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# Low-complexity GSC-RLS Beamformer with Self-Tuning Algorithm

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## ABSTRACT

Our research is to develop the low-complexity broadband microphone beamformer which has robustness to moving interference signal. In this paper, an improved broadband GSC-RLS beamformer structure with fast self-tuning algorithm is proposed. Also, the improved fast self-tuning algorithm for the proposed GSC-RLS structure is developed, based on the Song's method. The computational complexity of the proposed GSC-RLS structure with self-tuning algorithm is notably reduced. And the simulation result shows that the proposed beamformer has better performance than the conventional GSC-RLS algorithm and has lower computational complexity than conventional self-tuning GSC-RLS method.

## INTRODUCTION

Beamformers, which are microphone array applications, have been developed to receive more information from desired signals and cancel undesired signals, including the interference and noise found in electro-acoustics. In the speech processing field, broad-band beamformers are broadly used because the desired signals are broad-band signals. Notably, the adaptive beamformer is used if interference signals exist, as seen in Fig. 1. The adaptive beamformer shows its better performance through the adjustment of the beamformer weights used to cancel the interference signals.

Several investigators have proposed modifications to the adaptive LCMV broad-band beamformer found in the study done by Frost [1]. One of these modified beamformers is the Generalized Sidelobe Canceller (GSC) beamformer [2]. The GSC beamformer is equivalent to the LCMV beamformer [3]. The GSC structure using the Recursive Least Squares (RLS) method, which is called GSC-RLS, was developed. The equivalence between the GSC-RLS and the Constrained RLS (CRLS) was also proved by Werner [4].

The GSC-RLS beamformer has a good performance with fixed (or stationary) interference signals. However, the performance of the GSC-RLS is reduced if the interference signal sources are moving. The performance decline is caused by the fixed memory (or the forgetting factor) of the GSC-RLS. Therefore, a modified GSC-RLS beamformer with a variable forgetting factor is needed in order to cancel moving interference signals.

The AF-RLS method, which is the forgetting factor adjusting method, was introduced by Haykin [5]. However, the computational cost of the forgetting factor update process used by the AF-RLS is too heavy to be applied to a broad-band GSC structure. Therefore, a modified self-tuning method which has a low computational cost is needed. In our paper, we propose a modified GSC-RLS algorithm to cancel interference signals. To achieve this purpose, the AF-RLS [5], [6] is applied to the GSC-RLS in order to adjust the forgetting factor. After that, the modified forgetting factor update process is derived to reduce the computational cost and to improve the performance of the beamformer.

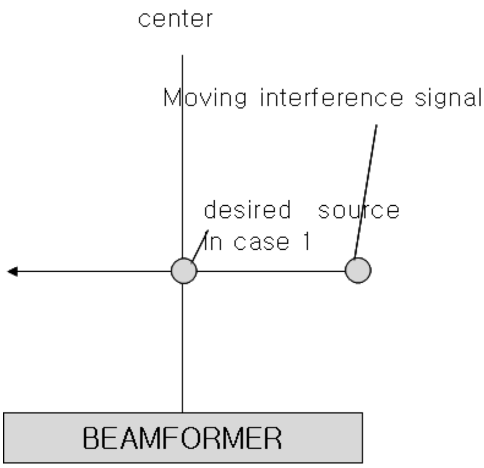


**Fig. 1.** An example of the moving interference environment.

## THE BROADBAND GSC-RLS

### The Broadband GSC Beamformer

The Generalized Sidelobe Canceller (GSC) is a broadly used structure in adaptive beamforming. The weight vector of the GSC is split into the fixed component $\underline{\underline{w}}_q$ by the constraints, and the variable component $\underline{\underline{w}}_a$ that is not affected by the

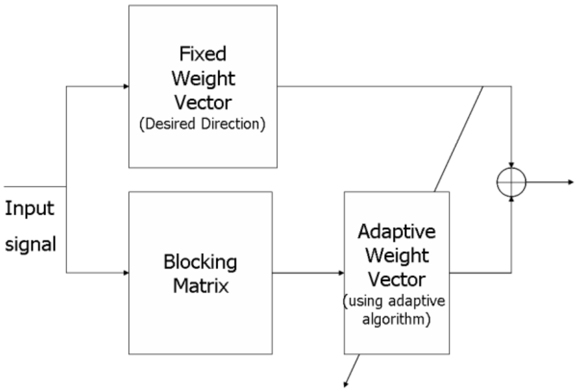


**Fig. 2.** The block diagram of the GSC structure.

constraints. To acquire the weights, let the columns of a matrix $C_a$ be defined as a basis for the orthogonal complement of the space spanned by the columns of the matrix $C$ [2]. According to definition of an orthogonal complement:

$$C^H C_a = O \quad (1)$$

or,

$$C_a^H C = O \quad (2)$$

where $O$ is a null matrix.

Let us define the $KJ \times KJ$ matrix $U$ and $KJ \times 1$ vector $q$ as:

$$U = \left[ C \;\vdots\; C_a \right], \quad (3)$$

$$\underline{q} = U^{-1} \underline{\underline{w}} = \left[ \underline{v}^T \;\vdots\; -\underline{\underline{w}}_a^T \right]^T \quad (4)$$

Therefore:

$$\underline{\underline{w}} = U \underline{q} = C \underline{v} - C_a \underline{\underline{w}}_a . \quad (5)$$

Using the constraints:

$$C^H \underline{\underline{w}} = C^H C \underline{v} - C^H C_a \underline{\underline{w}}_a = \underline{f} . \quad (6)$$

Using the above definition of an orthogonal complement:

$$\underline{v} = (C^H C)^{-1} \underline{f} . \quad (7)$$

Therefore, the fixed beamformer component $\underline{\underline{w}}_q$ is:

$$\underline{\underline{w}}_q = C \underline{v} = C (C^H C)^{-1} \underline{f} . \quad (8)$$

Finally, the weight vector of the GSC structure is:

$$\underline{\underline{w}} = \underline{\underline{w}}_q - C_a \underline{\underline{w}}_a . \quad (9)$$

The above GSC structure is equivalent to the LCMV beamformer [3]; a diagram of the GSC structure is presented in Fig. 2. The constraints matrix $C$ and the impulse response of the look-direction $\underline{f}$ are given design values; therefore the blocking matrix $C_a$ can be acquired from the constraints matrix $C$ by several methods. One of these methods is to use the singular value decomposition (SVD) method [7]. The fixed weight vector $\underline{\underline{w}}_q$ can be acquired with (8).

The main issue here is how the variable weight $\underline{\underline{w}}_a$ can be acquired. One of the more famous methods in solving this problem is the GSC-RLS method. The GSC-RLS method is based on the GSC structure and adjusts the variable weight vector $\underline{\underline{w}}_a$ using the Recursive Least Squares (RLS) method. The details of the GSC-RLS method are described in next section.

### The GSC-RLS Method

The RLS method is a recursive solution of the least squares. Using the RLS method, the weight vector of the GSC can be adjusted as [2], [4]:

$$\underline{u}(n) = C_a^H \underline{\underline{x}}(n) , \quad (10)$$

$$d(n) = \underline{\underline{w}}_q^H \underline{\underline{x}}(n) , \quad (11)$$

$$\xi(n) = d(n) - \hat{\underline{\underline{w}}}_a^H (n-1) \underline{u}(n) , \quad (12)$$

$$\underline{k}(n) = \frac{P(n-1)\underline{u}(n)}{\lambda + \underline{u}^H(n) P(n-1) \underline{u}(n)} , \quad (13)$$

$$\hat{\underline{\underline{w}}}_a(n) = \hat{\underline{\underline{w}}}_a(n-1) + \underline{k}(n) \xi^*(n) , \quad (14)$$

$$P(n) = \lambda^{-1} \left[ P(n-1) - \underline{k}(n) \underline{u}^H(n) P(n-1) \right], \quad (15)$$

where $\underline{u}(n)$ is the input and $d(n)$ is the desired signal of the RLS method. $\xi(n)$ is the a priori error, $\underline{k}(n)$ is the gain vector, the same as $P(n)\underline{u}(n)$, $P(n)$ is the inverse of the ensemble-averaged autocorrelation matrix, $\lambda$ is the 'forgetting factor' of the RLS method.

The GSC-RLS shows a high performance with a stationary interference source, but relatively low performance with a moving interference source. This degradation the result of a fixed forgetting factor (a fixed memory), therefore a forgetting factor adjust algorithm is needed. The forgetting factor adjust algorithm for the GSC-RLS method will be proposed in the next chapter.

## THE SELF-TUNING ALGORITHM FOR A BROADBAND GSC-RLS BEAMFORMER

### The AF-GSC-RLS Method

The fixed 'forgetting factor' of the GSC-RLS method causes performance degradation. To enhance the performance of the GSC-RLS, the forgetting factor must be variable. The forgetting factor can be adjusted with minimizing the following cost function [5], [6], [8]:

$$J_\lambda(n) = \frac{1}{2} E\left[ |\xi(n)|^2 \right] . \quad (16)$$

where $\xi(n)$ is the a priori estimation error referred to above. In order to minimize the cost function, we take a partial derivative of the cost function. The partial derivative of the cost function is:

$$\nabla_\lambda(n) = \frac{\partial J_\lambda(n)}{\partial \lambda} = -\mathrm{Re}\left[ \underline{\psi}^H (n-1) \underline{u}(n) \xi^*(n) \right], \quad (17)$$

where $\underline{\psi}(n) = \dfrac{\partial \hat{\underline{\underline{w}}}_a(n)}{\partial \lambda}$.

According to the "method of the steepest descent," we may adaptively compute the forgetting factor by using the

**Table 1.** The numerical complexity comparisons between the self-tuning methods.

| | Complex multiplications | Real multiplications |
|---|---|---|
| AF-GSC-RLS | $6.5(KJ)^2+4(KJ)$ | $(KJ)^2+KJ$ |
| MAF-GSC-RLS | $1.5(KJ)^2+2(KJ)$ | $(KJ)^2$ |
| Proposed GSC-RLS | $3(KJ)$ | $2(KJ)$ |

recursion [5], [6], [8]:

$$\lambda(n)=\lambda(n-1)+\alpha\,\mathrm{Re}\left[\underline{\psi}^H(n-1)\underline{u}(n)\xi^*(n)\right]. \quad (18)$$

Furthermore, $\underline{\psi}(n)$ can be updated with following equations:

$$S(n)=\left[\lambda(n)\right]^{-1}\left[I-\underline{k}(n)\underline{u}^H(n)\right]S(n-1)\left[I-\underline{u}(n)\underline{k}^H(n)\right]$$
$$+\left[\lambda(n)\right]^{-1}\underline{k}(n)\underline{k}^H(n)-\left[\lambda(n)\right]^{-1}P(n), \quad (19)$$

$$\underline{\psi}(n)=\left[I-\underline{k}(n)\underline{u}^H(n)\right]\underline{\psi}(n-1)+S(n)\underline{u}(n)\xi^*(n), \quad (20)$$

where $S(n)=\dfrac{\partial P(n)}{\partial\lambda}$.

The forgetting factor update calculation complexity of the AF-RLS method is shown in Table 1. The AF-GSC-RLS, which is the GSC-RLS beamformer using the AF-RLS, has a good performance, but the computational complexity of the forgetting factor update is very high. Therefore, a forgetting factor update algorithm which has a low complexity and a good performance is needed.

### MAF-GSC-RLS Method

AF-RLS method with a relatively low complexity was developed by Song [6], which is called the 'Modified AF-RLS (MAF-RLS)'. According to [6], $C(n)\overset{\Delta}{=}I-\underline{k}(n)\underline{u}^H(n)$ can be approximated as:

$$C(n)\approx\left[1-\frac{\underline{u}^H(n)\underline{k}(n)}{L}\right]I\overset{\Delta}{=}c(n)I\ , \quad (21)$$

where

$$c(n)\overset{\Delta}{=}1-\frac{\underline{u}^H(n)\underline{k}(n)}{L}. \quad (22)$$

The detailed proof of the above approximation is shown in [6]. Because there are so many $C(n)$ calculations in the forgetting factor update equation, this approximation can significantly reduce the computational cost.

Using the above approximations, the forgetting factor update equations are:

$$\hat{\lambda}(n)=\hat{\lambda}(n-1)+\alpha\,\mathrm{Re}\left[\underline{\hat{\psi}}^H(n-1)\underline{u}(n)\xi^*(n)\right], \quad (23)$$

$$\hat{S}(n)=\left[\hat{\lambda}(n)\right]^{-1}|c(n)|^2\hat{S}(n-1)-\left[\hat{\lambda}(n)\right]^{-1}c(n)P(n), \quad (24)$$

$$\underline{\hat{\psi}}(n)=c(n)\underline{\hat{\psi}}(n-1)+\hat{S}(n)\underline{u}(n)\xi^*(n)\ . \quad (25)$$

We call the GSC-RLS beamformer with the above MAF-RLS method the 'MAF-GSC-RLS'. The MAF-GSC-RLS beamformer has a relatively low complexity compared to the AF-GSC-RLS, which is the GSC-RLS with the AF-RLS method. However, the approximation can degrade the performance of the beamformer because an estimation error in $\nabla_\lambda(n)$ can occur using the approximation. Furthermore, the forgetting factor update complexity of the MAF-GSC-RLS is still heavy, because the complexity is proportionate to $(KJ)^2$. Therefore, an enhanced forgetting factor update algorithm will be developed in the next section.

### The Performance Enhancement of the MAF-GSC-RLS

The performance degradation of the MAF-GSC-RLS method is caused by 'the gradient noise amplification' problem. To overcome this problem, we use the normalization technique the same as the NLMS method for the MAF-GSC-RLS method [8]. The enhanced method is motivated by the fact that the both the NLMS and AF-RLS methods are based on the same method: the "method of the steepest descent." According to [8], a performance enhancement can be achieved with:

$$\hat{\lambda}_n(n)=\hat{\lambda}_n(n-1)+\frac{\alpha}{\left|\underline{\hat{\psi}}^H(n-1)\underline{u}(n)\right|^2}\mathrm{Re}\left[\underline{\hat{\psi}}^H(n-1)\underline{u}(n)\xi^*(n)\right]\Bigg|_{\lambda_-}^{\lambda_+}, \quad (26)$$

where $\hat{\lambda}_n$ is the forgetting factor of the proposed method.

### The complexity reduction of the MAF-GSC-RLS

The purpose of the MAF-GSC-RLS method is to reduce the forgetting factor update complexity of the AF-GSC-RLS. However, the forgetting factor update complexity of the MAF-GSC-RLS still will be $O\left((KJ)^2\right)$. To reduce the complexity, we modify the $\hat{S}(n)$ update equation (24) by multiplying the input vector $\underline{u}(n)$ in both sides as:

$$\hat{S}(n)\underline{u}(n)=\left[\lambda(n)\right]^{-1}\left[|c(n)|^2\hat{S}(n-1)\underline{u}(n)-c(n)P(n)\underline{u}(n)\right]. \quad (27)$$

By the definition of gain vector $\underline{k}(n)$:

$$\underline{k}(n)=P(n)\underline{u}(n)\ . \quad (28)$$

Therefore, equation (27) becomes:

$$\hat{S}(n)\underline{u}(n)=\left[\lambda(n)\right]^{-1}\left[|c(n)|^2\hat{S}(n-1)\underline{u}(n)-c(n)\underline{k}(n)\right]. \quad (29)$$

If we assume that the sampling frequency is relatively high so that:

$$\hat{S}(n-1)\underline{u}(n)\cong\hat{S}(n-1)\underline{u}(n-1)\ , \quad (30)$$

and define:

$$\underline{\hat{q}}(n)=\hat{S}(n)u(n)\ , \quad (31)$$

then the updated $\underline{\hat{\psi}}(n)$ equation is modified as:

**TABLE 2.** Summary of the proposed beamformer

| |
|---|
| Initialize : |
| $\underline{\underline{\hat{w}}}_a(0) = \underline{0}$ , |
| $\underline{\hat{q}}(0) = \underline{0}$ , |
| $\underline{\hat{\psi}}_n(0) = \underline{0}$ , |
| $P(0) = \delta^{-1} I$ , |
| where $\delta$ is a small positive constant for high SNR, and a large positive constant for low SNR. |
| For each instant of time, $n = 1, 2, \dots$ , |
| $\underline{u}(n) = C_a^H \underline{\underline{x}}(n)$ , |
| $d(n) = \underline{\underline{w}}_q^H \underline{\underline{x}}(n)$ , |
| $\xi(n) = d(n) - \underline{\underline{\hat{w}}}_a^H(n-1)\underline{u}(n)$ , |
| $\underline{k}(n) = \dfrac{P(n-1)\underline{u}(n)}{\hat{\lambda}_n(n-1) + \underline{u}^H(n)P(n-1)\underline{u}(n)}$ , |
| $\underline{\underline{\hat{w}}}_a(n) = \underline{\underline{\hat{w}}}_a(n-1) + \underline{k}(n)\xi^*(n)$ |
| $P(n) = \dfrac{P(n-1) - k(n)\underline{u}^H(n)P(n-1)}{\hat{\lambda}_n(n-1)}$ , |
| $\eta_n(n) = \underline{\hat{\psi}}_n^H(n-1)\underline{u}(n)$ , |
| $\hat{\lambda}_n(n) = \hat{\lambda}_n(n-1) + \dfrac{\alpha}{\lvert\eta_n(n)\rvert^2} \mathrm{Re}\left[\eta_n(n)\xi^*(n)\right]\Big\rvert_{\lambda_-}^{\lambda_+}$ , |
| $\underline{\hat{q}}(n) = \left[\lambda(n)\right]^{-1}\left[\lvert c(n)\rvert^2 \underline{\hat{q}}(n-1) - c(n)\underline{k}(n)\right]$ , |
| $\underline{\hat{\psi}}_n(n) = c(n)\underline{\hat{\psi}}_n(n-1) + \underline{\hat{q}}(n)\xi^*(n)$ . |

$$\underline{\hat{q}}(n) = \left[\lambda(n)\right]^{-1}\left[\lvert c(n)\rvert^2 \underline{\hat{q}}(n-1) - c(n)\underline{k}(n)\right], \tag{32}$$

$$\underline{\hat{\psi}}_n(n) = c(n)\underline{\hat{\psi}}_n(n-1) + \underline{\hat{q}}(n)\xi^*(n). \tag{33}$$

Using the equations (32) and (33), the forgetting factor is updated as:

$$\hat{\lambda}_n(n) = \hat{\lambda}_n(n-1) + \frac{\alpha}{\lvert\eta_n(n)\rvert^2} \mathrm{Re}\left[\eta_n(n)\xi^*(n)\right]\Big\rvert_{\lambda_-}^{\lambda_+}, \tag{34}$$

where,

$$\eta_n(n) = \underline{\hat{\psi}}_n^H(n-1)\underline{u}(n). \tag{35}$$

In the proposed method, the matrix calculations are eliminated by updating $\underline{\hat{q}}(n)$ instead of $\hat{S}(n)$. It is notable that there is no matrix calculation in the update equation of $\underline{\hat{q}}(n)$. That is, the proposed method has an $O(KJ)$ complexity instead of the $O\left((KJ)^2\right)$; the complexity reduction is very remarkable. The forgetting factor update complexity of the proposed method is compared to the other methods in Table 1. The conventional self-tuning methods are useless for the broadband GSC structure because they have a very high complexity, therefore, the comparison of the forgetting factor update complexity is very important.

The proposed improved MAF-GSC-RLS (IMAF-GSC-RLS) method is presented in Table 2. In addition, a block diagram of the IMAF-GSC-RLS method is presented in Fig. 3.

## THE SIMULATION

### The Experiment Settings

To verify the performance of the proposed IMAF-GSC-RLS beamformer, computer simulations using MATLAB were performed. The beamformer processor had four sensors on a line spaced at 38 centimeter intervals with 16 taps per sensor (thus $KJ = 64$). The sensors are assumed to have an omni-directional directivity and an identical sensitivity. The environment had a moving interference source which consisted of white noise. There were four interference source moving patterns: stationary, one-way, round-trip, and multiple. The desired signal and the interference signal were band-passed white noise, and the frequency response of the look-direction was determined by a low-pass filter.

The result of the each experiment was obtained by ensemble averaging over 100 independent trials. The performance of each beamformer was measured with the Signal-to-Interference-and-Noise-Ratio (SINR). The SINR denotes the energy ratio between the desired signal to the noise and interference signals. A formula expression of the SINR is:

$$SINR(n) = \frac{\left\lvert \underline{\underline{w}}^H(n)\underline{\underline{x}}_s(n)\right\rvert^2}{E\left\{\left\lvert \underline{\underline{w}}^H(n)\underline{\underline{x}}_i(n)\right\rvert^2\right\}}$$

$$= \frac{P_s\left\lvert \underline{\underline{w}}^H(n)\underline{a}(\theta_s)\right\rvert^2}{\underline{\underline{w}}^H(n)R_{i+n}(n)\underline{\underline{w}}(n)}, \tag{36}$$

where $\underline{\underline{x}}_s(n)$ is the stacked input vector from the desired signal source, and $\underline{\underline{x}}_i(n)$ is the stacked input vector from the interference source, $P_s$ is the power of the desired signal source, $\underline{a}(\theta_s)$ is the steering vector of the desired direction,

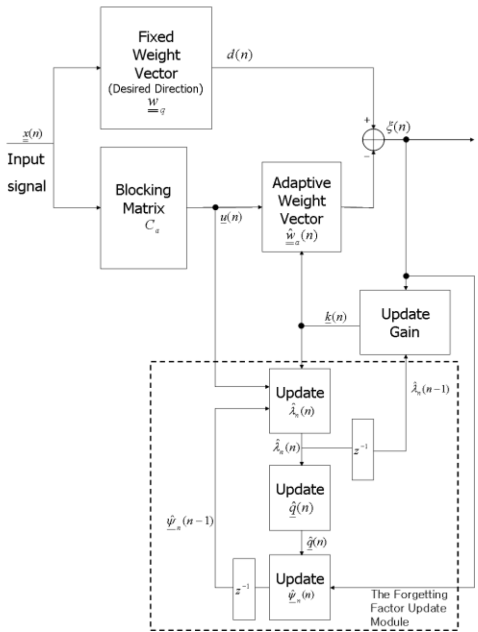


**Fig. 3.** The block diagram of the proposed beamformer

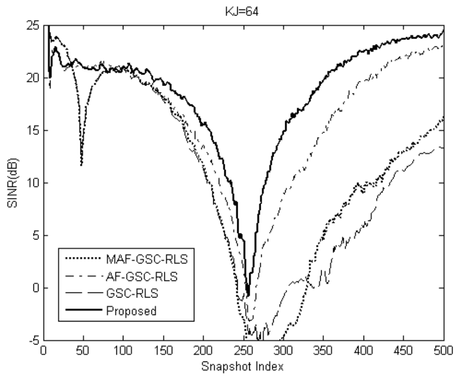


**Fig. 4.** The simulation result of the experiment.

and $R_{i+n}$ is the autocorrelation matrix of the noise and interference signal.

### Experiment result with a moving interference

In the experiment, the environment had a one-way moving interference, as shown in Fig. 1. The interference source started from the left at an angle of 45° and moved to the right until it reached the complementary angle of 45°, following the line parallel to the sensor array.

Fig. 4 shows the simulation results of Experiment 2. At $n = 250$ , the performance of every method was degraded because the incident angle of the interference source coincided with desired source. As shown in Fig. 6, the proposed beamformer showed the fastest recovery from the dip and the best performance after the recovery. The AF-GSC-RLS beamformer shows a similar performance to the proposed beamformer, but it is meaningless because the AF-GSC-RLS beamformer has a very heavy computational cost. The MAF-GSC-RLS beamformer shows a poor performance, only slightly better than the GSC-RLS beamformer.

## CONCLUSION

In this paper, a broadband GSC-RLS beamformer with a self-tuning method was proposed in order to enhance the interference cancellation performance. At first, the conventional self-tuning method is used to define the GSC-RLS structure, but the conventional method is inappropriate because the method has a heavy computational cost.

The improved self-tuning method has been developed in order to reduce the complexity and to improve the performance of the proposed GSC-RLS beamformer. The developed self-tuning method is based on the Song's method, but the performance has been improved and the complexity has been greatly reduced through our improvements. The proposed self-tuning RLS method can be used not only in the GSC structure but also in the other signal processing structures using RLS method.

The improved broadband GSC-RLS beamformer with the proposed self-tuning method was evaluated using a MATLAB simulation. The simulation environment had some moving interferences, and each method was evaluated for the interference cancellation performance under the simulation environment. According to the simulation results, the interference cancellation performance has been enhanced with the proposed beamformer.

## REFERENCES

1 O. L. Frost, Ⅲ, "An algorithm for linearly constrained adaptive array processing," *Proc. IEEE*, vol. 60, pp. 926-935, (1972).
2 Simon Haykin, *Adaptive Filter Theory (Fourth edition)* (New Jersey: Prentice Hall), pp. 120-126 (2002).
3 B. R. Breed, and Jeff Strauss, "A Short Proof of the Equivalence of LCMV and GSC Beamforming", *IEEE Signal Processing* Letters, vol. 9, no. 6, pp. 168-169, (2002).
4 Stefan Werner, Jose A. Apolinario, Jr., and Marcello L. R. de Campos, "On the Equivalence of RLS Implementations of LCMV and GSC Processors", *IEEE Signal Processing Letters,* vol. 10, no. 12, pp. 356-359 (2003).
5 Simon Haykin, *Adaptive Filter Theory (Fourth edition) (*New Jersey: Prentice Hall), pp. 662-663 (2002)
6 Seongwook Song, and Koeng-Mo Sung, "Reduced complexity self-tuning adaptive algorithms in application to channel estimation," IEEE *Trans. on Communications*, vol. 55, no. 8, pp. 1448-1452 (2007).
7 Todd K. Moon, and Wynn C. Stirling, *Mathematical Methods and Algorithms for Signal Processing (*Upper Saddle River: Prentice Hall), pp. 369-378 (2000).
8 Seokjin Lee, Jun-seok Lim, and Koeng-Mo Sung, "A low-complexity AFF-RLS algorithm using a normalization technique," *IEICE Electronics Express*, vol. 6, no. 24, pp. 1774-1780 (2009).