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An adaptive Approach to enhance Damping in a two dimensional Feed Forward Active Noise Control System using Velocity Microphones

Christian Kleinhenrich, Tobias Weigler and Detlef Krahé

University of Wuppertal, Wuppertal, Germany

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ABSTRACT

A well known method to build a feed forward active noise control (ANC) system for the damping in a certain area is based on the Kirchhoff-Helmholtz integral. The setup uses a combination of pressure and velocity microphones to measure the primary field (noise) and to reproduce a phase turned secondary field resulting in the attenuation of the noise. This paper presents a method to find a control input for an adaptive algorithm to enhance damping effects. This is achieved by the decomposition of the incident and the reflected waves on the system's borderline.

INTRODUCTION

The prototype of a two-dimensional active noise control (ANC) system at the laboratory for acoustics of the University of Wuppertal consists of an array of 24 microphones and 12 loudspeakers. It can attenuate a given sound field (primary field) inside a certain area by the superposition of a secondary sound field, which is a reproduction of the primary field with inverted phase. In the coaxial setup the microphones build the outer boundary and function as sensors for noise coming from the outside of the system. The speakers form the inner circle and synthesize the anti-noise. For signal processing purposes a Texas Instruments TMS320C6455+ digital signal processor (DSP) on a DSK-board was engaged. A special add-on card for the board was developed to provide conversion between the analogue and the digital domain for all input and output channels.

The performance of the system strongly depends on several parameters like exact geometry of the setup or the knowledge of the current speed of sound. Reflections from the ground or from walls can cause problems as well. Therefore, adaptive methods shall be integrated to enhance performance capabilities. A conventional approach would be to place error sensors inside the system to measure the deviance from an optimal damping. This solution would be inconvenient and not very practical. For that reason an adaptive approach is needed which engages the reference microphones on the outer boundary also as error sensors.

In [1] the mathematical background of the ANC system is derived using the Kirchhoff-Helmholtz-Integral. This integral is a variation of Kirchhoff's law which will be the basis of all calculations performed within the remainder of this paper. First, the mathematical formulation to obtain the speaker signals in order to attenuate noise inside the speaker array from the microphone pairs is presented. Second, Kirchhoff's law is used to affiliate the separation of incident and reflected waves to obtain an error signal on the borderline of the system.

BASIC THEORY

Achieving the speaker signals

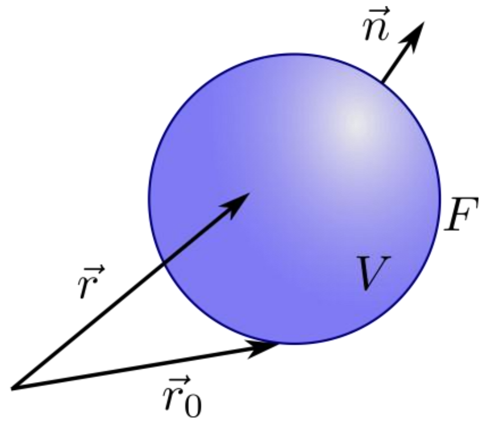

Figure 1: Solution area

Kirchhoff's law allows the prediction of sound pressure inside, or on the hull of a closed volume V when the pressure and its gradient on the hull $F(V)$ as well as all additional sources Ψ_i inside the volume are known (figure 1):

$$\oiint_{F(V)}\big(G(\boldsymbol{r}_0,\boldsymbol{r},\omega)\nabla P(\boldsymbol{r}_0,\omega) - \nabla G(\boldsymbol{r}_0,\boldsymbol{r},\omega)P(\boldsymbol{r}_0,\omega)\big)\boldsymbol{n}\,df + \iiint_V \Psi_i(\boldsymbol{r}_i,\omega)\,G(\boldsymbol{r}_i,\boldsymbol{r},\omega)dv = \begin{cases} P(\boldsymbol{r},\omega) & \text{if } \boldsymbol{r} \text{ is in } V \\ \frac{1}{2}P(\boldsymbol{r},\omega) & \text{if } \boldsymbol{r} \text{ is on the hull of } V \\ 0 & \text{if } \boldsymbol{r} \text{ is outside } V \end{cases} \quad (1)$$

Where G is Green's function and $\boldsymbol{n}$ is the normal vector on the hull of V.

Reduced to two dimensions and with [2]

$$\nabla P = -j\omega\rho_0 \boldsymbol{V} \quad (2)$$

equation 1 can be written as:

$$-\oint_{C(F)} (G_{2D}(\boldsymbol{r}_0, \boldsymbol{r}, \omega) j\omega\rho_0 \boldsymbol{V}(\boldsymbol{r}_0, \omega) + \nabla G_{2D}(\boldsymbol{r}_0, \boldsymbol{r}, \omega) P(\boldsymbol{r}_0, \omega)) \boldsymbol{n} ds + \oint_{C(F)} \Psi_i(\boldsymbol{r}_i, \omega) G_{2D}(\boldsymbol{r}_i, \boldsymbol{r}, \omega) ds = \begin{cases} P(\boldsymbol{r}, \omega) & \text{if } \boldsymbol{r} \text{ is in } F \\ \frac{1}{2} P(\boldsymbol{r}, \omega) & \text{if } \boldsymbol{r} \text{ is on } C(F) \\ 0 & \text{if } \boldsymbol{r} \text{ is outside } F \end{cases} \quad (3)$$

With

$$G_{2D} = \frac{1}{4j} H_0^{(2)}(k|\boldsymbol{r} - \boldsymbol{r}_0|) = \frac{1}{4j}\left(J_0(kr) - jN_0(kr)\right) \quad (4)$$

$$k = \frac{2\pi f}{c} = \frac{\omega}{c} \quad (5)$$

Where $H_0^{(2)}$ is the Hankel Function of the second kind and zeroth order which can be expressed by the Bessel Function J_0 and the Neumann Function N_0. The ANC system's setup is depicted in figure 2. Compared with equation 3 the microphones build the contour $C(F)$, the speakers represent the inner sources Ψ_i and the points of superposition are virtual points at the locations $\boldsymbol{r}_{SP}$. There, the pressure should be zero which can be expressed by:

$$P_{SP} = P(\boldsymbol{r}_{SP}, \omega) = P_{prim}(\boldsymbol{r}_{SP}, \omega) + P_{sec}(\boldsymbol{r}_{SP}, \omega) = 0 \quad (6)$$

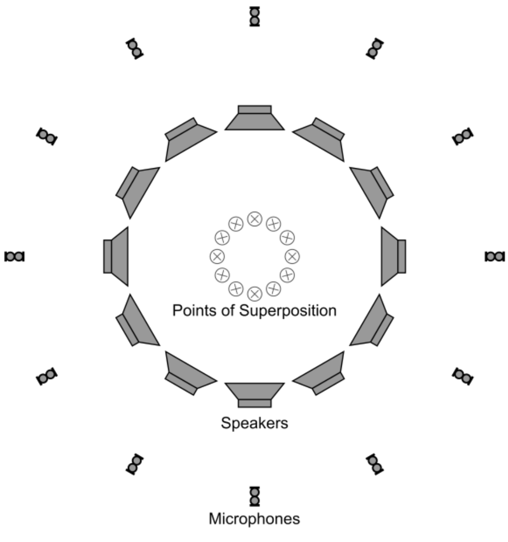

Figure 2: ANC system setup. The 12 microphone pairs consist of a velocity and a pressure microphone each.

The condition in equation 6 yields to:

$$P(\boldsymbol{r}_{SP}, \omega) = -\oint_{C(F)} (G_{2D}(\boldsymbol{r}_M, \boldsymbol{r}_{SP}, \omega) j\omega\rho_0 \boldsymbol{V}(\boldsymbol{r}_M, \omega) + \nabla G_{2D}(\boldsymbol{r}_M, \boldsymbol{r}_{SP}, \omega) P(\boldsymbol{r}_M, \omega)) \boldsymbol{n} ds + \oint_{C(F)} \Psi_i(\boldsymbol{r}_i, \omega) G_{2D}(\boldsymbol{r}_i, \boldsymbol{r}_{SP}, \omega) ds = 0 \quad (7)$$

Because of the finite number of 12 microphone pairs on $C(F)$ and 12 loudspeakers inside F equation (7) can be written in matrix form as:

$$\boldsymbol{l} \cdot \boldsymbol{S} = \boldsymbol{G}_{MSP} \cdot \boldsymbol{v}_M + \nabla \boldsymbol{G}_{MSP} \cdot \boldsymbol{p}_M \quad (8)$$

With the speaker signals

$$\boldsymbol{l} = \begin{pmatrix} l_1 \\ \vdots \\ l_{12} \end{pmatrix} \quad (9)$$

the secondary paths between every speaker and every point of superposition

$$\boldsymbol{S} = \begin{pmatrix} S_{L1,SP1} & \cdots & S_{L1,SP12} \\ \vdots & \ddots & \vdots \\ S_{L12,SP1} & \cdots & S_{L12,SP12} \end{pmatrix} \quad (10)$$

the transfer function matrix between every velocity microphone and every point of superposition

$$\boldsymbol{G}_{MSP} = j\omega\rho_0 \begin{pmatrix} G_{2D,M1,SP1}\boldsymbol{n} & \cdots & G_{2D,M1,SP12}\boldsymbol{n} \\ \vdots & \ddots & \vdots \\ G_{2D,M12,SP1}\boldsymbol{n} & \cdots & G_{2D,M12,SP12}\boldsymbol{n} \end{pmatrix} \quad (11)$$

the transfer function matrix between every pressure microphone and every point of superposition

$$\nabla \boldsymbol{G}_{MSP} = \begin{pmatrix} \nabla G_{2D,M1,SP1}\boldsymbol{n} & \cdots & \nabla G_{2D,M1,SP12}\boldsymbol{n} \\ \vdots & \ddots & \vdots \\ \nabla G_{2D,M12,SP1}\boldsymbol{n} & \cdots & \nabla G_{2D,M12,SP12}\boldsymbol{n} \end{pmatrix} \quad (12)$$

and the microphone signals for velocity and pressure

$$\boldsymbol{v}_M = \begin{pmatrix} v_1 \\ \vdots \\ v_{12} \end{pmatrix} \quad (13)$$

$$\boldsymbol{p}_M = \begin{pmatrix} p_1 \\ \vdots \\ p_{12} \end{pmatrix} \quad (14)$$

Therefore, the speaker signals can be achieved by:

$$\boldsymbol{l}^T = \boldsymbol{v}_M^T \cdot \boldsymbol{G}_{MSP} \cdot \boldsymbol{S}^{-1} + \boldsymbol{p}_M^T \cdot \nabla \boldsymbol{G}_{MSP} \cdot \boldsymbol{S}^{-1} \quad (15)$$

Borderline signals

To find a control input for an adaptive algorithm the reference microphones on $C(F)$ should be engaged. Hence, the signals on the borderline have to be predicted under the condition of perfect damping inside the circle. Taking Kirchhoff's law again, it can be written:

$$\frac{1}{2} P_C(\boldsymbol{r}_M, \omega) = -\oint_{C(F)} (G_{2D}(\boldsymbol{r}_M, \boldsymbol{r}_M, \omega) j\omega\rho_0 \boldsymbol{V}(\boldsymbol{r}_M, \omega) + \nabla G_{2D}(\boldsymbol{r}_M, \boldsymbol{r}_M, \omega) P(\boldsymbol{r}_M, \omega)) \boldsymbol{n} ds + \oint_{C(F)} \Psi_i(\boldsymbol{r}_i, \omega) G_{2D}(\boldsymbol{r}_i, \boldsymbol{r}_M, \omega) ds \quad (16)$$

And in discretized form:

$$\boldsymbol{p}_C^T = -2(\boldsymbol{v}_M^T \cdot \boldsymbol{G}_{MM} + \boldsymbol{p}_M^T \cdot \nabla \boldsymbol{G}_{MM}) + 2 \cdot \boldsymbol{l}^T \cdot \boldsymbol{F} \quad (17)$$

With the pressure on the contour at the microphone positions

$$\boldsymbol{p}_C = \begin{pmatrix} p_{c,M1} \\ \vdots \\ p_{c,M12} \end{pmatrix} \quad (18)$$

the transfer functions matrix between the velocity microphones

$$\boldsymbol{G}_{MM} = j\omega\rho_0 \begin{pmatrix} 1 & \cdots & G_{2D,M1,M12}\boldsymbol{n} \\ \vdots & \ddots & \vdots \\ G_{2D,M12,M1}\boldsymbol{n} & \cdots & 1 \end{pmatrix} \quad (19)$$

the transfer functions matrix between the pressure microphones

$$\boldsymbol{\nabla G}_{MM} = \begin{pmatrix} 1 & \cdots & \nabla G_{2D,M1,M12}\boldsymbol{n} \\ \vdots & \ddots & \vdots \\ \nabla G_{2D,M12,M1}\boldsymbol{n} & \cdots & 1 \end{pmatrix} \quad (20)$$

and the matrix of all feedback paths

$$\boldsymbol{F} = \begin{pmatrix} F_{L1,M1} & \cdots & F_{L1,M12} \\ \vdots & \ddots & \vdots \\ F_{L12,M1} & \cdots & F_{L12,M12} \end{pmatrix} \quad (21)$$

With equation 15 the signal on the contour becomes

$$\boldsymbol{p}_C^T = -2(\boldsymbol{v}_M^T \cdot \boldsymbol{G}_{MM} + \boldsymbol{p}_M^T \cdot \boldsymbol{\nabla G}_{MM})$$
$$+2 \cdot \boldsymbol{v}_M^T \cdot \boldsymbol{G}_{MSP} \cdot \boldsymbol{S}^{-1}\boldsymbol{F} + 2 \cdot \boldsymbol{p}_M^T \cdot \boldsymbol{\nabla G}_{MSP} \cdot \boldsymbol{S}^{-1}\boldsymbol{F} \quad (22)$$

Error signals

For an optimization routine it is necessary to compare predicted signals with measured signals on the outer contour. In equation 22 the prediction depends on several transfer function matrices. To reduce complexity the paths $\boldsymbol{G}_{MM}$ and $\boldsymbol{\nabla G}_{MM}$ can be measured and the associated term subtracted from the equation. The measurement could be done during an offline phase before the operation of the system. The prediction then only depends on the transfer functions of the ANC system which are estimations of the real paths

$$\widehat{\boldsymbol{T}}_v = \widehat{\boldsymbol{G}}_{MSP} \cdot \widehat{\boldsymbol{S}}^{-1} \quad (23)$$

$$\widehat{\boldsymbol{T}}_p = \boldsymbol{\nabla}\widehat{\boldsymbol{G}}_{MSP} \cdot \widehat{\boldsymbol{S}}^{-1} \quad (24)$$

and the feedback paths. From this follows that 22 can be reduced to

$$\boldsymbol{p}_{C,d}^T = 2 \cdot \boldsymbol{v}_M^T \cdot \widehat{\boldsymbol{T}}_{\boldsymbol{v}} \cdot \boldsymbol{F} + 2 \cdot \boldsymbol{p}_M^T \cdot \widehat{\boldsymbol{T}}_{\boldsymbol{p}} \cdot \boldsymbol{F} \quad (25)$$

If the reduced prediction $\boldsymbol{p}_{C,d}^T$ is correct it is equal to a measured signal on the contour $\boldsymbol{p}_{C,m}^T$ from which the influences of $\boldsymbol{G}_{MM}$ and $\boldsymbol{\nabla G}_{MM}$ also have been subtracted. The error then can be written as:

$$\boldsymbol{e}^T = \boldsymbol{p}_{C,m}^T - \boldsymbol{p}_{C,d}^T \quad (24)$$

Under the assumption that also the feedback paths $\boldsymbol{F}$ can be measured accurately enough follows:

$$\boldsymbol{e}^{*\boldsymbol{T}} = \boldsymbol{e}^T \cdot \boldsymbol{F}^{-\boldsymbol{1}} = \boldsymbol{p}_{C,m}^T \cdot \boldsymbol{F}^{-\boldsymbol{1}} - \boldsymbol{v}_M^T \cdot \widehat{\boldsymbol{T}}_v - \boldsymbol{p}_M^T \cdot \widehat{\boldsymbol{T}}_p \quad (25)$$

When the error vector $\boldsymbol{e}^{*\boldsymbol{T}}$ equals $\boldsymbol{0}$ the estimated transfer functions $\widehat{\boldsymbol{T}}_v$ and $\widehat{\boldsymbol{T}}_p$ are identical to the real transfer functions and the pressure inside the area should be zero. Finally, it has to be figured out what transfer functions are influenced by which error signal. Comparing the left and the right side in equation (25) delivers:

$$\begin{pmatrix} e_{M1}^* \\ \vdots \\ e_{M12}^* \end{pmatrix} \to \begin{pmatrix} t_{Mv1,L1} + t_{Mv2,L1} + \cdots + t_{Mv12,L1} \\ \vdots \\ t_{Mv1,L12} + t_{Mv2,L12} + \cdots + t_{Mv12,L12} \end{pmatrix} \quad (26)$$

$$\begin{pmatrix} e_{M1}^* \\ \vdots \\ e_{M12}^* \end{pmatrix} \to \begin{pmatrix} t_{Mp1,L1} + t_{Mp2,L1} + \cdots + t_{Mp12,L1} \\ \vdots \\ t_{Mp1,L12} + t_{Mp2,L12} + \cdots + t_{Mp12,L12} \end{pmatrix} \quad (27)$$

With the transfer functions for the velocity and pressure microphones of the ANC system respectively

$$\boldsymbol{T}_v = \begin{pmatrix} t_{Mv1,L1} & \cdots & t_{Mv1,L12} \\ \vdots & \ddots & \vdots \\ t_{Mv12,L1} & \cdots & t_{Mv12,L12} \end{pmatrix} \quad (28)$$

$$\boldsymbol{T}_p = \begin{pmatrix} t_{Mp1,L1} & \cdots & t_{Mp1,L12} \\ \vdots & \ddots & \vdots \\ t_{Mp12,L1} & \cdots & t_{Mp1,L12} \end{pmatrix} \quad (29)$$

Equations 26 and 27 show that one error signal influences several transfer paths at a time, namely all paths from the microphones to the associated speaker. Obviously the influence should not be same since a microphone is more dominant if it is near a speaker. This circumstance can be taken into account by multiplying the error with a factor which depends on the distance and the angle between the error microphone and the other microphones.

REFERENCES

1 D. Krahé, “A MIMO-System for a 2-dimensional active noise control application”, *The Fourth International Workshop on Multidimensional Systems*, *NDS*, 123-128 (2005)

2 M. Zollner and E. Zwicker, *Elektroakustik* (Springer, Berlin, 1998)