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Quantitative numerical approach for ultrasound imaging with Time Domain Topological Energy

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ABSTRACT

Time Domain Topological Energy (TDTE) is a new method of imaging that comes from the field of shape optimization under constraints and corresponds to an approximate resolution of the inverse problem. TDTE has been first developed for Non-Destructive Testing where its performances have shown a better ability for imaging defects in complex materials than classical tools. For acoustic imaging purpose, the rationale utilizes the following steps: an inspected medium is compared to a numerical reference where geometrical and physical properties can be iteratively modified. This comparison is realized using the ultrasonic field recorded by an array of transducers. A forward field is numerically obtained by simulations of the acoustic propagation in the reference medium where properties of velocity and density are chosen to be close to those of the inspected medium. That means the whole ultrasonic field is known inside the whole reference medium and at the location of the transducer array during the recording time. Under the constraint of the wave equation, an adjoint problem coming from an optimization process in time domain leads to a time reversal formulation where a signal difference is time reversed and propagated through the reference medium, giving the complete adjoint field. As the comparison involves the minimization of a cost function, the first term of the asymptotic expression of this function often called topological derivation or topological gradient can be used to draw an acoustical image of the medium. A more stable quantity called "topological energy" is computed by integrating the product of the squared, the forward and the adjoint fields. This modified version of the topological gradient avoids processing an iteration to limit instabilities and to improve the convergence.

A. INTRODUCTION

Quantitative ultrasonic imaging is mainly based on ultrasound propagation in time domain and aims at determining the physical parameters of the medium like elasticity [1], wave velocity and mass density, through the response of an inspected medium to an ultrasound pulse and the resolution of the inverse problem of wave propagation. Such inverse problem is difficult to solve because the response of the medium is only known on the limited surface of observation of the transducer array.

In order to improve the processing of the information contained in the recorded signal, several technics have been developed in recent years such as Time Reversal Mirrors TRM [2,3]. Through the invariance of the wave equation by time reversal, TRM has shown a remarquable ability to perform adaptative or iterative focusing through inhomogeneous media.

To solve inverse problems, the topological gradient coming from shape optimization has been developed by Schumacher [4]. It has been been applied by Masmoudi *et al.* [5,6] to the case of electromagnetism waves and then to the case of elasticity in frequency domain.

Dominguez *et al.* [7] has defined a new imaging method coming from this concept to solve numerically the inverse problem in Non Destructive Testing (NDT) in time domain for composite materials. This approach using Fubini's theorem and Parseval identity leads to a formulation where Time Reversal appears naturally. In this paper, the acoustical numerical simulations necessary to obtain images are coming from a Difference Time Domain (FDTD) solver. They could be obtained with any other numerical solver as well. It is worth noting that the same approach has been also performed by Bonnet [8] independently but in a very similar form. An important difference is that Dominguez has used a new quantity called topological energy instead of topological gradient (also known as topological derivative). The Time Domain Topological Energy (TDTE) gives a better dynamic of the resulting image and a better stability without any iterative process.

The topological energy determination needs steps in the same virtual medium: 1) to solve the forward problem, namely the reference propagation of an acoustic wave and 2) to solve the adjoint problem, namely the adjoint propagation linked to the ultrasound measurement of the response of the "unknown" inspected medium. In other words, these two problems present two different time domain excitations of the same virtual domain. For the adjoint propagation, the constraints of the wave equation in frequency domain lead to a formulation in time domain where time reversal clearly appears. This formulation has been obtained for objects presenting Neumann or Dirichlet boundary conditions but can easily be extended to any reflective object.

TDTE method has been first applied in Non Destructive Testing (NDT) of composite materials, where the aim was to detect and to localize well defined defects. TDTE applied on these defects presenting a high impedance contrast [7,9] has given very good results. Then TDTE has been tested on biological tissues [10] presenting low impedance contrasts. Whereas the method has been developed for Neumann or Dirichlet conditions, this application has given good results.

In section 1, we briefly summarize the principle of the TDTE method and ask the interested reader to use references for more detailed approach. In section 2, we present numerical results to study the ability of the method to detect a defect, to determine its position and to give hints of the physical properties of an inspected medium mimicking the low impedance contrasts present in biological tissues. Finally, a conclusion and prospects are given.

B. PRINCIPLE OF TDTE METHOD

B.1. Cost function in topological optimization

The rationale of the TDTE imaging method is described on figure 1 and comes from topological optimization. The spatial distribution of physical properties on a numerical domain (Ω) is optimized to tend toward the unknown domain (Ω_m). These two media are submitted to the same transient ultrasonic excitation and their responses (pressure fields) are recorded on a transducer array noted Γ_m. The optimization is based on these responses and is derived from the calculation of the cost function (1) that quantifies the distance between the trial domain Ω (also named reference) and the real one Ω_m:

$$j(\Omega) = J\left(U\left(\vec{r} \in \Gamma_m, t\right)\right) = \frac{1}{2} \int_{\Gamma_m} \left(\int_0^T \left\| U\left(\vec{r} \in \Gamma_m, t\right) - U_m\left(\vec{r} \in \Gamma_m, t\right) \right\|^2 dt \right) d\vec{r} \quad (1),$$

where $U(\vec{r} \in \Gamma_m, t)$ and $U_m(\vec{r} \in \Gamma_m, t)$ are respectively the acoustic fields recorded during the interval [0,T] by the transducer array Γ_m of known Ω and unknown Ω_m media excited by the same ultrasound pulse, corresponding to the propagation in the expected realistic reference medium and the inspected medium.

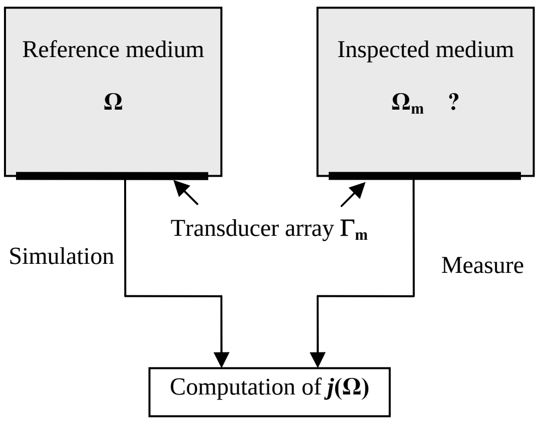

Figure 1. Principle of topological optimization with the cost function $\boldsymbol{j(\Omega)}$: comparison of the acoustic fields obtained on a transducer array between the reference medium and the inspected medium.

Practically, during the first step of optimization, the physical properties of the reference medium are chosen homogeneous and expected to be as close as possible to those of the inspected medium. This means that only the excitation of Ω is recorded by the transducer array on the contrary to Ω_m for which the signals coming from the heterogeneities are also recorded. The difference between these two signals extracts the response of the heterogeneities that corresponds to the distance between the two media.

Then, the optimization is realized by an iteration process on Ω to find spatial distributions that minimize the cost function. The spatial distributions of the tested medium are built by including geometrical or physical "holes" in the numerical reference domain. The term of hole is used to refer to an abrupt impedance contrast such as Neumann or Dirichlet conditions. In NDT of composite materials, the impedance contrast corresponds to defects in the materials and consequently to a high impedance contrast. In this case, an iterative process on the reference medium can be used with the holes to find the spatial distribution of defects.

Nevertheless, in biological ultrasound imaging, the contrast between media is low. Therefore, the term of hole corresponds in our case to low contrast heterogeneities. The iteration process then needs a quantification of the impedance distribution that is not evident to obtain.

B.2. Topological gradient

B.2.1. Forward problem

The cost function uses a forward propagation to solve the inverse problem linked to the response of the inspected medium. This forward problem corresponds to the pressure field in the reference domain. With a numerical FDTD solver, the solution $U(\vec{r} \in \Omega, t)$ of the propagation fields is known in the whole domain Ω and on the transducer array $U(\vec{r} \in \Gamma_m, t)$.

The more the trial reference domain is near to physical properties of the inspected medium, the more the distance represented by the cost function is minimized. This minimization can be obtained by an iteration of the process through the study of a variation $d\Omega$ produced by the introduction of an infinitesimal "hole" in the reference domain Ω, as described in figure 2. We determine the sensitivity to the variation $d\Omega$ of the domain Ω through an asymptotic expansion (2) of the cost function (1):

$$j(\Omega + d\Omega) = j(\Omega) + f(d\Omega) g(\vec{r}) + o\left[f(d\Omega)\right] \quad (2),$$

where $\forall d\Omega$, $f(d\Omega) > 0$ and $\lim_{d\Omega \to 0} f(d\Omega) = 0$.

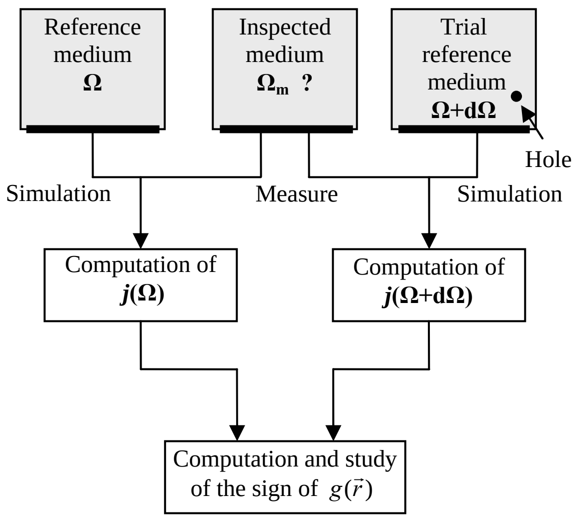

Figure 2. Principle of topological optimization with the topological gradient: comparison of the cost functions between the reference medium and a variation in this reference medium.

We verify that the cost function decreases if the heterogeneity has a closer description of the inspected medium using the condition (3):

$$j(\Omega + d\Omega) - j(\Omega) = f(d\Omega)g(\vec{r}) + o[f(d\Omega)] > 0 \quad (3),$$

with $\forall d\Omega$, the boundary conditions $f(d\Omega) > 0$.

The first order term, only function of the space variable, is the topological gradient $g(\vec{r})$. If the insertion of a hole $d\Omega$ in Ω corresponds to the real shape of Ω_m so $j(\Omega + d\Omega)$ decreases and $g(\vec{r})$ verifies the following condition:

$$g(\vec{r}) < 0 \quad (4).$$

The locations where the condition (4) is true correspond to heterogeneities (holes). After successive iterations, the reference medium is close to the inspected medium and the resulting cartography identifies the position and the shape of the interfaces of the heterogeneities present in the inspected medium.

B.2.2. Adjoint problem

Dominguez *et al.* [5,7] have shown that the topological gradient uses two numerical problems of propagation in virtual medium: the one named forward and the other named adjoint. The adjoint problem is the second numerical problem of ultrasound propagation linked to the ultrasound measurement $U_m(\vec{r} \in \Gamma_m, t)$ in the unknown medium Ω_m.

The source of the adjoint field is given by (5):

$$\begin{cases} V(\vec{r},T) = 0, & \forall \vec{r} \in \Omega, \quad \textbf{(a)} \\ \partial_{tt}^2 V + c^2(\vec{r}) \Delta V(\vec{r}, T-t) = 0, & \forall \vec{r} \in \Omega \backslash \Gamma_m, \forall t \in [0,T], \quad \textbf{(b)} \\ \partial_{tt}^2 V + c^2(\vec{r}) \Delta V(\vec{r}, T-t) = -(U - U_m)(\vec{r}, T-t), & \\ & \forall \vec{r} \in \Gamma_m, \forall t \in [0,T], \quad \textbf{(c)} \end{cases} \quad (5),$$

where T is the recording time period of the acoustic fields on the transducer array Γ_m and $V(\vec{r} \in \Omega, t)$ is the adjoint acoustic field. For the initial condition (eq. 5a) at $t = T$, the adjoint field is equal to zero in the whole medium. In equation (5b), the excitation source of the domain during the computation is equal to zero in the whole medium outside of Γ_m. In equation (5c), the source emitted through Γ_m, corresponds to the comparison of the measurement $U_m(\vec{r} \in \Gamma_m, t)$ and the reference $U(\vec{r} \in \Gamma_m, t)$. This signal $(U_m - U)(\vec{r} \in \Gamma_m, T-t)$, only containing the response coming from inhomogeneities, is time reversed [2,3] and propagates numerically through Ω during [T,0], giving the whole field $V(\vec{r} \in \Omega, t)$, solution of the adjoint problem. By this way we show the link between the topological optimization and time reversal giving a physical meaning to the mathematical solution of shape optimization.

The topological gradient:

$$g(\vec{r}) = \int_0^T U(\vec{r} \in \Omega, t) V(\vec{r} \in \Omega, t) dt \quad (6),$$

is equal to product of the forward and adjoint acoustic fields integrated with respect to the recording time. In the equation (6), the adjoint field propagates during [0,T] in the direction of the transducer array on the contrary to the reference field.

In the vicinity of defects, the topological gradient oscillates around zero that indicates the localization of the defects present in the inspected medium. The iterative process of the topological gradient can be performed on equation (6) with the negative values (4) and gives a map of the interfaces of the heterogeneities present in the inspected medium.

B.3. Topological energy

B.3.1. From topological gradient to topological energy

A modified version (7) of the topological gradient uses the square of forward and adjoint fields integrated as follows:

$$ET(\vec{r}) = \int_0^T \left\| U(\vec{r} \in \Omega, t) \right\|^2 \left\| V(\vec{r} \in \Omega, t) \right\|^2 dt \quad (7).$$

Consequently, integrated squares of the acoustic fields in pressure give the image named Topological Energy. In only one computation, this new quantity gives a good representation of the medium with a minimization of the oscillation compared to the topological gradient. Instead of iterating the process to obtain the cartography including holes of the inspected medium, in one computation, TDTE images is providing a good detection of the impedance contrast of an inspected medium.

B.3.2. From NDT to biological imaging

TDTE method has shown efficient results on composite materials [5,7]. Defects in these materials present an important impedance contrast and are localized. Accordingly, this contrast is easy to detect from an ultrasonic measurement and provides an important value of the topological energy and then a good detection of defects.

The use of TDTE in the case of biological media implies the detection of low impedance variations and continuously variable properties. Accordingly, these contrasts are lower in an ultrasonic measurement and provide a lower value of the topological energy than in NDT. This is why, it seemed easier with TDTE to detect significant defects in NDT than variable biological medium. However, the results obtained on synthetical and real biological media [10] have shown a good ability of the methods to image these media.

The iterative process of the method applied to the biological medium requires the creation of acoustic impedance cartographies instead of holes. A quantitative approach of the topological energy must be realized on these media to find their physical properties and iterate the process.

B.3.2. Link with a reflection coefficient

The physical properties of the reference medium are taken as closer as possible to the inspected medium Ω_m except the heterogeneities. The source Γ_m is the same both for reference and for inspected media so the incident fields are also the same. Since the forward field is computed in a homogenous medium, we can consider this field as the incident field $U_i(\vec{r} \in \Omega, t)$ of the inspected medium before being modified by the heterogeneities. Then, in the inspected medium, these heterogeneities create reflected waves propagating in all directions of the space depending on their shape and their orientation with respect to the transducer array. The reflected signals recorded by the transducer array only contain a part of the whole reflected field. Therefore the adjoint field can been also considered like the reflected fields $U_r(\vec{r} \in \Omega, t)$ repropagated in the reference medium during [0,T].

The topological energy sums the incident field $U_i(\vec{r} \in \Omega, t)$ multiplied by the reflected fields $U_r(\vec{r} \in \Omega, t)$ in the time domain. However, $U_r(\vec{r} \in \Omega, t)$ is the recorded and reflected part of the incident field and can be written for plane waves like $R(\vec{r} \in \Omega) \times U_i(\vec{r} \in \Omega, t)$ with $R(\vec{r} \in \Omega)$ the reflection coefficient in intensity of the heterogeneities of the inspected medium. Equation (7) is proportional of these two acoustic intensity fields and can be linked to the reflection coefficient $R(\vec{r} \in \Omega)$ of the heterogeneities in the first simplified approach of the topolological energy formulation like it is shown in (8):

$$ET(\vec{r}) = R(\vec{r} \in \Omega) \int_0^T \left\| U^+{}_i(\vec{r} \in \Omega, t) \right\|^2 \left\| U^-{}_i(\vec{r} \in \Omega, t) \right\|^2 dt \quad (8),$$

where the signs + or – point out the direction of the propagation of the field: + for the fields moving away and – approaching the transducers.

This topological energy of the incident field can be considered like a totally reflecting object with a reflection coefficient $R_i(\vec{r} \in \Omega)$ (9) localized in the reference domain:

$$R_i(\vec{r}) = \int_0^T \left\| U^+{}_i(\vec{r} \in \Omega, t) \right\|^2 \left\| U^-{}_i(\vec{r} \in \Omega, t) \right\|^2 dt \qquad (9).$$

With a theorical normalization of the equation (9) computing in each spatial point of the mesh the reflection coefficient (total reflection) of the whole reference medium $R_{1i}(\vec{r} \in \Omega)$, the reflection coefficient of heterogeneities (8) can be estimated by the following equation (10):

$$R(\vec{r}) = \frac{ET(\vec{r})}{R_{1i}(\vec{r})} \qquad (10).$$

C. QUANTITATIVE RESULTS

C.1. Detection of an object

The topological energy is computed from the acoustic propagation fields modelled with a finite difference solver. The simulation of the acoustic propagation is realized on a numerical domain Ω discretized by a mesh in two spatial dimensions and time dimension. The performance of the method to detect a variation of the impedance depends on both the values of the spatial and time step.

Indeed, the topological energy is computed in each spatial point of the mesh of the reference medium that determines the spatial discretization of the resulting image. Moreover, the sum of the topological energy should be computed with a value of a time discretization small enough to observe the crossing of the forward field with the adjoint field [7].

Figure 3 shows the importance of the time step (Dt = 0.01 µs) in the detection of a low mass density variation (R = 0.074) in a simple medium: the more the value of the time pitch increases the more the sum of the topological energy decreases.

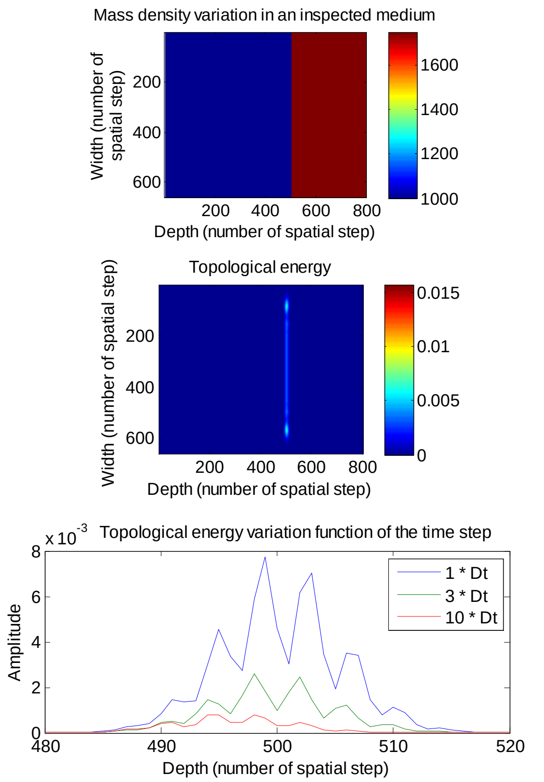

Figure 3. Study of the topological energy variation: upper image, the mass density variation of an inspected medium (constant celerity 1540 m.s^{-1}) in spatial step (Dx = 0.03 mm). Middle image: topological energy of the medium in spatial step computed with a time step Dt = 0.01 µs. Bottom curves: the axial shape of the topological energy in spatial step computed at the center of the medium for three values of the time step: blue curve 1 Dt, green curve 3 Dt and red curve 10 Dt. Implementation: PML boundary conditions, 64 transducers of 3.5 MHz.

C.2. Position and shape of an object

The adjoint field corresponds to the reflected signals of the heterogeneities, the shape of this field depends on the duration and the wavelength of the incident field. In this way, the final axial shape of the topological energy is function of the duration and the wave length of the incident field.

Figure 4 shows the result of the incident field multiplied by the reflected or adjoint field at each time step for the inspected medium studied in figure 3. The periodicity of this image is linked to the wavelength of the fields. The computation of the topological energy is realized with the square of the pressure acoustic fields. That means the periodicity of these variables corresponds to one half of a wavelength. When these variables are multiplied by each other, it appears they are in phase at each quarter wavelength. Accordingly, the sum gives a result with periodic oscillations of a quarter of wavelength.

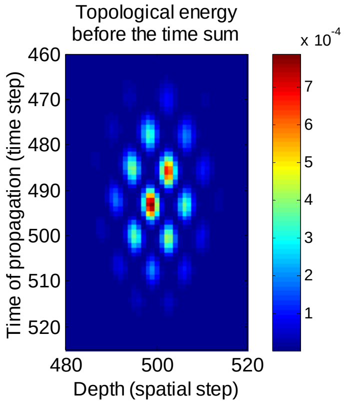

Figure 4. Study of the topological energy axial shape in the middle of the inspected medium described in figure 3 before the sum with a computation using a time step Dt = 0.01 µs.

The adjoint field is time reversed, that means the two fields cross at the interface location, so the topological energy is maximum at that place. The maxima in the image of topological energy indicate the position of the set of the impedance contrast variation. However, when the heterogeneities have variation in term of waves speed, the repropagation of the reflected signals is realized in a homogenous medium with constant velocity. That means the size of the heterogeneity determined in topological energy is modified compared to the real size in the inspected medium. This bias in topological energy appears on the second interface. As figure 5 demonstrated, if the heterogeneity has lower (resp. higher) celerity of propagation than the reference medium, the reflected signals will propagate faster (resp. slower) at the heterogeneity location and the maximum of the topological energy will moved in depth after (resp. before) the real position.

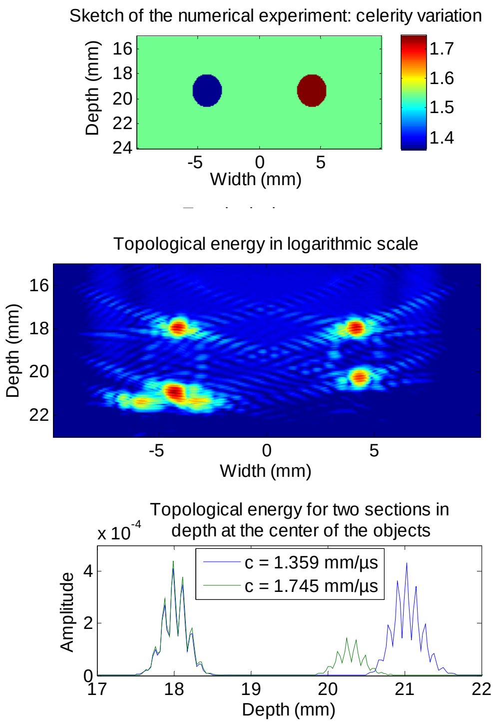

Figure 5. Example of a numerical medium mimicking inclusions surrounded by water. Upper image: map celerity of the inspected medium in mm.µs^{-1}. Middle image : visualization of the shape of spherical objects in topological energy in logarithmic scale. Bottom curves: two axial sections of the topological energy centered on the objects.

Implementation: constant mass density (1000 kg.m^{-3}), PML boundary conditions, 64 transducers of 3.5 MHz.

With the use of an array of transducers on one side of the medium and absorbing boundary conditions, it's more difficult to obtain precisely the shape of spherical objects. In the figure 5, high values of the topological energy correspond to the specular reflection (forward echo) and the low topological energy coming from the side of the objects can inform on the shape of these objects. With this representation in logarithmic scale, the diffraction with numerical interference is also visible but differentiates from the objects.

C.3. Determination of the reflection coefficient of an object

The first localized high values of the topological energy give the position of the first interface of the objects and an estimation of their impedance variations with equation (10).

In figure 5, the impedance contrast is almost the same for the two objects (Z_{left} = 1.359 MRay, Z_{right} = 1.745 MRay and Z_{medium} = 1.540 MRay). That means their reflection coefficient in intensity is also the same ($R_{left/medium}$ = 0.0040 and $R_{right/medium}$ = 0.0039). The last part of the figure 5 shows that the first area of the topological energy is the same for the two objects. Indeed, the axial sum of the topological energy in the image gives the reflection coefficient of the object ($ET_{left/medium}$ = 0.0038 and $ET_{right/medium}$ = 0.0039). Knowing the reflection coefficient in intensity and the impedance of the surrounding medium, the impedance of these heterogeneities can be found. However, the question is whether the physical parameters of the objects correspond to low or high impedance. With the computation of the topological gradient added to the topological energy, it is possible to determine the sign of the reflection coefficient in pressure. At the location of the topological energy maximum, the sign of the topological gradient, presented in figure 6, is positive for the right object and negative for the left object.

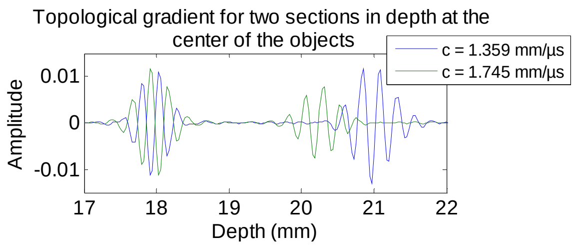

Figure 6. Topological gradient for two sections centered in depth on the objects presented in figure 5.

In this case, we can determine if the objects have an impedance higher (positive reflection coefficient) or lower (negative reflection coefficient) than the environment medium.

The celerity computed for the left object (figure 5) is 1.362 mm.µs^{-1} instead of 1.359 mm.µs^{-1} and the celerity computed for the right object is 1.745 mm.µs^{-1} for the same value. In this numerical test, the celerity of right object is perfectly retrieved and the error made for the left object can be considered as negligible. Knowing the respective velocities of the objects, the errors of localization of the second interface of the objects can be corrected.

Figure 7 shows the impedance contrast of successive plane objects in mass density (Z_1 = 1.540 MRay, Z_2 = 1.848 MRay and Z_3 = 2.156 MRay). The reflection coefficients in intensity are $R_{1/2}$ = 0.0083 and $R_{2/3}$ = 0.0059. Each axial sum of the topological energy at the defects location in the image is equal to the reflection coefficient of the objects and are $ET_{1/2}$ = 0.0083 and $ET_{2/3}$ = 0.0058. Accordingly, the equation (10) is validated for successive impedance variations for the determination of reflection coefficients.

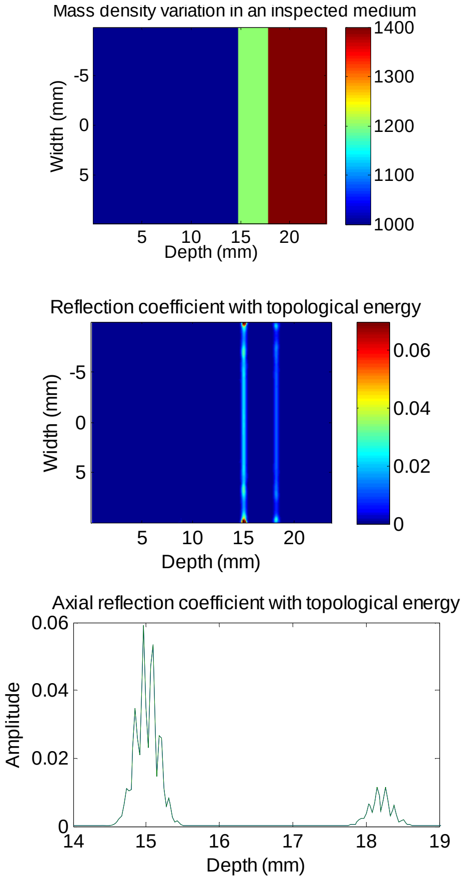

Figure 7. Study of the topological energy variation for successive plane objects: upper image, the mass density variation of an inspected medium (constant celerity 1540 $m.s^{-1}$). Middle image: the topological energy of the medium computed with a time step Dt = 0.01 µs and a spatial step Dx = 0.03 mm. Bottom curves: the axial shape of the topological energy computed at the center of the medium. Implementation: PML boundary conditions, 64 transducers of 3.5 MHz.

The main difficulty of this quantitative approach is the choice of physical parameters because heterogeneities can present both mass density and velocity variation.

D. CONCLUSION AND PROSPECTS

TDTE method uses the topological optimization to solve numerically the inverse problem for ultrasound imaging. With a normalization of the topological energy equation, reflection coefficients of the heterogeneities present in a inspected medium have been retrieved. These coefficients are low like in the biological tissues.

With a good spatial and time discretization of the domain, this method is able to detect, to position, and to determine impedance variation of an inspected numerical medium with a good accuracity. Even if this medium presents heterogeneities with a low impedance contrast, their determination is possible with the resulting image.

The topological energy provides image with better dynamic and contrast than the topological gradient. Nevertheless, the topological gradient informs about the sign of the reflection coefficients, and thus completes the topological energy in the determination of the physical properties of objects.

In prospect, an iterative process of the method can be considered to determine the physical properties of media presenting impedence variation both in mass density and in velocity. Cartography of both properties of the inspected medium can be expected. To achieve this goal, an improvement of the computation time of the adjoint field is necessary to increase the test amount for this quantitative approach.

Another prospect will consist in testing this quantitative approach on synthetic tissues where properties are well known and finally apply it to real biological tissues.

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