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Numerical analysis on 2D and 3D edge tones in terms of aerodynamic sound theory

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ABSTRACT

Edge tone is numerically reproduced by compressible LES for 2D and 3D models. To detect the aerodynamic sound source, Lighthill's sound source as well as Howe's vortex sound source are calculated. We find a marked difference in distribution between the two types of sources originating from the difference in formula: Howe's vortex sound theory is framed based on the concept that the total enthalpy instead of pressure or air density is true sound, so that the source generating fluctuation of the total enthalpy is different from that generating acoustic pressure. We also calculate mutual correlations among acoustic pressure and the two types of sound sources, i.e., Lighthill's and Howe's sources, so as to examine details of interaction among them.

INTRODUCTION

Edge tones are acoustic fluctuations generated by the oscillation of a jet emanating from a flue and colliding with an edge. The study of edge tones has a long history and many authors have contributed to this problem(1, 2, 3, 4). It is considered that some feedback mechanism, fluid and/or acoustic feedback, sustains the jet oscillation whose frequency mainly determines the frequency of the edge tone emitted by aerodynamic sound sources, so-called Lighthill's source(5). However, the detailed mechanism of the edge tone is still not understood completely.

The aim of our study is to specify positions of the sound sources and to clarify how they are created in turbulence and how the sound is emitted from them, in terms of the aerodynamic sound theory. For the first step, we numerically reproduce the jet oscillation as a sound source and the edge tone as a product simultaneously for 2D and 3D models with compressible Large-eddy Simulations(6). In a previous work(7, 8), we have really succeeded in reproducing sound vibrations of 2D and 3D air-reed instruments with a numerical scheme provided as a free software, OpenFOAM, by OpenCFD Ltd(9).

In this paper, we concentrate ourselves on a simple case of a symmetrical edge without a resonator and calculate edge tones for 2D and 3D models with changing the jet velocity. Lighthill's sound sources are obtained numerically and their behavior is analyzed using statistical methods. Actually mutual correlations among the sound source and the sound field are calculated so as to examine details of interaction among them. With those results, we try to specify the most dominant area of sound sources distributed around the jet and the eddies behind the edge which are generated by collision of the jet with the edge.

We also compare Lighthill's sound source with the sound source of the vortex sound theory formulated by Howe(4, 10). In the vortex sound theory, the sound wave is considered as a propagation of fluctuation of the total enthalpy instead of the air pressure or air density. Thus, the formulae are different and so are the source terms. We will clarify the difference of source distribution between Lighthill's and Howe's formulae and will discuss why such a difference occurs.

EDGE TONE

As shown in Fig.1, edge tones are a sort of aerodynamics sound generated by the unsteady but mostly periodical oscillation of jet emanated from the flue and collied with the edge, which is considered as the sound source of air-reed instruments(4, 11, 12). Understanding the mechanism of edge tones has been a long standing problem in the fields of aero-acoustics and musical acoustics and details of its mechanics are not completely understood yet. However, its features have been well captured by semi-empirical equations introduced based on experimental results. To the authors' knowledge, the first pioneer work was done by Brown, who introduced the following empirical formula for the frequency of edge tone(1):

$$f = 0.466j(100V - 40)(1/(100l) - 0.07), \tag{1}$$

where f denotes the frequency, V the speed of jet and l the distance between the flue and the edge. The number j is taken as $j = 1.0, 2.3, 3.8, 5.4$. For $j = 1$, it gives the fundamental frequency and others denote overtones. With increase of V, the fundamental oscillation is excited and its frequency increases in proportion to V. But it jumps to one of overtones, if V exceeds a threshold value, and the process, i.e, the transition from one to other overtone at a threshold value, is repeated with increase of V. The transitions are hysteretic and the the threshold values of V in the downward process are usually different from those in the upward.

After Brown's work, several authors proposed different formulae. Among them, the formula given by Holger *et al.* is well known and more precise(2):

$$f = 0.925\frac{\sqrt{d}}{l^{3/2}}V(n+\alpha_n)^{3/2} \tag{2}$$

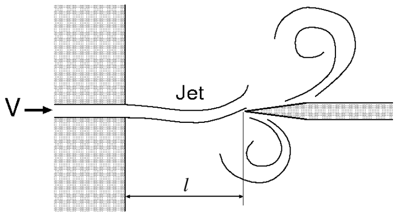


Figure 1: Edge tone

where d denotes the height of the flue, n is a natural number and $\alpha_1 = 0.4$, $\alpha_2 = 0.35$, $\alpha_3 = 0.50$. Similar formulae were also given by Crighton and Howe independently with theoretical arguments(3, 4).

LIGHTHILL'S THEORY

The sound generated by turbulence is usually called aerodynamic sound, which is a very small byproduct of the motion of unsteady flows of high Reynolds number. The source of aerodynamic sound was given the exact form by Lighthill(5). Lighthill exactly transformed the set of fundamental equations, Navier-Stokes and continuity equations, to an inhomogeneous wave equation whose inhomogeneous term plays the role of the source:

$$\left(\frac{\partial^2}{\partial t^2} - c_0^2\nabla^2\right)(\rho - \rho_0) = \frac{\partial^2 T_{ij}}{\partial x_i \partial x_j}, \tag{3}$$

where the tensor T_{ij} is called Lighthill's tensor and is defined by

$$T_{ij} = \rho v_i v_j + ((p - p_0) - c_0^2(\rho - \rho_0))\delta_{ij} + \sigma_{ij}. \tag{4}$$

Here, c_0 denotes the speed of sound in a stationary acoustic medium, p the air pressure with the average p_0, ρ the air density with the average ρ_0, and σ_{ij} the viscous stress tensor. It is considered that the sound wave is generated by the quadrupole source distribution in turbulence given by the inhomogeneous term in RHS of eq.(3) and propagates like that in the stationary acoustic medium, even though turbulence exists. This interpretation is called Lighthill's acoustic analogy.

Since the dissipation by σ_{ij} can be ignored for a high Reynolds number and adiabaticity is well held as

$$(p - p_0) - c_0^2(\rho - \rho_0) = 0, \tag{5}$$

then the first term of eq.(4), $\rho v_i v_j$, becomes the major term of the source. Further, particle velocities of the sound are usually sufficiently small compared with those of the real flow and so the source term is well approximated by that obtained from incompressible fluid with $\rho = \rho_0$ and div $\mathbf{v} = 0$. Then, the sound source is given by

$$\begin{aligned}\frac{\partial^2 T_{ij}}{\partial x_i \partial x_j} &\sim \rho_0 \frac{\partial^2 v_i v_j}{\partial x_i \partial x_j} \\ &= \rho_0(s_{ij}^2 - w_{ij}^2) \\ &= \rho_0 \mathrm{div}(\omega \times \mathbf{v}) + \rho_0 \nabla^2\left(\frac{1}{2}v^2\right),\end{aligned} \tag{6}$$

where s_{ij} and w_{ij} are respectively given by

$$s_{ij} = \frac{1}{2}\left(\frac{\partial v_j}{\partial x_i} + \frac{\partial v_i}{\partial x_j}\right), \tag{7}$$

$$w_{ij} = \frac{1}{2}\left(\frac{\partial v_j}{\partial x_i} - \frac{\partial v_i}{\partial x_j}\right). \tag{8}$$

For two dimensional(2D) fluid, it is further reduced into

$$\frac{\partial^2 T_{ij}}{\partial x_i \partial x_j} \sim -2\rho_0\left(\frac{\partial v_1}{\partial x_1}\frac{\partial v_2}{\partial x_2} - \frac{\partial v_2}{\partial x_1}\frac{\partial v_1}{\partial x_2}\right). \tag{9}$$

In calculation of Lighthill's source for 3D and 2D models, we will use the above formulae later.

For exactly incompressible fluid, an analogue to Lighthill's equation is written by a Poisson equation

$$-\nabla^2 p = \rho_0 \frac{\partial^2 v_i v_j}{\partial x_i \partial x_j}. \tag{10}$$

As the analogy to the static electric field, a static pressure field is created by the source term in RHS corresponding to the main term of Lighthill's quadrupole source, but the propagation speed of pressure distortion is infinite due to incompressibility. For compressible fluid, the pressure distortion propagates at a finite speed, then the term $\frac{1}{c_0^2}\frac{\partial^2 p}{\partial t^2}$ should be added to LHS of eq.(10) and Lighthill's equation eq.(3) with the approximations eqs.(5) and (6) is obtained again. Since the compressible portion of a dynamical variable is extremely small compared with its incompressible portion, then Lighthill's equation (3) may be well approximated in turbulence by eq.(10).

HOWE'S VORTEX SOUND THEORY

After Lighthill's paper was published(5), several authors pursued the physical meaning of Lighthill's acoustic analogy(3, 10, 13, 14, 15). Powell indicated the role of vorticity as a sound source with reducing Lighthill's source term as the last line in eq.(6)(14), which has since been followed by Howe(4, 10). Actually Howe reformulated Lighthill's equation in terms of the total enthalpy (or stagnation enthalpy) B defined by

$$B \equiv \int dh + \frac{1}{2}v^2, \tag{11}$$

where the enthalpy h is given by

$$dh = \rho^{-1}dp + TdS. \tag{12}$$

For homentropic flow with $dS = 0$, B is approximated by

$$B \sim \frac{p}{\rho} + \frac{1}{2}v^2. \tag{13}$$

Then, fluctuation of B is regarded as a deviation from Bernoulli's equation due to compressibility. Howe considered that B is the true expression of sound and introduced a reformulated equation in terms of B:

$$\begin{aligned}&\left\{\frac{\mathrm{D}}{\mathrm{D}t}\left(\frac{1}{c^2}\frac{\mathrm{D}}{\mathrm{D}t}\right) + \frac{1}{c^2}\frac{\mathrm{D}\mathbf{v}}{\mathrm{D}t}\cdot\nabla - \nabla^2\right\}B \\ &= \mathrm{div}\{\omega \times \mathbf{v} - T\,\mathrm{grad}S - \sigma\} \\ &- \frac{1}{c^2}\frac{\mathrm{D}\mathbf{v}}{\mathrm{D}t}\cdot\{\omega \times \mathbf{v} - T\,\mathrm{grad}S - \sigma\} \\ &+ \frac{\mathrm{D}}{\mathrm{D}t}\left(\frac{1}{c^2}T\frac{\mathrm{D}S}{\mathrm{D}t}\right) + \frac{\partial}{\partial t}\left(\frac{1}{c_p}\frac{\mathrm{D}S}{\mathrm{D}t}\right) \\ &+ \frac{1}{c^2}\frac{\mathrm{D}}{\mathrm{D}t}(\mathbf{v}\cdot\sigma) - \frac{1}{c^2}\frac{\partial \mathbf{v}}{\partial t}\cdot\sigma.\end{aligned} \tag{14}$$

where the specific heat at constant pressure c_p is defined by $c_p = T(\frac{\partial S}{\partial T})_p$, the sound speed c by $\frac{1}{c^2} = (\frac{\partial \rho}{\partial p})_S$ and the vector σ by $\sigma_i = (1/\rho)\partial\sigma_{ij}/\partial x_j$.

For a homentropic and low Mach but high Reynolds number flow, eq.(14) is well approximated by

$$\left(\frac{1}{c_o^2}\frac{\partial^2}{\partial t^2} - \nabla^2\right)B \sim \mathrm{div}(\omega \times \mathbf{v}). \tag{15}$$

This equation indicates that the sound in terms of B is generated by the moving vortices as shown in RHS. Using the approximation eq.(13) and ignoring the term $c_0^{-2}\frac{\partial^2}{\partial t^2}v^2/2$ which is considered to be smaller than other terms in both fluid and acoustic regimes, we get

$$\frac{1}{\rho c_o^2}\frac{\partial^2}{\partial t^2}p - \nabla^2 B \sim \mathrm{div}(\boldsymbol{\omega}\times\mathbf{v}). \qquad (16)$$

Eq.(16) with eq.(13) is equivalent to Lighthill's equation eq.(3) with the approximations eqs.(5) and (6).

(a)

(b)

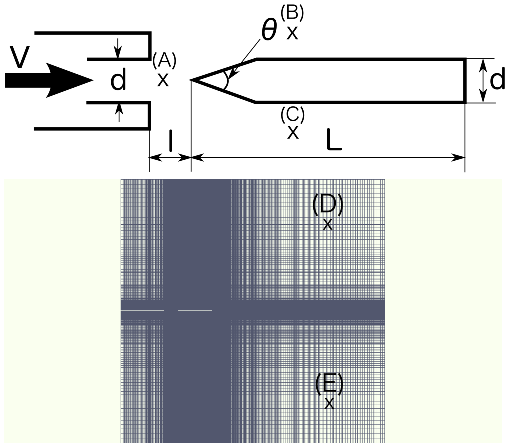


Figure 2: 2D model (a)Location of flue and edge. (b) Numerical mesh.

MODEL AND NUMERICAL SCHEME

For the numerical analysis of edge-tones, we need to simultaneously calculate flow dynamics of the jet and a sound field generated by it. The sound speed c of about 340m/s is much higher than the jet velocity V, which is at most several tens in MKS units. For reproducing sound, an extremely smaller time step is required compared with ordinary numerical calculations of fluid dynamics. On the other hand, spatial scales used for calculations of fluid dynamics with those vortices, some of which may be smaller than 1mm, are much smaller than wave lengths of sound, e.g., 34mm even at 10kHz. Therefore, in numerical calculations of edge tones we must satisfy both the requirements, a sufficiently small time step to describe sound propagation and spatial meshes fine enough to reproduce vortices in fluid. Further, particle velocities of sound(or energies of sound) are usually much less than those of a flow. Indeed, sound energies in the living environment are 10^{-4} times as small as or smaller than those of fluid. Then, it is not easy to numerically calculate sound propagation gradually dissipating for a long distance with a high degree of accuracy.

For numerical calculation, we use a compressible LES(Large eddy simulation), which is very popular in numerical simulations of aero-acoustics(6). Actually the scheme we adopt is the compressible LES solver of OpenFOAM, provided as a free software by OpenCFD Ltd(9). LES is very stable for a long time simulation, while it involves some ambiguities in boundary layers due to the statistical assumption for dynamics of eddies smaller than a given grid size. It however makes sense in the statistical point of view.

By using the compressible LES, we calculate the edge-tone for a 2D and 3D dimensional models. Fig.2 (a) shows geometry of the 2D model we adopt, where the height of flue (and the height of the splitter) d is taken as $d = 1$mm, the distance of the edge from the flue aperture $l = 5$mm, the edge angle $\theta = 20°$ and the length of splitter $L = 35$mm. Fig.2(b) shows the spatial area of the numerical mesh, which is taken large enough to reproduce sound in a near-field. Table 1 shows parameters of the mesh. The 3D model has an uniform width added to the 2D model's geometry and it is bounded by two slip walls. Hence, on every cross section parallel to the slip walls, the location of the flue and the splitter is the same as that of the 2D model. The distance between the slip walls is 10mm and the number of grid points between them is 40.

Table 1: Parameters of mesh

points	cells	faces
210214	104116	417455

The averages of pressure and temperature are taken as $p_0 =$ 100kPa and $T_0 = 300$K, respectively. The time step of numerical integration is $\Delta t = 10^{-7}$sec. Time evolution is calculated up to 0.05sec for the 2D model, but up to 0.02sec for the 3D model. For the 2D model, the velocity of the jet emanating from the flue V is changed as a control parameter in the range $(5 \le V \le 30\text{m/s})$, though it is fixed at $V = 10.0$m/s for the 3D model. Observations are done as follows. The acoustic pressure p is observed at the symmetric points (D) and (E) in Fig.2(b). The vorticityω and Lighthill's and Howe's sound sources are observed at the points (A), (B) and (C) in Fig.2(a). The point (A) is on the center line of the jet flow to detect the vorticity of the jet and the sound sources generated by the jet. On the other hand, the symmetric points (B) and (C) are respectively located behind the edge over and under the splitter to observe the vorticity and the sound sources related to the eddies caused by the collision of the jet to the edge.

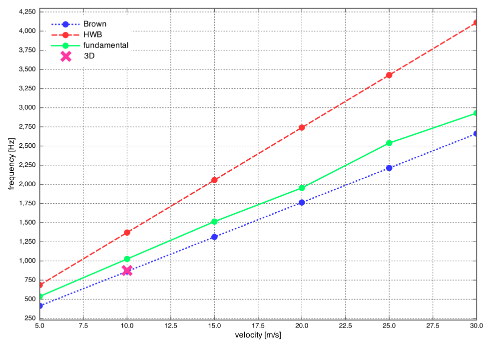


Figure 3: Jet velocity vs. frequency of acoustic vibration. Solid line: numerical result. Dotted line: Brown(eq.1). Broken line: Holger *et al.*(eq.2).

NUMERICAL RESULTS

Change of frequency with jet velocity

First, we discuss the relation of the frequency of sound observed at point (D) or (E) to the jet velocity V. Fig.3 shows change of the sound frequency for the 2D model as a function of V together with the lines given by eq.(1)(Brown) and eq.(2)(Holger *et al.*). The sound frequency at $V = 10$m/s for the 3D model is also depicted by an 'x' in this figure.

The frequency of the edge tone for the 2D model increases with increase of V. It well follows Brown's equation rather than Holger's one, though it exists between the two lines given by eq.(1) and eq.(2).

The frequency at $V = 10$m/s for the 3D model is slightly lower than that for the 2D model and almost coincides with that predicted by Brown's equation eq.(1). It means that the sounding mechanism of the edge tone for the 3D model is essentially the same as that for the 2D model, but the 3D model is more realistic than the 2D model so that it well follows the Brown's equation. For the 3D model, even when taking slip boundary walls, eddies may decay more rapidly, which makes the result more realistic. It is also interesting to study the problem what extent the frequency changes if the boundary walls are changed, for example, to solid walls on which $\mathbf{v} = 0$.

Thus, it is concluded that the compressible LES well reproduces the edge tone vibrations, but we don't unfortunately explain the reason why Brown's equation is well fit for our results rather than the equation given by Holger et al..

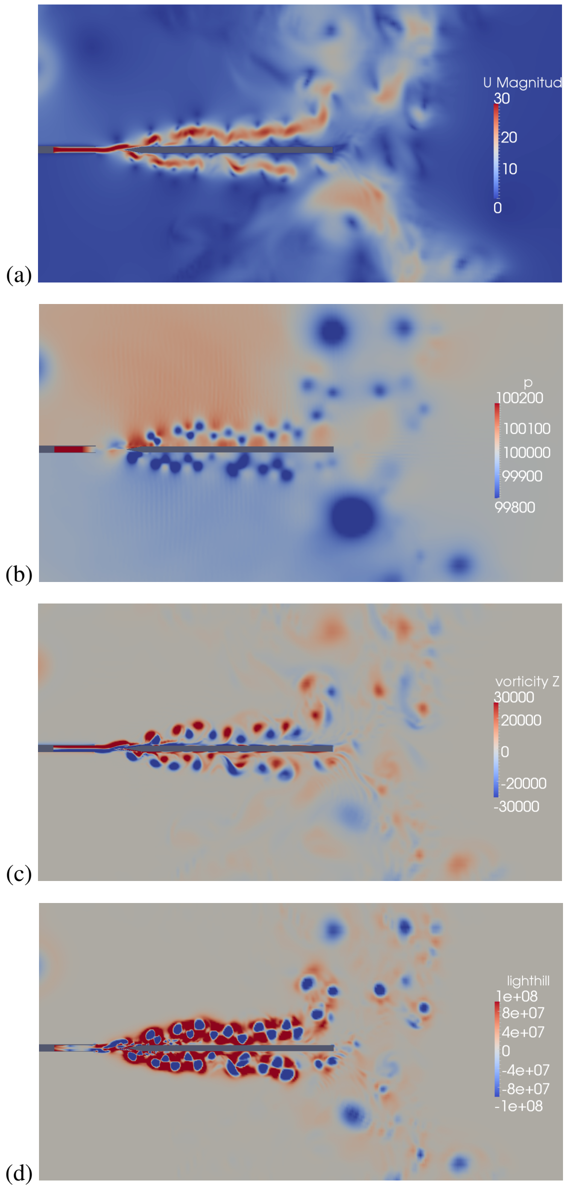


Figure 4: Spatial distributions of characteristic dynamical variables of a stable oscillation at $V = 30\mathrm{m/s}$ for 2D model. (a) Velocity. (b) Pressure. (c) Vorticity. (d) Lighthill's source.

Spatial distributions of characteristic dynamical variables

Fig.4 shows spatial distributions of characteristic dynamical variables at $V = 30\mathrm{m/s}$ for the 2D model, i.e., velocity, pressure, vorticity and Lighthill's source term. As shown in Fig.4 (a), the jet oscillates between the flue aperture and the edge, collides with the edge and is alternately poured into the upper and lower parts of the splitter generating eddies almost periodically. The eddies produced go along the splitter gradually separating from it. As a result, almost anti-symmetric distributions are formed in the upper and lower sides of the splitter, though the anti-symmetricity becomes ambiguous in a far-side.

The anti-symmetricity is more clearly found in the pressure distribution in Fig.4(b). At the moment at which the distribution is detected, the acoustic pressure becomes positive in the upper side, while it is negative in the lower side. At the center of the eddy, pressure takes a local minimum value. Thus, local minima of negative values almost periodically appear forming a regular pattern in the upper or lower side of the splitter.

Fig.4(c) shows the vorticity distribution. The anti-symmetric distribution is formed in the upper and lower sides of splitter.

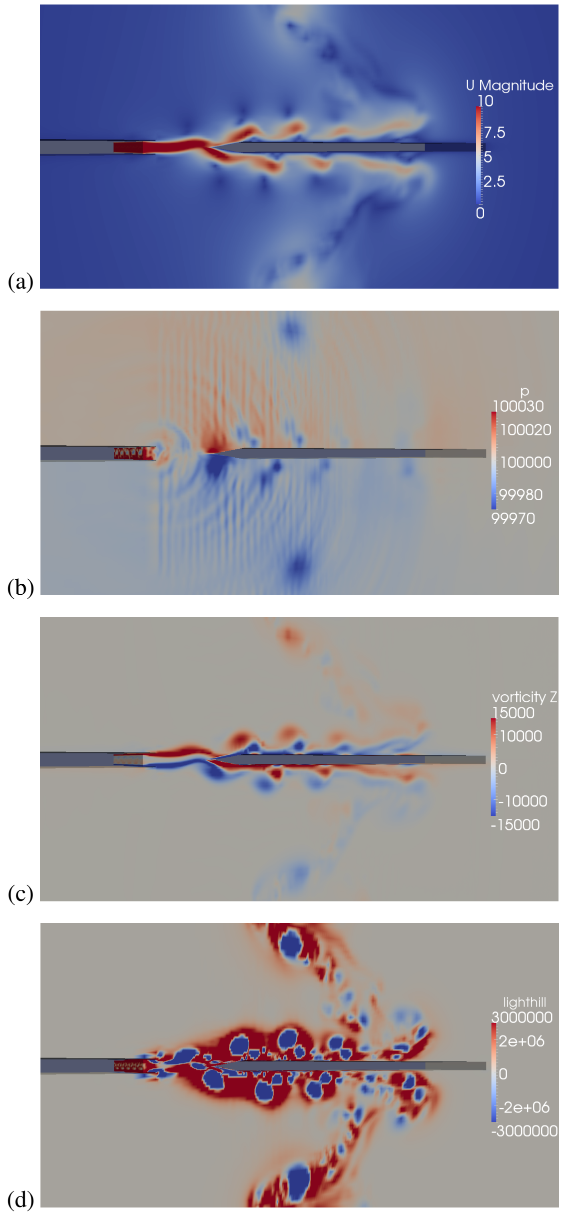


Figure 5: Spatial distributions of characteristic dynamical variables of a stable oscillation at $V = 10\mathrm{m/s}$ for 3D model. (a) Velocity. (b) Pressure. (c) Vorticity. (d) Lighthill's source.

At the center of the eddy, vorticity take a positive or negative value depending on its rotational direction. Eddies with a positive center and ones with a negative center alternately appear along the splitter in each side, though the acoustic pressure takes a negative value at the center of each eddy. On the other hand, the vorticity takes positive and negative values along the upper and lower parts of the jet, respectively.

The distribution of Lighthill's source $\frac{\partial^2 v_i v_j}{\partial x_i \partial x_j}$ shown in Fig.4(d) almost overlaps with the vorticity distribution. However, it always takes a negative value at the center of each eddy, which is surrounded by a positive circumference. Lighthill's distribution forms an array of sound sources along the splitter in both upper and lower sides. On the other hand, Lighthill's distribution makes a different pattern along the jet flow from that of vorticity. Namely, for a part of the jet wave with a positive slope, it takes negative values along the upper part, while it has positive values along the lower part. On the other hand, for another part with a negative slope, the opposite pattern appears: positive along the upper part and negative along the lower. Namely, Lighthill's source lags behind and leads by $\pi/2$ in phase the jet wave in its upper and lower parts, respectively. Then the different patterns opposite to each other alternately appear along the wavy jet. Those patterns move toward the edge with the propagation of the wave of the jet and are broken due to the collision with the edge.

In Fig.5, spatial distributions of the characteristic dynamical variables at $V = 10\text{m/s}$ for the 3D model are drawn. The characteristic feature of distribution of each dynamical variable in Fig.5 is very similar to that of the same dynamical variable for the 2D model drawn in Fig.4. Further, all the distributions of velocity, pressure, vorticity and Lighthill's source are almost the same to those for the 2D model of the same jet velocity at $V = 10\text{m/s}$, respectively, though they are not drawn. Therefore, it can be said that the acoustic mechanism of edge tone for the 3D model is well captured by using the 2D model at least for the case of slip boundary walls. However, for solid boundary walls on which the velocity is zero, some of the features might change.

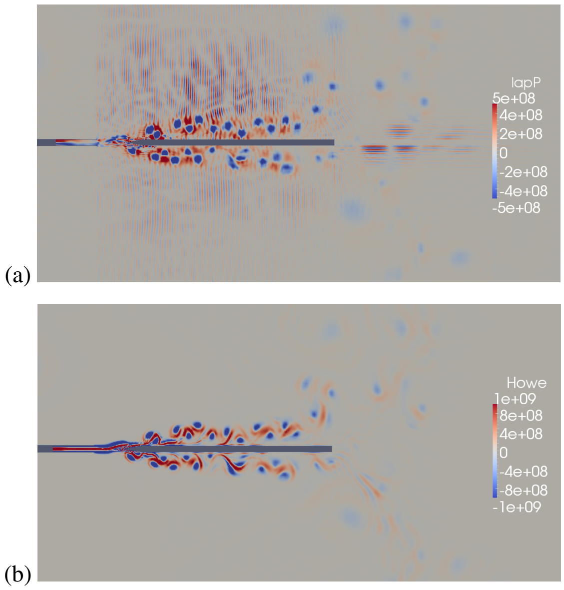


Figure 6: Spatial distributions of sound source for 2D model. (a) $-\nabla^2 p$. (b) Vortex sound source.

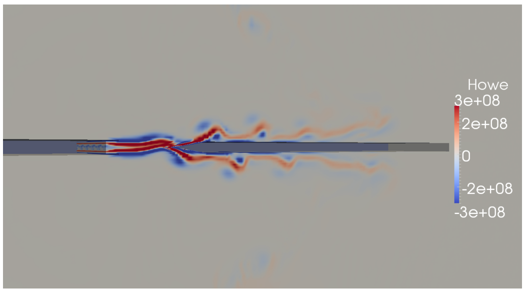


Figure 7: Spatial distribution of Vortex sound source for 3D model.

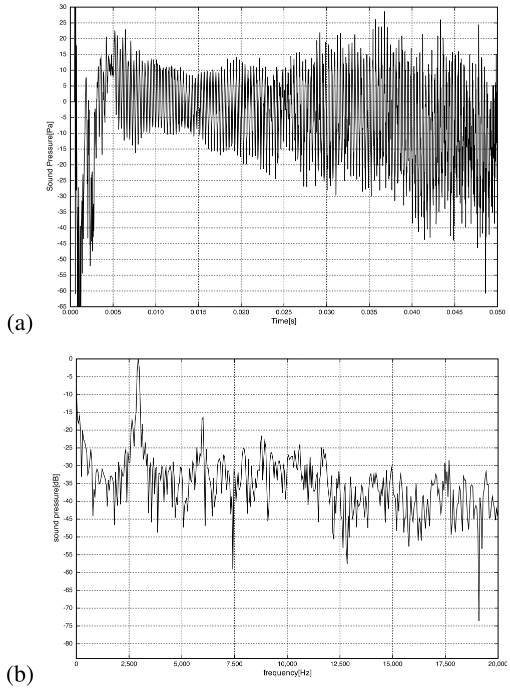


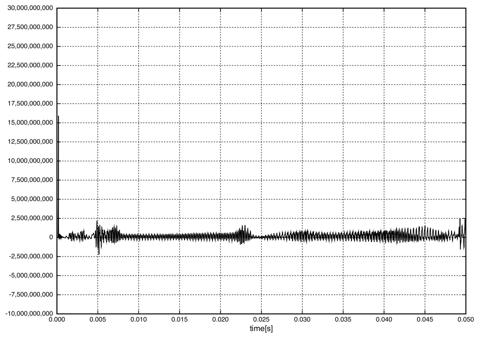
Figure 8: Sound vibration at point (D) for 2D model. (a) Acoustic pressure. (b) Power spectrum.

Lighthill's sound source vs. Howe's vortex sound source

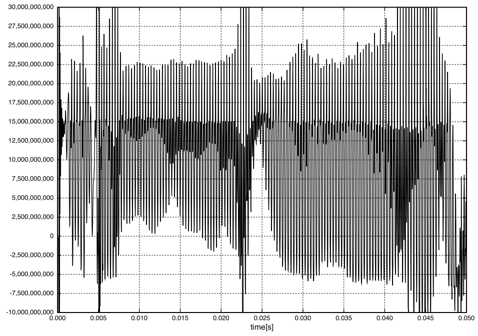
Fig.6(a) and (b) show spatial distributions of $-\nabla^2 p$ and Howe's sound source $\text{div}(\boldsymbol{\omega} \times \mathbf{v})$, respectively. As shown in Fig.6(a), the spatial distribution of $-\nabla^2 p$ is very similar to that of Lighthill's sound source, which is theoretically predicted with comparison of eq.(10) with Lighthill's equation in eq.(3) with the approximations of eqs.(5) and (6). It turns out that Lighthill's equation is well approximated by eq.(10) of incompressible fluid in the turbulence region in which sound is generated, namely sound source area.

Although we don't show a result, at a distance from the turbulence region, i.e., acoustic region without sound sources, $-\nabla^2 p$ well reproduces the wave front of sound, though components of higher wave numbers are amplified as $-\nabla^2 p \sim k^2 p$. In this regime, the source term of Lighthill's equation eq.(3) can be ignored, but the term of second partial derivative of p at t in LHS must be included, namely propagation of sound wave is written by the wave equation.

Fig.6 (b) and Fig.7 show distributions of Howe's sound source for the 2D and 3D models, respectively. The features of Howe's vortex sound source are similar to those of Lighthill's sound

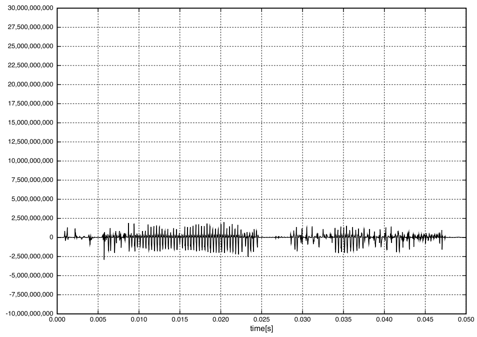


(a)

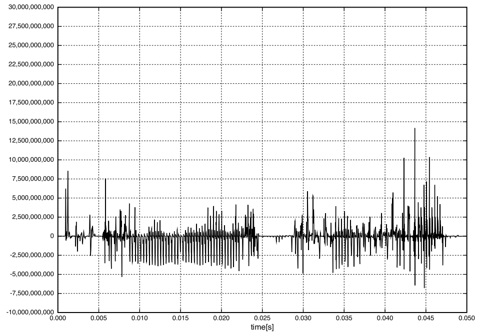


(b)

Figure 9: Sound sources at point (A) for 2D model. (a) Lighthill. (b) Howe.



(a)



(b)

Figure 10: Sound sources at point (B) for 2D model. (a) Lighthill. (b) Howe.

source in some points, but are different in other points. As Lighthill's sound source, it takes negative values near the center of each eddy. However, the positive areas surrounding the negative centers of eddies for Lighthill's source distribution contract to wavy bands for Howe's source distribution, which weave through the negative centers as a snake dance. A remarkable difference between Lighthill's and Howe's source distributions is found in the jet flow. Indeed, Howe's sound source takes positive values along the center line of the jet flow, though it takes negative values along the upper and lower edges of it. Such a difference comes from dropping the term $\nabla^2 v^2/2$ in Howe's source term, but it is apparent and is not essential. In Howe's formula shown before, the effect of the term $\nabla^2 v^2/2$ is included in the wave equation of the total enthalpy in LHS of eq.(15) (or eq.(16)). It is important that the term $\nabla^2 v^2/2$ is in the same order as $\mathrm{div}(\boldsymbol{\omega}\times\mathbf{v})$ in the turbulence, although it is often assumed that $|\nabla^2 v^2/2| \ll |\mathrm{div}(\boldsymbol{\omega}\times\mathbf{v})|$. Our numerical result suggests that that assumption is not true in the turbulence in which sound is generated.

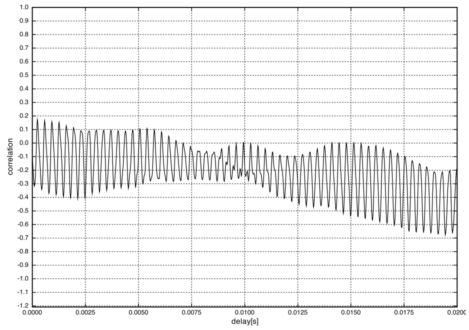
Correlation among jet oscillation, eddies, sound

In this section, we investigate time evolution of dynamical variables observed at points (A), (B) and (D) at $V = 30\mathrm{m/s}$ for the 2D model and mutual correlations among them. Fig.8 (a) and (b) show the time evolution of the acoustic pressure observed at point D and its power spectrum, respectively. The acoustic pressure oscillates almost periodically at the frequency of 2929.7Hz, except for an initial transient oscillation in the range $(0 < t < 0.005\mathrm{s})$. At the point (E), there is observed an oscillation with a similar envelope but in out of phase due to the antisymmetry.

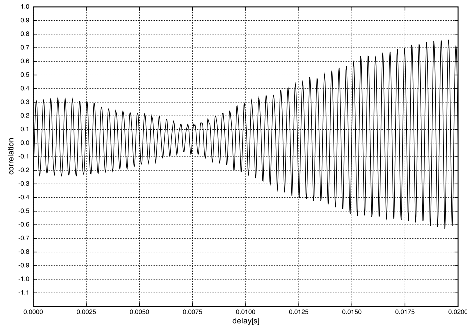
Fig.9 (a) and (b) show the time evolution of Lighthill's sound source $\frac{\partial^2 v_i v_j}{\partial x_i \partial x_j}$ and that of Howe's vortex sound source $\mathrm{div}(\boldsymbol{\omega}\times\mathbf{v})$ at point (A), respectively. Except for the initial transitions, they oscillate almost periodically with the same frequency as the acoustic pressure at point (D). However, Lighthill's sound source takes much smaller values than those of Howe's vortex sound source. Discrepancy of the two sources comes from the term $\nabla^2 v^2/2$, which is lost in the source term but included in the total enthalpy in Howe's formula. As shown in previous sections, Lighthill's distribution takes almost zero along the center line of the jet flow, while Howe's distribution forms a ridge along it. Therefore the discrepancy is enhanced along the center line of the jet flow. Amplitudes of both oscillations shrink momentarily near to $t = 0.025\mathrm{s}$. This is due to the effect of that sudden vertical blow caused by a complex unsteady motion of the fluid surrounding the flue and the edge, which disturbs the jet motion and makes it go to an upside missing the edge, but it recovers soon and takes a regular oscillation, again.

Fig.10 (a) and (b) show the time evolution of Lighthill's sound source $\frac{\partial^2 v_i v_j}{\partial x_i \partial x_j}$ and that of Howe's vortex sound source $\mathrm{div}(\boldsymbol{\omega}\times\mathbf{v})$ at point (B), respectively. They oscillate almost periodically with the same frequency as the acoustic pressure at point (D) except for some regions in which oscillation amplitudes are extremely small. One of those motions which appears just after $t = 0.025\mathrm{s}$ is of the aftereffect of the sudden blow which disturbs the jet motion near to $t = 0.025\mathrm{s}$. The amplitudes of the two types of sources are in the same order, but Howe's vortex sound source is normally larger in amplitudes than Lighthill's sound source. So the contribution of the term $\nabla^2 v^2/2$ is still not negligible.

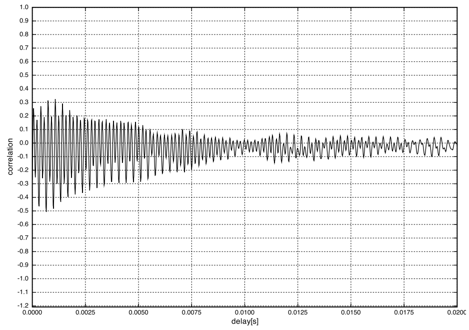
Fig.11 shows the correlations among the oscillations of Lighthill's sound source at points (A) and (B) and that of the acoustic pressure at point (D). As shown in Fig.11 (a) and (b), the correlation between Lighthill's sound source at point (A) and the acoustic pressure at point (D) oscillates quite regularly at the same frequency as the acoustic pressure, and so does that between Lighthill's sound source at point (B) and the acoustic pressure at point (D). However, the correlation between the points (A) and (D) is unstable compared with that between the points (B) and (D), especially in the latter half, and the for-



(a)



(b)



(c)

Figure 11: Correlations among dynamical variables observed at points A, B and D. (a) Lighthill at point (A) .vs. sound pressure at point (D). (b) Lighthill at point (B) .vs. sound pressure at point (D). (c) Lighthill at point (A) .vs. Lighthill at point (B).

mer has smaller amplitudes in the latter half than the latter. It means that the eddies behind the edge make dominant contribution to generating the acoustic pressure compared with the jet flow. However, if the jet velocity is taken at a smaller value, the correlation between the points (A) and (D) becomes a little larger in amplitude than that between the points (B) and (D). Even in that case, the array of the sources caused by the eddies behind the edge exists, for example see Fig.5(d), and so the total contribution from the array may be larger than that of the jet flow.

As shown in Fig.11(c), the correlation of Lighthill's sound source between points (A) and (B) rapidly decays and never grows. Similar decay is also observed for the correlation of Howe's vortex sound source as well as that of vorticity between points (A) and (B). It means that the interaction between the jet flow and the eddies behind the edge is not so strong. It is usually consider that the feedback from the eddies behind the edge is essential to sustain the oscillation of the jet flow(3). Then it is expected that the interaction between the jet and the eddies is strong enough to make the feedback and so our result is somewhat questionable and insufficient. We need to answer the question if such a small interaction is enough to sustain the jet oscillation or not. We leave this problem for future works.

DISCUSSION AND CONCLUSION

We numerically study 2D and 3D edge tones in terms of aerodynamic sound theory with compressible LES. Our main results are as follows.

First the edge tone is well reproduced by the 2D model as well as the 3D model. Namely the frequency of acoustic pressure is proportional to the jet velocity following Brown's equation. The 3D model is more realistic, since it almost coincides with Brown's equation, though we reported only one example at $V = 10\mathrm{m/s}$. Ignoring details, the mechanisms of edge tone for 2D and 3D models are almost the same. Thus, the 2D model is the minimal model for study of the edge tone.

The sound sources are located along the jet flow and in neighborhoods of those eddies behind the edge, which form arrays of sources over and under the splitter. The distribution of Howe's vortex sound source is different in detail from that of Lighthill's sound source in the neighborhoods of the eddies, while they are markedly different from each other in shape and strength near the jet flow. Indeed, Howe's vortex sound source takes much larger values compared with Lighthill's sound source near the jet. This comes from the difference of formula: Howe's vortex sound theory is framed based on the concept that the total enthalpy instead of pressure or air density is true sound, so that the source generating fluctuation of the total enthalpy loses the term $\nabla^2 v^2/2$, though it is included into the nonlinear wave equation of the total enthalpy. It is found from a more precise analysis that the vortex sound source $\mathrm{div}(\boldsymbol{\omega} \times \mathbf{v})$ takes almost opposite values to the term $\nabla^2 v^2/2$ along the jet. So the little remainder, namely the sum of $\mathrm{div}(\boldsymbol{\omega} \times \mathbf{v})$ and $\nabla^2 v^2/2$, contributes to Lighthill's sound source. Elucidating the mechanism of making such a difference between the two type of sound sources and explaining the physical meanings of it are postponed for future works.

From the analysis of correlation functions among the two types of sound sources, i.e., Lighthill's and Howe's sources, and acoustic pressure, we expect that the arrays of eddies behind the edge make dominant contribution as the sound source rather than the jet flow, though the contribution of the jet flow may relatively increase with decrease of the jet velocity. The arrays of sound sources may make directional sound fields in a far field, while the sound field emanating from the jet might not show a strong directionality. Then observations of far field sounds may reflect which part, the arrays of eddies or the jet flow, dominantly contributes to the sound generation.

For air-reed instruments, it is considered that the jet contribution, so-called volume flow mechanism, often dominates the contribution of eddies, i.e., the momentum drive mechanism, especially in a low range of the jet velocity(8, 11). As shown in ref.(8), the arrays of eddies mostly disappear for air-reed instruments due to the operation of strong sound field created by the resonance of a pipe, especially inside the pipe. Actually, only a few eddies rolled up exist in a vicinity of the open mouth inside the pipe and the other part of the pipe is occupied by the strong sound field, so that the momentum drive caused by those eddies makes relatively small contribution. Therefore, it is important to clarify the difference in sound generation between the pure edge tone and air-reed instruments.

Related to this problem, it is also important to study the feedback mechanism which sustains the oscillation of the jet. It should be different between the pure edge tone and air-reed instruments: the motion of eddies behind the edge may plays the key role for making the feedback for the edge tone, but the strong sound field in the pipe dominantly controls the motion of the jet for air-reed instruments. To do this, more comprehensive investigation using analytical, numerical and experimental methods is required.

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