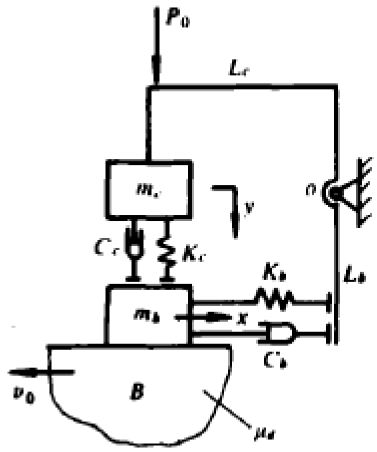


Figure 9 structural frictional closed-loop coupling model [14]

The dynamical equation for the system vibration is

$$[m]\{\ddot{z}\}+[c]\{\dot{z}\}+[k]\{z\}=\{F_f\} \tag{1}$$

Where

$$\{z\}=\begin{Bmatrix} x \\ y \end{Bmatrix},\ [m]=\begin{bmatrix} m_b & \\ & m_c \end{bmatrix},$$

$$[c]=\begin{bmatrix} c_b & -c_b L_b/L_c \\ -c_b L_b/L_c & c_c+c_b L_b^2/L_c^2 \end{bmatrix},$$

$$[k]=\begin{bmatrix} k_b & -k_b L_b/L_c \\ -k_b L_b/L_c & k_c+k_b L_b^2/L_c^2 \end{bmatrix},$$

$$\{F_f\}=\pm\mu_d\left(P_0+[G_c]\{\dot{z}\}+[G_k]\{z\}\right)$$

$$G_c=\begin{bmatrix} 0 & c_c \\ 0 & 0 \end{bmatrix},\ G_k=\begin{bmatrix} 0 & k_c \\ 0 & 0 \end{bmatrix}$$

Since P_0 is a constant, it can be eliminated by transforming the coordinate system. The equation of free vibration is obtained.

$$[m]\{\ddot{z}\}+\left[C_g\right]\{\dot{z}\}+\left[K_g\right]\{z\}=\{0\} \tag{2}$$

Where $\left[C_g\right]$ and $\left[K_g\right]$ are unsymmetric.

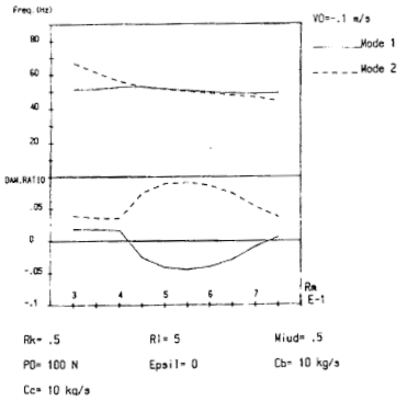
$$\left[C_g\right]=\begin{bmatrix} c_b & -c_b L_b/L_c-c_c \\ -c_b L_b/L_c & c_c+c_b L_b^2/L_c^2 \end{bmatrix}$$

$$\left[K_g\right]=\begin{bmatrix} k_b & -k_b L_b/L_c-\mu_d k_c \\ -k_b L_b/L_c & k_c+k_b L_b^2/L_c^2 \end{bmatrix}$$

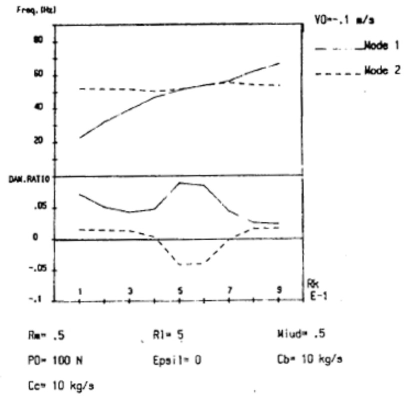
According to the system unsymmetric characteristic matrix under certain condition, the complex eigenvalues can be obtained. Corresponding with the positive real part of eigenvalue, the unstable mode of vibration is identified.

Letting $R_m=\dfrac{m_c}{m_b}$, $R_k=\dfrac{K_c}{K_b}$, $R_l=\dfrac{L_c}{L_b}$ and μ_d=constant .

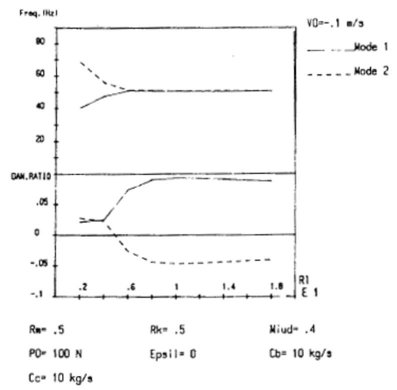
Figure 10(a,b,c) are the analytical results of system vibration frequency and damping ratio vs system parameters matching (R_m , R_k and R_l). It can be seen that under certain parameters matching, the unstable mode with negative damping ratio is obtained. It exhibits the importance of matching the structural parameters appropriately for keeping the dry frictional system stable. The analysis is made by neglecting the damping factor.



(a) Rm – Freq & Damp



(b) Rk – Freq & Damp



(c) Rl – Freq & Damp

Figure 10. Analytical results of system vibration frequency and damping ratio vs system parameters matching[14]

This model is more closed to the real brake system in principle and can be referred to for practical brake vibration noise modeling. [2,3,5,6,14] works did it in this way, which will be presented later.

The closed-loop couple theory is verified by an experimental record on a drum brake low frequency vibration[14].

The mode coupling model established by Hoffmann and Gaul[21](see Figure 11)

Based on the linearised equations of motion

$$\begin{bmatrix} m & \\ & m \end{bmatrix}\begin{Bmatrix} \ddot{x} \\ \ddot{y} \end{Bmatrix} + \begin{bmatrix} k_{11} & k_{12} - \mu k_3 \\ k_{21} & k_{22} \end{bmatrix}\begin{Bmatrix} x \\ y \end{Bmatrix} = 0 \qquad (3)$$

The stability analysis is made for a specific case.

$$\begin{bmatrix} 1 & \\ & 1 \end{bmatrix}\begin{Bmatrix} \ddot{x} \\ \ddot{y} \end{Bmatrix} + \begin{bmatrix} 2 & 1-\mu\Delta \\ 1 & 2 \end{bmatrix}\begin{Bmatrix} x \\ y \end{Bmatrix} = 0$$

When $\Delta > 1$, a pair of unstable and stable mode results are obtained.

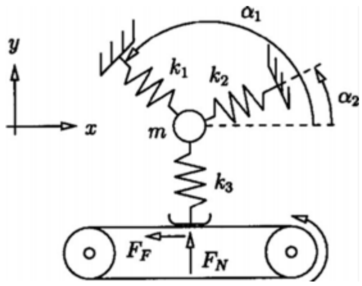


Figure 11 Minimal single mass two degree of freedom model [21]

It explores how is the phase shifts between in-and-out-of-plane oscillations is formed physically, which is necessary for energy feed-in of self-excite vibration. Such an explanation is suitable for all systems, having unsymmetric stiffness matrix without damping element considered. But whether the peculiar structure characteristics of the model is suitable for practical brake is uncertain.

Rohinowicz[22] also had put forward the idea of memory-dependent friction in which a time lag of friction response is obtained.

The principal schematic models help people to understand the mechanism of brake vibration & noise occurrence, but cannot be used to solve the practical problems occurred with the real brakes.

EXPERIMENTAL AND COMPUTATIONAL INVESTIGATION ON BRAKE VIBRATION AND NOISE

EXPERIMENTAL INVESTIGATIONS

At the early time the experimental investigations are in the majority of study because of oversimplified models are not correlated to actual brakes. The experimental investigations can be divided into three categories roughly;

1) The aim of investigation is to find the brake vibration & noise phenomenon at every aspect and try to suppress it.

Felske[1,23] exposed both disk and drum brakes vibration pattern on test rig by Holographic interferometry measurement found a series of possibilities of eliminating squeal and made a success in suppressing a drum brake squeal by stiffening it's backing plate. Felske's work enlightens investigators as to the structural dynamics of components which make much influencing on brake vibration & noise occurrence.

The phenomenon that brake vibration and squealing frequency is non of any resonance frequency of its components was incomprehensive to the investigators. [24] suggested that the brake squeal is closely related to the resonance of coupled brake system as a whole did an excitation test on non working whole disc brake assembly (see Figure 12). It is found the resonance frequency by excitation test, which agree with squealing frequency of studied brake (7.8 Hz). And the measures based on the investigation to suppress the disc brake squealing are found.

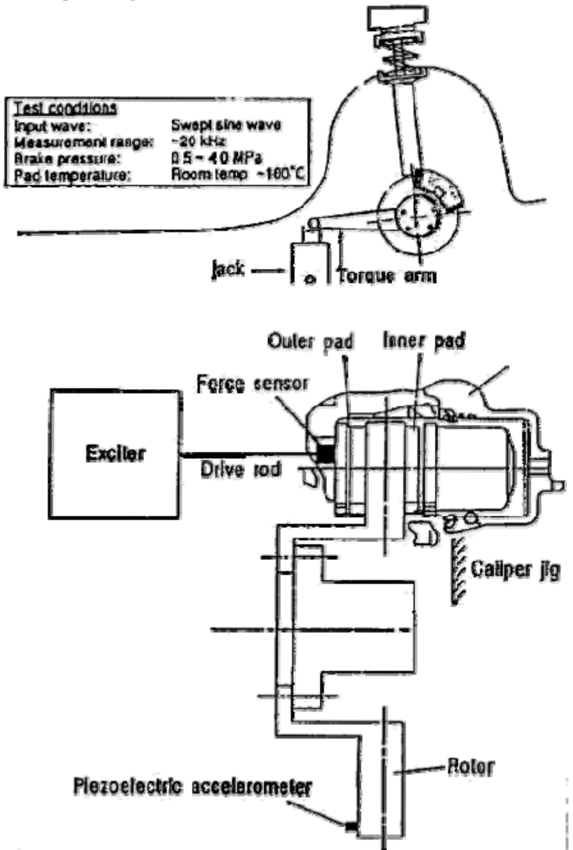


Figure 12.Excitation test apparatus for disc brake assembly[24]

Baba[4] suggests that the mounting bracket of a disc brake is the vibration translation component of the brake squealing. 10 examples of structural modified mounting bracket are made (see Figure 13 from A to K).

Mounting bracket No.	Vehicle test	Results of FEM analysis			
	Coefficient of squeal ξ	Vibration mode			Natural frequency (Hz)
		Max. of amplitude δ max	Amplitude difference $\delta_{out} - \delta_{in}$	Phase difference θ ($\delta_{out} / \delta_{in}$)	
A	1.00	1.00	1.00	1.00	2255
B	0.94	0.89	0.57	0.71	2233
C	0.55	0.73	0.23	1.01	2340
D	0.51	0.67	0.42	0.19	2355
J	0.04	0.25	0.04	0.05	2278
K	0.00	0.28	0.07	0.15	2358

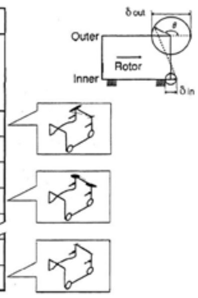


Fig. 12 Relationship between Mode of Vibration and Squealy Tendency (2)

Figure 13. Relationship between mode of vibration and squeal tendency[4]

Based on the vehicle road comparison test of them an empirical of squeal tendency equation is given. It indicates that when the mounting bracket is excited, the smaller the maxim amplitude and the difference between the inner and outer amplitudes, the less likely to squeal.

Above presented examples of experimental investigation on brake vibration & noise reveal different aspects of brake vibration & noise phenomenon, which provide useful evidence for theoretical and modeling analysis.

The common feature of measures taken for eliminating brake vibration and noise occurrence is the modification of dynamic characteristics of related components of brakes. It can be summarized that the component structural dynamics is the most active factors, which influencing the brake vibration & noise occurrence.

Nevertheless, the experimental investigation is time, finance and labor heavily consumed and difficult to give the general rules to guide the quiet brake design at the product design stage.

So the theoretical and computational (simulational) study on brake vibration & noise is developed in parallel or in co-operation with the experimental investigation.

2) Another kind of experimental investigation are developed at more earlier time. They are focused on different much more simple schematic models of mechanism of brake vibration & noise occurrence.

In 1972, North[25] published the agreement of its eight-degree of freedom model with the experiment results. Before North the similar studies were made by Jarvis[26] and Earles[27] on pin-disc system modeling in noise generation.

In beginning of the 21st century continued the study on experimental point contact model by [28-30]. It emerged that incipient squeal frequencies observed experimentally could be predicted in 75% of the cases using the simplest formalism presented in [29]. This study also reveals that the over changing nature of friction- induced noises can, to a good extent, be explained by slight structural variation undergone by any mechanical system[30].

3) With the progress of research on brake vibration and noise problem into the realities, a specific kind of experimental analysis is performed by Francesco Massi[31] named Tribo Brake setup. It has three main advantages: (1) it can generate squeal noise easily; (2) it has a dynamic behavior that can be easily measured; and (3) it has a dynamic behavior that can be easily adjusted.

The proposed experimental setup simplifies the geometry of a real brake extremely, so that it is possible to have repeatable and consistent measurements. A series of brake vibration and noise occurrence mechanism can be conformed based on the experimental measurement analysis. Moreover, based on such kind of experimental analysis, different modeling analysis approaches can be investigated. For example[32], the linear and nonlinear numerical approaches are investigated based on them and very important conclusions are made: " a linear one, useful to predict the squeal onset in a wide range of driving parameters and a nonlinear model, able to reproduce the squeal phenomena in the time domain.", and " The obtained results confirm the need for both numerical approaches; i.e. the use of both a linear and a nonlinear analysis for the study of brake squeal."

THEORETICAL AND COMPUTATIONAL MODELING AND ANALYSIS

It seems that the simplified models are not correlated to practical brake vibration & noise occurrence. They are can not be used to analysis the influencing factors of a concrete brake system which should be modified to suppress the vibration generation. Chen[14] for a drum brake low frequency vibration (judder) model and Liles[33] finite element disc squealing model are the earliest relative complete models.

Figure 14 shows the scheme of a drum brake low frequency model[14]. The model is established according to the structure frictional closed-loop coupling theory of brake vibration & noise occurrence.

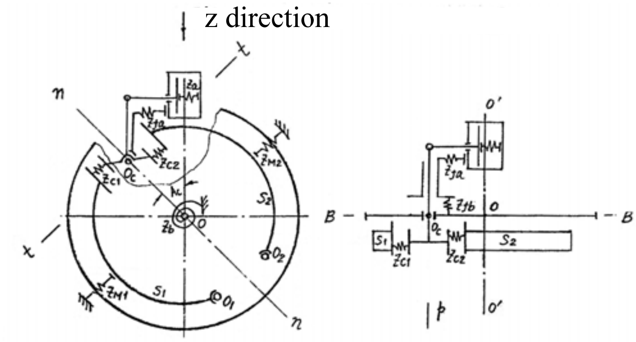


Figure 14. Structural diagram of drum brake model[14]
B-B: back plate flat; o'-o': rotation axis of drum; p-p: axis of cam shaft; o: intersection point of o'-o' and B-B; oc: intersection point of p-p and B-B; n-n: straight line through o and oc; t-t: straight line through oc and perpendicular to n-n

The approximate scheme of the model closed-loop path is shown in Figure 15.

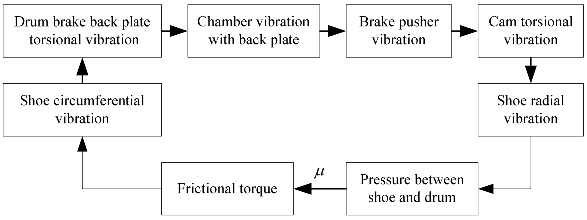
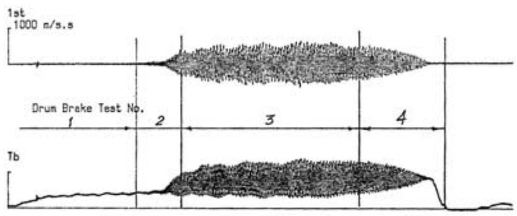


Figure 15. Approximate scheme of the model closed-loop path [14]

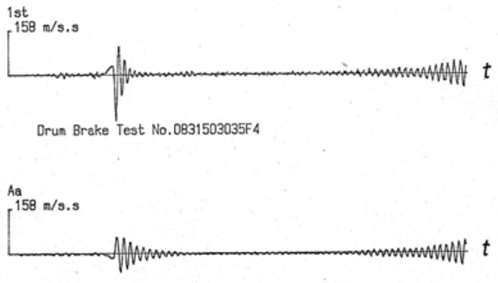
The theoretical analysis results well agreed with experimental one in many aspects. They are:

1) The computational vibration frequency is 76 Hz, while the experimental one is 73~76 Hz;

2) Figure 16 shows a whole braking and vibrating process. It can be seen that the vibration frequency is considerably stable. The process can be categorized into 4 stages (Figure 16(a)). Figure 16(b) is the enlarged picture of the preformation and beginning of the emergence stages. It can be seen that the system is stable, when the system is not completely closed coupled. The original disturbance is always depressive. Afterwards the system is becoming to an unstable one and under any slight disturbance the vibration is emerged and keeping going on until the drum stops.

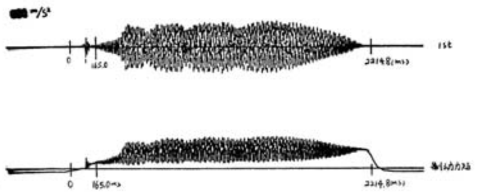


(a) The whole process of brake vibration oscillation
1. preformation 2.emergence 3. continuation 4.depression



(b) Enlarged picture of vibration acceleration of lining and air chamber at first and beginning of second stage of vibration
Figure 16. A whole braking and vibrating process [14]

2) The backward drum rotation operation makes the vibration much more gravely than that of forward rotation operation (see Figure 17). It can be seen that the divergence tendency of backward rotation vibration is much more stronger than that of the forward rotation. Fitting to experimental results the coefficient of divergence ratio are 0.02 and 0.102 for the forward and backward rotation operations respectively. The rate is ~5.



(a) Forward rotation

(b) Backward rotation
Figure 17. Comparison of oscillations of brake vibration of forward and backward rotation [14]

The corresponding calculated by model is 0.069 and 0.47. The rate is ~ 6.5. So there is a good agreement of the analytical with experimental results.

Figure 18 shows the picture of the broken holder of the air chamber due to the violent vibration at the backward rotation test.

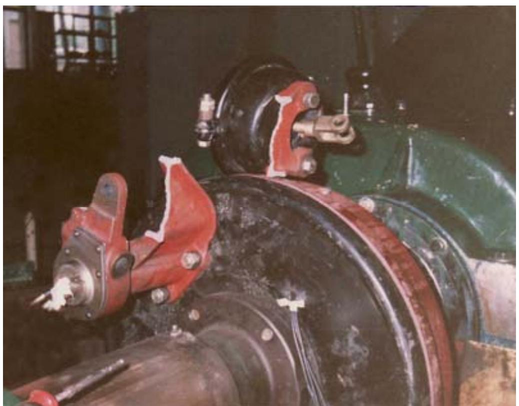


Figure 18. Broken holder of the air chamber [14]

3) It is very interest to point out that the stick-slip phenomenon appeared only at the end of braking process (see Figure 7).

It is concluded that the stick-slip related to neither triggering nor the maximum amplitude (intensity) of the vibration.

4) A series of influence factors on vibration occurrence are simulated. For examples: (Figure 19) shows the simulative curves of the eccentricity of the leading shoe (Ecc1) vs the modal frequencies and damping of the system (only three modes of lower model frequencies are shown in the figure). And Figure 20 shows the simulation results of backward rotation operation. They agree with the experimental results well.

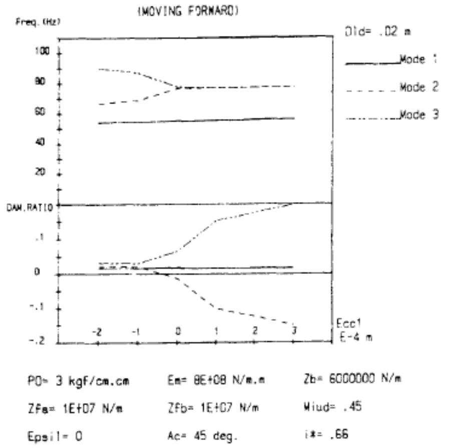


Figure 19. Simulation results of forward rotation

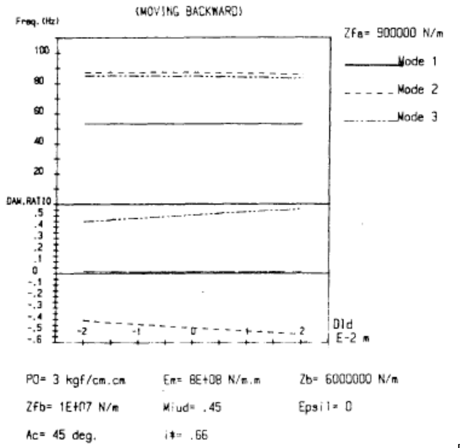


Figure 20. Simulation results of backward rotation[14]

Comparing Figure 18 and 19 with Figure 10 of two degree-freedom model, it can be seen that they have the same feature. When the unstable mode is emerged, the two modes frequencies of the system are approaching with each other and becoming to an unstable mode frequency.

Liles's[33] finite element disc brake squealing model is the earliest relative consummate model. A series of single design parameters which make influence on brake squealing is studied by the model through complex eigenevalue analysis. The parameters studied are coefficient of friction as constant; lining length; lining thickness; lining stiffness; caliper stiffness; rotor stiffness and structural damping.

Recently a series of more consummate finite element model with modal synthesis developed. For example, the similar model as Liles is constructed by [5,6] (see Figure 21) for a car disc brake with squealing problem at 2300Hz. In fact it is also a structural frictional closed-loop coupling model.

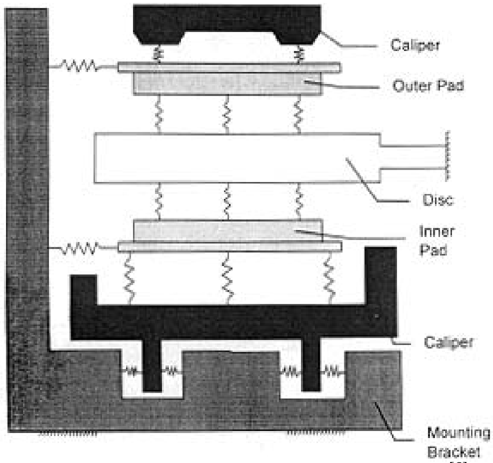
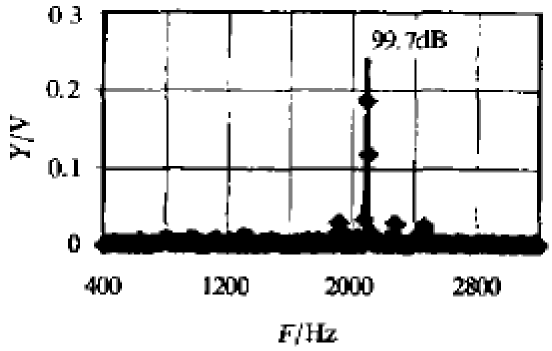


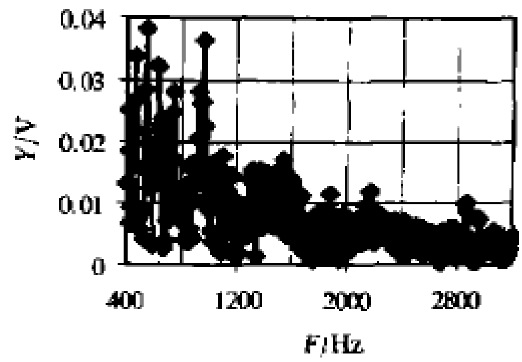
Figure 21. Scheme of the disc brake model [6]

But in this paper the analysis method to suppress brake squeal is the analysis of substructure modal contribution factors to the noise mode referring to the work [2] (1991). The three biggest contribution factors to the noise mode are the mounting bracket modals of frequencies 3106Hz with contribution factor coefficient (0.42); 2481Hz –(0.30) and 2884Hz –(0.23).

The modification design is made to the mounting bracket by increasing the modes nature frequencies. The experimental investigation reveals the positive result of the measure taken[34] (see Figure 22).



(a) Original design



(b) Modification design

Figure 22. Experimental results of the original and modification design of mounting bracket [34]

[35,36] continued Liles work by construction of large scale finite element models under steady sliding conditions with constant coefficient of friction.

[2,3] constructed finite element drum brake squeal model with modal synthesized part of back plate assembly. The

model is one of the earliest representative researches of drum brake squeal model. By using substructural modal contribution analysis method, a specific modal of back plate assembly has been found as the key influence factor on squealing. Stiffening back plate makes the squealing eliminated. This case coincides with the example of Felske's experimental work [1].

A four components finite element drum brake squeal model is constructed by [37]. Based on the model, the method of estimation of critical friction coefficient value for brake squeal analysis is provided to determine the stability boundary. The sensitivity analysis of lining stiffness per unit area on system stability is made. The shortage of the model is treating the back plate as a rigid one, which the both experimental[1] and analytical[2,3] investigations on drum brake squeal show that it is important component of drum brakes, making big influence on squeal occurrence.

The above mentioned models are some of the presentative for the practical brakes including disc squeal and drum of low and high frequency vibration & noise. Based on which some brake vibration & noise problems can be solved by different analyzing methods.

Another kind of way modeling is for transient analysis in time domain. The prominent feature in this model is it can simulate the nonlinear properties, especially the frictional property as multiple factors' functions (sliding speed, temperature lining pressure, humidity etc.). And the disk is modeled as rotating one under follower acting forces[38~41].

To compare the main positive and negative aspects of the two squeal analysis approaches (or modeling), a table is given (see table 1) by [42]

Table 1. The main positive and negative aspects of the two squeal noise analysis approach[42].

Squeal noise analysis approach	Advantages and detections	Disadvantages and inaccuracies
Complex eigenvalue analysis	All unstable eigenvalues are found in one run (low computational time with respect to transient analysis)	Limited accuracy in transient states (the method linearises the non-linearities)
	All unstable vibration modes of the brake system can be predicted (modes with a real part) but in the actual case, not all diverge versus a squeal tendency	Nonstationary features, such as material properties (time-dependent) cannot be included
		The real magnitude of a motion cannot be known
Transient analysis	The finite amplitude of a limit-cycle motion is known	Runs occur to find a limit-cycle moton
	Time-varying properties of the brake system (introduced by moving loads) can be taken account	Time-domain solutions require a lot of memory storage
		Design changes are hard to perform on

Source: (C. Carlo, 2009)

Concrete comparison between linear complex eigenvalue analysis and nonlinear time domain analysis were done by [31,32] based on Tribobrake setup experiments. An unstable mode is obtained at 3.5kHz frequency by both methods under the same parameters conditions, which agree with experimental results. Besides, it is pointed out that the linear complex eigenvalue analysis gave some more unstable modes under different parameters, which are not realistic. Because of its specially designed Tribobrake setup of the experimental investigation and structural geometrical simplification for analytical investigation, the results should have high confidence.

In fact, the brake vibration and squealing only occurs under certain operation speed range called critical speed range. The brake is a parameter varying system (at least the speed parameter varies). It should and also could be investigated in frequency domain by speed perturbation method and treating the rubbing force as the function of speed.

The common point of view of the two kind of modeling is brake vibration & noise is the friction included self-excited vibration.

THE ANALYTICAL METHODOLOGY

Because of immerse complexity of brake dynamics system successive establishment of modeling is only the first step of analysis. The ultimate aim is practical suppress brake vibration and noise occurrence tendency. The analytical methodology investigation is developed.

The complex eigenvalue analysis to identify unstable made is the dominate means. The time domain simulation method uses it even as the associated method. Except the analysis on the schematic model the some earliest literatures using complex eigenvalue analysis on practical models are [14,33]. And due to its simplest algorithm it is favorable to the industry. Moreover, based on the complex eigenvalue analysis the general systematic analyzing method can be developed.

At the earliest stage investigators analyzed the relative single design parameter influencing vibration occurrence by changing parameters directly in the model through repeated eigen analysis without predicted target. Such a method is very time consume and difficult to find the optimization design measure.

THE SUBSTRUCTURAL MODAL CONTRIBUTION ANALYSIS METHOD[2,3] (1991)

The analyzing equation is

$$\{r\} = [\beta]\{q\} \tag{4}$$

Where $\{r\}$ is the normalized modal coordinate vector of the brake model system; $[\beta]$ – characteristics matrix of the system; $\{q\}$ – the substructural modal vector coordinate.

Supposing r_i is the system's unstable mode, which is constructed by $\{q\}$

$$r_i = \sum_j \beta_{ij} \cdot q_j \tag{5}$$

Where β_{ij} represent contribution coefficient of substructure j^{th} modal to r_i.

According to the amplitude of β_{ij}, the influencing lever of substructures modal can be judged. In this way of analysis, the disc brake squeal is suppressed by modifying 11th order model frequency of mounting bracket [5,6]. But it is noticeable that the substructural modal contribution factor analysis is not a completed one because of the coefficient factor β_{ij} is a complex value. The amplitude of the coefficient indicates the relative level of influence vs others. In other words, the modal has its eigenvalue and modal shape. Which factor makes the key role on unstable modal occurrence is not clear. So what kind of modification should be done, modal frequency or modal shape is unknown.

ENERGY FEED-IN ANALYSIS METHOD[43,44]

When the modification of mounting bracket was done for the studied car disc brake, two schemess were made by increasing the 11th order modal frequency of mounting bracket called A and B. But according to computational analysis on model one (scheme B) is available and A is not. So the scheme B design is verified by practice [34].

For a comprehensive grasp of the phenomenon, a feed-in energy analysis method is derived[43,44]. The dimensionless value of feed-in energy between frictional pairs is equivalent to the real part of the unstable model eigenvalue. According to the analysis, the correlated modal shape coefficient influencing factors can be found accurately. It is found that the successive modification scheme B was made by modification of related modal shapes coefficients actually. And feed-in energy value of modification scheme A is even a bit larger than that of the original design. So the feed-in energy analysis method is an effective method which can analyze the modal shape influencing factors accurately, and is a general method of all brake vibration noise occurrence problems.

SENSITIVITY ANALYSIS METHOD OF SUBSTRUCTURAL MODAL PARAMETERS TO BRAKE SQUEAL [45]

Nevertheless, it is still not the overall general analyzing method to extract all the influencing factors and their influencing sensitivity to brake vibration and noise occurrence tendency. The sensitivity analysis of substructural modal parameters to brake squeal occurrence tendency method is promoted by [45]. This analyzing method should be the overall analyzing methodology which can give the sensitivity of brake vibration and noise occurrence tendency to all the substructural modal eigenvalues and eigenvectors influences.

In the results of analysis by this way, the optimization modification design target function can be formed in detail. Then the optimization design of concrete components concern can be done by finite element model of components with target function directly. In this way, two steps of optimization design of brake system to suppress vibration and noise occurrence is formed. This method is verified by a car disc analysis and corresponding practice presented in [34].

What are the relative single design parameter influencing brake vibration and noise occurrence, such as frictional coefficient, modulus of lining material concerned and etc, they have been studied by many investigators directly because they are apparent in the model independently. The sensitivity analysis usually made by small design parameters perturbations with graphical displayed form [33,46~49].

Above presented analytical methodology is confined in the scope of linear models. But it could be quite useful for the industry to solve practical problems.

It is a worthy reference comment adapted from [42]: “The only analytical approach at the moment for an industrial application is complex eigenvalue extraction in a full FEM braking system model. It seems to be (although having theoretical limitations) for practical issues, the only possibility for a robust design investigation. Brake squeal solutions in fact, can be better evaluated through a robust design approach.”

THE THERMOELASTIC INSTABILITY OR HOT POT THEORY OF BRAKE VIBRATION & NOISE OCCURRENCE

Based on finite element disc brake model[50,51] the critical speeds correlating to different order hot spots distributions as unstable rotation speeds can be found. The disk brake should have different frequencies at corresponding critical speeds. Besides, the analyzed critical speeds are quite high. It seems is not correlated to the phenomenon happened with practical brakes. Brake vibration & noise of a certain brake has a specific peak at certain frequency and at relatively low speed range usually. And the analysis made only on the disc geometry (thickness) influencing factor vs critical speeds.

A very recent review about Thermo-Elastic Dynamic Instability (TEDI) is made by [52].

What is concerned in brake vibration and noise occurrence, it is pointed out that the most important consequence of thermo-elastic instability (TEDI) is the formation of hot spots at the sliding interface, which can cause in turn material damage and wear and also a source of undesirable frictional vibration, known in automotive disc brake as "hot roughness" or "hot judder”[9,53~56].

But it is arguable whether the "hot spot" is the cause of brake vibration and noise or oppositely just is the result formed in the vibration process. According to author's experience, it should be the latter one. When the modal experimental analysis is done on the tire, the modal shape can be measured by thermal imaging equipment easily, like "hot spots". In the case of modal experimental analysis, the input energy should be much less than that when friction force is rubbing on disc plate. So it is reasonable to treat the hot spot as the result of disc vibration.

Until now, very few or almost none information were published about the successful explanation about the brake vibration and noise occurrence and the measure to solve the problems directly. [52] suggested that although the coupling of dynamic and thermal terms is generally weak, thermal are capable of making otherwise neutrally stable dynamic modes unstable. So the new form of instability TEDI is to be studied.

It is suggested that the thermo-elastic theory analysis is still not developed completely enough for brake vibration and squeal. Maybe it should be analyzed with establishment of model as a whole brake system.

DAMPING ADDED TO SUPPRESS BRAKE VIBRATION & NOISE OCCURRENCE

The viscoelastic damping and different damping materials are used to control brake vibration and noise [57~60]. The typical structure is that there are ARS (Adhesive Rubber-steel) sticked to the back plate of the pad. But in practice, it can’t control vibration and noise occurrence completely. The experimental investigation points out that " Two different roles of the modal damping are described: a large modal damping can reduce the response of the damped mode and consequently prevent its participation in squeal coupling; however, a high damped mode has more probability to couple with others modes that fall close to its natural frequencies because its tune-in range is larger. This suggests careful use of the addition of damping in order to reduce squeal."[31,32]

The modeling analysis[61] also shows that where and how high the damping should be added to suppress vibration occurrence is to be studied. Otherwise, while one kind (mode) of vibration is suppressed, a new one might be occurred.

THE ACTIVE CONTROL MEASURES STUDIED

1) The electronic control canceling system for a disc brake and noise control [62].

2) Active control of automotive disc brake rotor squeal using dither [63].

CONCLUSIONS

1) Friction induced brake vibration and noise is an elusive problem. Its occurrence has strong random characteristics because the frictional properties are functions of many factors, such as temperature, humidity and operation conditions (braking speed, pad pressure and etc) which are difficult to indentify properly.

2) Recently big progress has been reached by experimental and theory analytical methodology based on linearized model using complex eigenvalue analysis. Based on such method, the sensitivity analysis to unstable mode on all parameters involved in the model can be studied. The optimization modification target function can be formed for following stape optimization design of concrete components to eliminate brake vibration and noise occurrence.

3) Presented methodology requires full brake modeling, i.e. the model should involve all the factors that can be treated as a certain one mentioned in [42] influencing on brake vibration and noise occurrence. Otherwise, it can not be found as the influence factor. So for further advance of the investigation, the study on developing full brake model in cooperation with experimental investigation is very important.

4) Both analytical approaches, i.e. linear complex eigenvalue analysis and transient nonlinear time domain analysis, are necessary. To advance future investigation of brake vibration and noise problems, for the linear complex eigenvalue analysis, the over-predicted unstable modes should be studied. It is important for virtual development of new quiet brake product accurately.

5) Until now, according to publication, the most active method to eliminate the brake vibration and noise occurrence by experimental and analytical investigation is modification design of correlated components of brakes.

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