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Linear string-soundboard coupling in pianos

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ABSTRACT

The coupling between strings and soundboard at the bridge is essential in determining the quality of piano tones. The objective of the work is to investigate the coupling properties through time domain modeling of a string triplet coupled to the input admittance. For this purpose, various input admittances were measured on a strung upright piano soundboard and modelled as a parallel set of oscillators. These oscillators are coupled to a linear finite difference modeling of slightly detuned strings. The model accounts for the reaction of the bridge on string waveforms, and thus is different from usual methods were the bridge velocity is obtained through convolution between string forces and admittance impulse response. A comparison between both methods is presented at the end of the paper. Each oscillator of the admittance is defined by three parameters: amplitude, frequency and damping factor from which, equivalently, mass, stiffness and mechanical resistance are derived. Modifying the amplitudes allows studying the effects of variations of the global mobility pattern, without affecting individual frequencies and damping. Simultaneous variations of mass and stiffness can be related to variations of thickness of the soundboard. Variations of selected frequencies simulate selected variations of modes, as in the addition of stiffeners. Finally, variations of frequency-dependent losses in the soundboard material can be simulated through modifications of the set of damping factors. Simulated string and bridge waveforms yield a better understanding on the effects of both string tuning and bridge mobility on the amplitude, duration and decay pattern of the tone. As it is well-known among piano makers and tuners, the bridge show large variations of input admittance along its compass. As a consequence, tuners try to compensate these variations by acting on string tension, in order to avoid to much irregularities in sound power and duration in the piano register.

INTRODUCTION AND MOTIVATIONS

The work presented in this paper is an intermediate step of a complete modeling of the piano, which is currently in progress. The leading idea is to isolate the subsystem, composed of a string triplet with realistic boundary conditions at bridge and agrafe, from the rest of the instrument. The goal is to investigate, through simulations, the bridge-strings coupling and its influence on piano tones.

In the present study, we restrict ourselves to the linear behavior of the string. The modeling of the nonlinear piano string, due to large displacement during the attack transient, is presented in a companion paper [1]. We also neglect the motion of the string at the agrafe: measurements performed on an upright piano show an amplitude level of -20 to -30 dB at the agrafe, compared to the bridge, and thus this simplification is justified. Finally, we ignore the transverse lateral motion of the string. As a consequence, the term "bridge admittance" in what follows refers to the ratio

$$Y(\omega) = \frac{V(x_B,\omega)}{F(L,\omega)} \quad (1)$$

where $V(x_B,\omega)$ is the vertical transverse bridge velocity at bridge point x_B corresponding to the attachment point of the main string lentgh, and where $F(L,\omega)$ is the vertical component of the force at the string end L due to its transverse vertical motion. Within the context of linear approximation of the string motion, this force is equal to:

$$f(L,t) = -T\frac{\partial w}{\partial x}(L,t), \quad (2)$$

where T is the string tension at rest and $w(x,t)$ is the transverse vertical component of the string's displacement (see Figure 1). For a triplet of strings, the total force acting of the bridge is the sum:

$$f_{tot}(L,t) = \sum_{s=1}^{3} T_s \frac{\partial w_s}{\partial x}(L,t). \quad (3)$$

Remark: in this paper, time-dependent variables (such as $f(L,t)$) are written in ordinary letters whereas frequency-dependent quantities (such as $F(L,\omega)$) are written in capital letters. The admittance Y is usually defined in the frequency domain. Its inverse Fourier transform is the *impulse response* denoted $h(t)$ below.

In a 1-D model of string-bridge coupling, it is customary to assume $x_B = L$ which means that the coupling is reduced to one single point of fixation. The real situation is rather different since the string passes across the bridge (see Figure 2). The actual segment ℓ_B of the string in contact with the bridge is determined by two needles (or pins). One of these pins (the left one p_1 on Figure 2) fixes the normal length L of the main part of the string. It forms an angle $\alpha < \pi/2$ with the horizontal plane so that the string is supposed to be blocked. The second

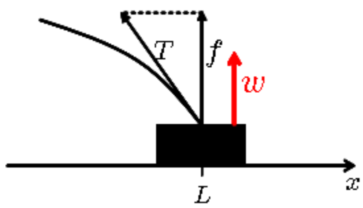


Figure 1: Piano subsystem: a linear 1D model of transverse string coupled to a punctual bridge admittance at position $x = L$. T is the string tension, $w(L,t)$ is the bridge displacement and f is the transverse string force exciting the bridge.

pin p_2 is bent in the opposite direction to improve the blocking. It fixes the length ℓ_D of the "dead ends" (duplex scale) of the string, the other end being finally attached on the tuning pin. In some pianos, these "dead ends" are carefully tuned (for example to a partial of the main string) whereas they are damped with felt in others [2]. Because of this geometry, it is plain that the real boundary conditions of a piano string differ substantially from the crude 1-D model. Some assumptions can be

Figure 2: Real boundary conditions. In a real piano, the bridge-string contact is not punctual but is distributed over a length ℓ_B between two pins.

made for a more realistic physical description of the end conditions that would need further studies and more experimental evidence:

- Pins p_1 and p_2 ensure a zero vertical and lateral displacement of the string, but do not seem to be efficient in ensuring the nullity of the longitudinal displacement. As a consequence, the string length to consider for calculating the longitudinal frequencies should be $L+\ell_B+\ell_D$ rather than L, as it is usually done. This property is of almost no consequences in the context of linear modeling of the string, but becomes important with a nonlinear string, where transverse and longitudinal motions are coupled [1].
- One cannot totally exclude that some dissipation due to sliding friction between string and pins exist, since the pins do not prevent from longitudinal string motion.
- The balance of forces at each pin should be reconsidered taking into account the reaction of the pin, which is not perpendicular to the bridge.

The theory of strings-bridge coupling has been developed by Weinreich [3]. This author showed, among other things, the influence of string detuning and bridge admittance on the temporal envelope of piano tones. In this work, arbitrary values were given to both the real and imaginary parts of the admittance. In more recent studies, the admittance is modeled as a small number of mechanical oscillators [2]. Physically speaking, the real part of the admittance governs the transfer of energy from strings to the soundboard and the decay times of the strings. The imaginary part primarily affects the detuning of the strings. Both parts thus influence the temporal envelope.
Because of various sources of nonlinearities, which will be progressively introduced in future versions (hammer-string contact, variation of string length and tension in time), our model of piano tones was developed in the time domain. In pianos, the spectrum of the notes is wide with an increasing number of salient spectral components from treble to the bass range. Each component of the strings vibrations interacts with those soundboard modes which are close in frequency. Thus, in order to get an idea of the large simultaneous number of coupling between strings and soundboard, it is of interest to develop a wideband model of admittance at the bridge. In summary, we wish here to extend the above mentioned previous models to a more general situation where all string modes interact with an admittance that compares well with measurements, at least in a significant frequency range. In practice, as shown in the next section, we are dealing with an admittance model composed of a set of about 100 mechanical oscillators in parallel, the result agreeing well with measurements in the range 0 to 4 kHz. Each oscillator is defined by a set of three mechanical parameters (mass, stiffness, resistance) which can be adjusted independently from the others, thus allowing a large variety of coupling situations with direct link to both the physics and making of the instrument.
The second objective of the present paper is to test whether it is justified or not to model the string-soundboard interaction by a simple filtering of the string force, as it is currently done in several studies. For that purpose, the bridge velocity resulting from a finite difference modeling of the string coupled to a physically modeled admittance is compared to the bridge velocity obtained from a direct convolution between a calculated force at the bridge at its impulse response. In other words, we compare a bidirectional model (where the motion of the bridge influences back the motion of the string) with an unidirectional model which assumes a given string force as input of an "admittance filter" whose output is the bridge velocity.

MEASUREMENTS AND MODELING

Measurements

Measurements were performed on a strung soundboard of a PLEYEL upright piano hold vertically (see Figure 3). The keys, hammers and dampers are removed.

Figure 3: Strung PLEYEL upright piano used for the experiments. The strings are damped with felt during measurements of the bridge velocity.

Measurements on strings include measurements of length (from agrafe to bridge) and diameter. The density is derived from the mass of an element of string measured on a weighing machine of high precision. The Young's modulus is derived from measurements of string's inharmonicity on the first 20 partials. Each successive string is set into vibrations by means of various mallets, all other strings being damped with felt. An accelerometer on the mallet's head allows to derive the impact force. The period of the string is estimated from measurements of the string velocity by means of a laser vibrometer. This yields an estimation for the string tension. An estimation of the string's internal damping was obtained in a previous study where piano strings were stretched between two rigid supports [4]. For each impact, the impact position is measured.
In order to check whether the motion of the string can be considered as transverse, perpendicular to the bridge, at least during the initial transient motion, string waveforms are reconstructed from the measured impact force (see Figure 4). The reconstructed string velocity is obtained by deriving the initial velocity pulses from the mallet's force, and by adding (or substracting) the successive pulses taking the reflections at bridge and agrafe and the propagation times into account, in the theoretical context of a single vertical polarisation of the string motion. In this simple reconstruction, attenuation at the bridge and dispersion along the string are ignored. The comparison between measured and reconstructed waveforms yields a visual justification for a transverse vertical motion of the string. The procedure of admittance measurements was presented in

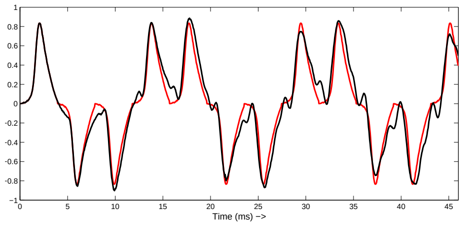


Figure 4: Reconstruction of string waveforms. String C2. The black curve represents the string velocity excited at $x = L/4$ with a mallet and measured with a laser vibrometer at position $x = L/2$. The red curve is reconstructed from measurements of the mallet's force pulse and addition of the reflected pulses at bridge and agrafe, with the assumption of perfect reflexion.

detail in a previous paper [5] and will be only briefly reviewed here. In all measurements, the strings are damped with felt. The bridge velocity is measured with a laser vibrometer. Measurements of admittance in the low-frequency range (below 750 Hz) are obtained through bridge excitation with an impulse hammer. The impulse response is then obtained through inverse Fourier transform of the transfer function between velocity and force. The effective frequency range is limited by the width of the force impulse. This impulse can be reduced by using a smaller (miniature) impact hammer though, in this case, the quality of admittance determination is mainly limited by the signal-to-noise ratio, a consequence of the little energy contained in the force impulse. From 0.75 to 4 kHz, the best results were obtained when the bridge is excited by a shaker LDS V406 driven by random noise. Because the soundboard is rigid and heavy, a powerful shaker is necessary in order to set the bridge with sufficient velocity amplitude. on an upright piano, the admittance is of the order or magnitude of 10^{-3} s/kg (or even less at some locations where ribs are fixed to the soundboard). The magnitude of the force delivered by the shaker is in the range 50 to 100 N. An initial calibration procedure is conducted in order to obtain the velocity and force gains G_V (in m.s^{-1}/V) and G_F (in N/V) of each measurement channel. Here again, the impulse response is the inverse Fourier transform of the transfer function between velocity and force. The force is measured by an impedance head fixed on the shaker. The shaker is decoupled from the soundboard by means of a rigid rod. The quality of the transfer function determination is assessed by estimating the coherence between force and velocity signals. This allows to detect nonlinearities and/or insufficient signal-to-noise ratio.

Model

Admittance

In future versions of the piano model, the load of the strings at the bridge will be represented as a 2-D linear vibrating system composed of a prestressed soundboard with bridges and ribs. In this context, and assuming that the damping is small enough, the velocity at a given point (and thus the admittance, for a unitary driving force) can be represented by the sum of modal contributions, where the magnitude V_i of each contribution depends on both excitation and measurement locations, of the form [6]:

$$V(\omega) = \sum_{i=1}^{N} \frac{V_i}{\omega^2 - \omega_i^2 + 2j\zeta_i\omega\omega_i}. \qquad (4)$$

where ω_i and ζ_i are the modal frequencies and damping factors, respectively. In the time-domain, the corresponding impulse response is of the form:

$$h(t) = \sum_{i=1}^{N} A_i e^{-\alpha_i t} \sin(2\pi f_i t + \phi_i) \ \text{ with } \ \tan\phi_i = \frac{\alpha_i}{4\pi f_i}. \qquad (5)$$

Therefore, the subsystem of the piano composed of a string coupled at the bridge can be represented by the string with a boundary impedance (or admittance) with impulse response $h(t)$ similar to (5). It is well-known that the impulse response of any SDOF mechanical oscillator made of the association of a mass, a stiffness and a resistance, is a damped sinusoid. Thus, $h(t)$ can be viewed as the sum of impulse responses of N oscillators, each oscillator being defined by three mechanical parameter. Assuming the condition $\alpha_i \ll 2\pi f_i$, we have:

$$M_i = 1/A_i \ ; \ K_i = \frac{4\pi^2 f_i^2}{A_i} \ ; \ R_i = \frac{2\alpha_i}{A_i}, \qquad (6)$$

where M_i is the mass (in kg), K_i the stiffness (in N/m) and R_i the resistance (in Ns/m).
Several strategies are possible for determining the number N of appropriate oscillators and the $3N$ of associated mechanical parameters [5]. One general class of methods consists in adjusting a FIR filter (in time or frequency domain) close to the measured admittance. The parameters are then derived from the complex poles and magnitude of the pole decomposition of the admittance. These methods give fair results, but the main difficulty, in the context of plate vibrations, is related to the difficulty of distributing the poles adequately over the frequency range under examination, in view of the modal density of the physical system. In addition, the order of the filter increases rather dramatically if the admittance contains lightly damped modes with a sharp frequency peak.
Here, we use alternatively the ESPRIT method [7] which is particularly suitable for the analysis and modeling of free vibrations and transients, and whose main advantage is that only a small portion of the signal is necessary. This method is very efficient in modeling rapidly decaying sinusoids corresponding to highly damped modes, as those encountered in the vibrations of wooden plates like soundboard. With this method the signal is directly modeled as a sum of N sinusoids with $4N$ parameters corresponding to amplitude, frequency, damping factor and phase for each term.
The main difference with $h(t)$ in (5) is that, in the latter case, the phase is imposed, which leads to $3N$ independent parameters. In fact, input admittance at the bridge are determined with force and velocity measured at the same point (in practice, very close points) so that the phases should be near zero. Based on these physical considerations, we thus nullify the phases after application of the ESPRIT method to the impulse response, which is equivalent to consider that the measurement noise primarily affects the phase coefficients, the amplitudes, frequencies and damping factors being more robust. Finally, the number N of sinusoids can be estimated by various methods: preliminary Fourier transform, estimation of the modal density from the typical geometrical and material parameters of the soundboard [5], or signal processing techniques through the ESTER criterion [8].

String model. Finite differences method

The transverse vertical motion $w(x,t)$ of a linear string of length L is modeled by the standard string wave equation, with initial tension T and linear mass density μ. Standard refinements such as lineic resistance r and stiffness EI can be added without any difficulty. The action of the exciter (mallet or hammer) is represented by a force density term $f(x,t)$ in the wave equation [9]. Depending on the purpose, the force can be either derived from measurements or mathematically imposed. In either cases, the force impulse is distributed over a small portion of the string and its duration is comparable to those measured on usual hammers (typically 1 ms). A condition of zero displacement and bending moment is assumed at the agrafe. At the bridge, each oscillator is modeled by a second-order differential equation driven by the string force. The string motion at the bridge is equal to the sum of the displacement contributions w_{Bi} of the N oscillators. Because of this moving boundary condition, the motion of the string is influenced back by the motion of the bridge. The observed quantities of interest are essentially the string force at the bridge and the bridge velocity. In summary, the continuous model of string coupled to the soundboard is the following:

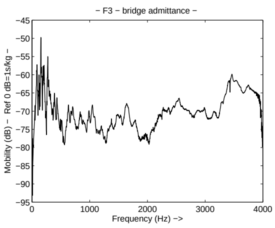
$$\begin{cases} \mu w_{tt} = T w_{xx} + f(x,t) + r w_t + EI w_{xxxx}, \\ w(0,t) = 0 \ ; \ w_{xx}(0,t) = 0, \\ \text{for i=0 to i=N :} \\ M_i \dfrac{d^2 w_{Bi}}{dt^2} + K_i w_{Bi} + R_i \dfrac{d w_{Bi}}{dt} = -T \dfrac{\partial w}{\partial x}(L,t), \\ w(L,t) = \displaystyle\sum_{i=0}^{N} w_{Bi}(t). \end{cases} \tag{7}$$

where w_{tt} (resp. w_{xx}) means second-order derivative vs time (resp. space). The set of equations (7) is solved with a second-order explicit finite difference scheme [10]. For such a scheme, the stability is guaranteed is the following so-called CFL condition is fulfilled:

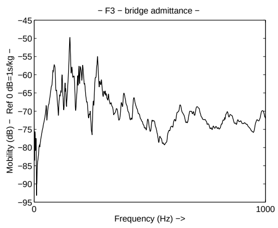
$$\sqrt{\frac{T}{\mu}} \frac{N_s}{f_s L} < 1 \tag{8}$$

where N_s is the number N_s of discrete elements on the string, and f_s the sampling frequency in time. In order to limit as much as possible the risk of numerical dispersion (frequency warping), N_s is selected in order so that the CFL condition (8) is as close as possible to unity. Notice that the CFL condition becomes more severe if the string equation contains a stiffness term [10].

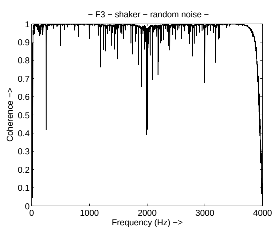
RESULTS AND DISCUSSION



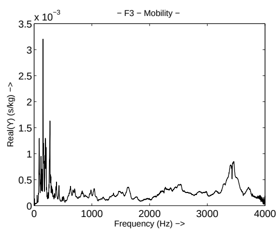
(a) Admittance (0-4 kHz).



(b) Admittance (0-1 kHz).



(c) Coherence.

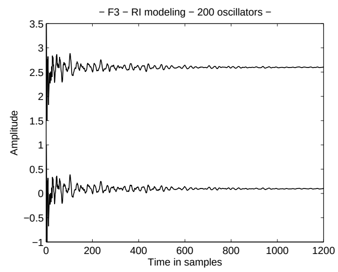


(d) Admittance - Real part.

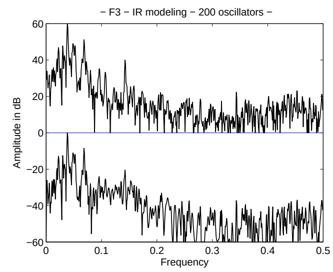
Figure 5: Bridge admittance measurements - string F3 -

Figure 5-(a) shows the measured bridge admittance at the end of string F3, whose measured fundamental frequency is $f_1 = 169$ Hz. The string length is $L = 0.95$ m and the diameter is Φ=1.083 mm. Assuming a density ρ = 7850 kg/m^3 (steel), this yields an estimated tension T=746 N and a linear mass density μ=7.23 g/m. The admittance is measured with random noise excitation. The coherence displayed in Figure 5-(c) assesses that the measurements are valid in the range 70 Hz-4 kHz. The negative slope near 4 kHz in both figures is due to low-pass filtering of the signal. This admittance is typical of those measured on an upright piano [5]. It exhibits sharp peaks due to well-separated soundboard modes below 500 Hz. The admittance decreases slightly between 500 and 2000 Hz, followed by a slight increase between 2 and 4 kHz. Figure 5-(b) is an expanded view of the (a)-plot between 0 and 1 kHz. The order of magnitude of the admittance (around -70 dB re. 1s/kg, or, equivalently nearly 4 10^{-4} s/kg) is consistent with previous measurements [11]. It has been shown in a previous study that it is also coherent with typical geometrical and material data of piano soundboard [5]. Figure 5-(d) shows the real part of the admittance, which is positive between 0 and 4 kHz. The positivity of this term is a supplementary confirmation of the validity of the measurements, since it accounts for the transfer of energy from strings to soundboard and air.

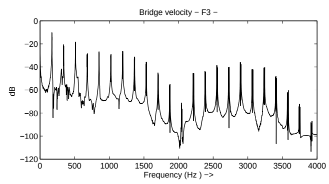
Figure 6-(a) shows a comparison between the measured impulse response (top) at the end of string F3, which corresponds to the inverse Fourier transform of the admittance shown in Figure 5, and its modeling as a sum of 200 impulse responses of decoupled SDOF damped oscillators (bottom) as in Equation (5). The corresponding spectra are shown in Figure 6-(b).



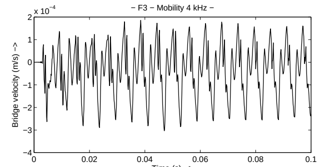
(a) Impulse Response ; measured (top) and model with 200 oscillators (bottom).



(b) Admittance ; measured (top) and model with 200 oscillators (bottom).



(c) Simulated bridge velocity ; spectrum.

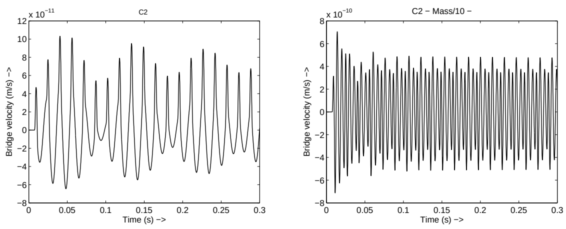


(d) Simulated string velocity ; time-domain.

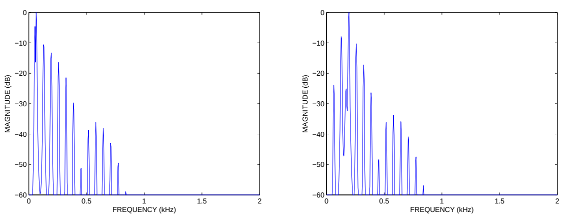
Figure 6: Impulse response modeling and synthesis through convolution - string F3.

From this model, the mechanical parameters of the bridge admittance are derived, following the procedure described in the previous Section. The string motion is simulated with explicit finite differences, the number of spatial steps being equal to N_s=141, with a sampling rate f_s=48 kHz. This yields a CFL number equal to 0.993, and thus the condition of stability (8) is fulfilled. Figure 6-(d) shows the bridge velocity simulated through convolution between the total force acting on the bridge due to three slightly detuned strings, following Equation (3), and the impulse response. The corresponding spectrum is shown in Figure 6-(c). The spectral envelope is coherent with the fact that the excitation impulse is located at x_0=10 cm from the agrafe, which corresponds roughly to L/10. As a consequence, the string partials which are integer multiples of 10 are only very weakly excited. A zoom on each peak would show, in addition, the presence of three close sharp peaks, each of them corresponding to one string of the triplet. The coupling with the soundboard is only visible below 500 Hz. However, modifications of the admittance above this frequency, which cannot be easily displayed, are clearly audible.

In Figure 7, the bridge velocity (for string C2) is directly



(a) Reference. (b) Reduced soundboard mass.

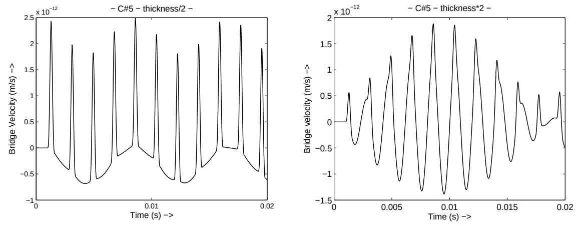


(c) Reference spectrum. (d) Reduced mass - spectrum.

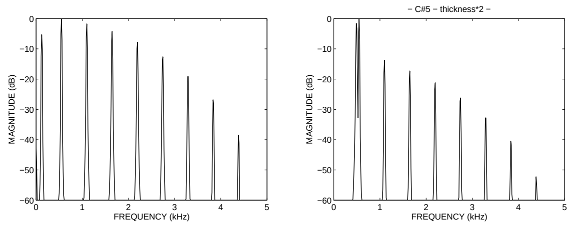
Figure 7: Bridge velocity String C2 - Finite differences simulations and mass variations.

computed with finite differences, with the bridge admittance modeled as a set of oscillators. Unlike in the previous example, there is no convolution between simulated string force and measured (or modeled) impulse response at the bridge. Figures 7-(a) is the reference velocity signal, and 7-(b) shows the change in waveform consecutive to a division of all mass coefficients of the oscillators by a factor 10. This would correspond to a use of a much lighter soundboard. This intentionally exaggerated factor has been selected in order to emphasize the effects on the figures, although smaller factors also induce clear audible effects. The corresponding spectra are shown in Figure 7-(c) and Figure 7-(d). As expected, the decrease in mass soundboard increases the impedance matching of string and bridge which induces a stronger coupling between both elements. This can be viewed, for example, in Figure 7-(d) where the coupling between string and soundboard modes are visible below 500 Hz. These simulations are made for a single string C2, with fundamental frequency $f_1 = 64$ Hz, tension T=844 N, length L=1.07 m and linear string mass density μ = 45.2 g. The excitation position is at x_0= 15 cm, roughly equal to $L/6$. With a sampling frequency f_s=10 kHz and a number N_s=78 spatial points, we obtain a CFL coefficient of 0.9959. The bridge, here, is the "small" bridge of the soundboard, where only the lowest strings are attached. The modeling of the bridge admittance is restricted here to the low frequencies (below 750 Hz) and the impulse response is fairly reproduced with a set of 56 oscillators.

The results shown in Figure 8 are aimed at illustrating the ef-



(a) Thickness divided by 2. (b) Thickness multiplied by 2.

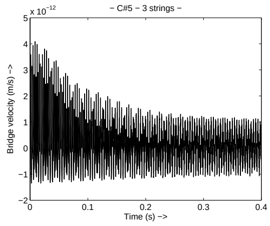


(c) Thickness/2 - spectrum. (d) Thickness × 2 - spectrum.

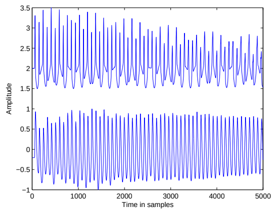
Figure 8: Bridge velocity - String C#5 - Variations of soundboard thickness.

fects of change in soundboard thickness on string-bridge coupling and bridge velocity. These effects are simulated for the string C#5 of length L=32.8 cm, with fundamental frequency f_1=535 Hz. The diameter of the string is Φ=0.922 mm, with a linear mass density μ = 5.24 g. The estimated tension is T =649 N. A force pulse excites the string at position x_0=3.28 cm (L/10). With sampling frequency f_s=48 kHz, and N_s=44 spatial points, the CFL coefficient is 0.9835. The low-frequency part of the bridge admittance (below 750 Hz) is modeled with 58 oscillators. The reference bridge velocity is shown in Figure 9-(d). For a given surface, the mass of the soundboard varies as the thickness h whereas the stiffness varies roughly as h^3. Therefore, in order to compare the reference bridge velocity with the one resulting from the coupling of the string with a soundboard of thickness divided by 2, the mass coefficients of the oscillators are divided by 2, while the stiffness coefficients are divided by 8. This results in the waveform shown in Figure 8-(a) with corresponding spectrum in Figure 8-(c). A similar process for thickness doubling results in the waveform shown in Figure 8-(b) with spectrum shown in Figure 8-(d). It can be seen that the change in thickness modifies both the spectral and temporal envelope of the sound.

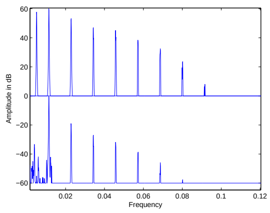
Figure 9-(a) shows an example of simulation of bridge velocity excited by three slightly detuned strings for the note C#5. The length is identical for all three strings, and the tensions (estimated from measurements) are 649, 647.5 and 652 N, respectively. A double decay is clearly identified on the temporal envelope. Finally, Figure 9-(b) shows a comparison between the results obtained for computing the bridge velocity, using two different methods. The waveform on top is calculated with a finite difference model of the string loaded by the system of diiferentail equations that represents the admittance, as in Equation 7. The waveform at the bottom is obtained by convoluting the string force (obtained with finite differences) with the measured impulse response. This comparison show some obvious differences. The corresponding spectra in Figure 9-(c) show that the periodicity of both signals are identical, as the spectral envelope above the second partial. The main differences are located below the fundamental of the string, in



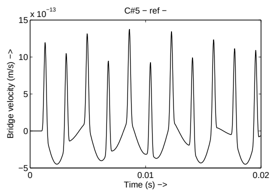
(a) Three detuned strings.



(b) Top: FD model ; bottom: convolution.



(c) Top: FD model ; bottom: convolution.



(d) Reference.

Figure 9: Simulation of bridge velocity - String C#5.

the frequency domain where the soundboard modes are weakly damped. In this domain, the underlying approximations made with the use of convolution are probably not justified, due to amplitude of bridge motion relative to the string. More numerical experiments are needed here for supporting this claim with more evidence.

CONCLUSION AND FUTURE WORK

In this paper, a model of strings-soundboard coupling has been presented and discussed. This model is based on a linear model of transverse string displacement coupled at one end to a mechanical representation of the bridge admittance. The admittance model is derived from measurements on an strung upright piano.

Admittance measurements at the piano bridge are difficult, mainly because of the inertia of the soundboard which, in turn, induces low values of the bridge velocity. For that reason, measurements are usually limited to the range 0-4 kHz. A standard modal analysis approach would yield an accurate modeling of the admittance up to a maximum of 1.0 to 1.5 kHz, but would fail for higher frequencies, due to the internal losses in the material. This frequency range is not appropriate in the context of the piano, where the fundamental frequencies are within the interval 27-4200 Hz, and in view of the particular sensitivity of the human ear in the range 300 Hz-3 kHz, among other things. Therefore, another approach was preferred here, where the the inverse Fourier transform of the measured admittance (the impulse response IR at the bridge) is directly represented as a sum of exponentially decaying sinusoids, using the ESPRIT method [7] [8]. The quality of this representation is assessed by a comparison between measured and simulated IR, with constraints on the phase inspired by the physical situation where force and velocity are measured at the same point on the bridge. This comparison is based on a least-square estimation: a least-square error of -20 dB (or less) is considered as acceptable.

The major aim of the modeling is to investigate the effect of structural changes in the soundboard on the bridge velocity. For that purpose, a simple finite differences model of the string is constructed where the end condition at the bridge is modeled as a set of N second-order differential equations (DE). Each DE represents a SDOF damped oscillator whose IR is a damped sinusoid, the sum of the N IR being equal to the modeled admittance IR. In this paper, a few examples of the effects of variations of mass and stiffness parameters of these oscillators were presented. These variations are related to specific geometrical and/or material modifications of the soundboard. Here, the variations of parameters are intentionally exaggerated for visual purposes. Examples of synthesis with more realistic conditions will be played during the oral presentation.

In real pianos, one can observe large variations of bridge admittance from bass to treble. This results in significant differences in decay time from one note to the next: such fluctuations are generally judged as undesirable by players. In the near future we wish to use our model of three strings coupled to various admittances, in order to account quantitatively for the procedure used by piano tuners for compensating, at least partially, these unwanted fluctuations. As shown theoretically by Weinreich, the temporal envelope of the sound depend, in fact on both bridge admittance and string detuning.

Preliminary experiments were also made in order to compare the bridge velocity obtained through convolution between string force and bridge IR (see, for example, [12]), with the one resulting from full “bidirectional” finite differences model. It turns out that some differences exist, if the magnitude of the bridge displacement becomes comparable to the one of the string. This might be the case for particular weakly damped low-frequency modes of the soundboard, and, in general, for light soundboards.

Finally, the present “subsystem” model is intended to serve as reference case of a more complete modeling of the piano that couples togther a nonlinear model of strings with a 2D model of bridge and soundboard.

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