Proceedings of 20th International Congress on Acoustics, ICA 2010

23-27 August 2010, Sydney, Australia

Effects of Extensional Wave propagating through Shell Structure on Vibration Energy Flow

Hirofumi Take (1), Motofumi Nishijima (1), Naoki Shigemura (1), Naoya Kojima (1) and Masato Mikami (1)

(1) Graduate School of Science and Engineering, Yamaguchi University, 2-16-1 Tokiwadai, Ube, Yamaguchi, 755-8611, Japan

PACS: 43.40.Ey

ABSTRACT

Shell structures are often employed in mechanical structures from viewpoints of light weight and high rigidity. Typical vibration countermeasures in mechanical structures are a direct damping of the vibration energy source and cutting off transmission paths of the main vibration energy. It is necessary to identify vibration transmission paths in either case of them. The Vibration Intensity (VI or Structure Intensity: SI) method is one of the techniques to visualize vibration transmission paths. In the vibration transmission in curved shells, the extensional wave occurs in addition to the flexural wave, which directly affects the noise from the shell. Previous researches reported that the extensional wave little affects the noise but interacts with the flexural wave in the curved shell and shows complicated transmission paths of the vibration energy. However, there are still unknown characteristics of vibration transmission paths in the curved shell. The purpose of this research is to elucidate the characteristics of vibration transmission paths in curved shells by VI method. The finite element method analysis was conducted for L-shaped shells consisting of two flat parts I and II and a curved part between the flat parts. The curvature radius ranged from 20 to 100 mm. The flat part I was excited in the out-plane direction by a sine wave with the amplitude of 1 N. The excitation frequency was varied from 10 to 3000 Hz. The results of the flexural VI and extensional VI show that effects of extensional wave propagating in Curve part on vibration energy flow is lager for smaller curvature radius.

1. Introduction

It is important to control the vibration in mechanical structures to improve living environment because the phenomenon causes noise radiation to the surrounding enviroment and fatigue failure of structures. Effective countermeasures for vibration are a direct treatment for the vibration energy source and cutting off transmission paths of the main vibration energy. In each case, it is necessary to identify vibration transmission paths. The Vibration intensity (VI) method is one of the techniques to visualize vibration transmission paths [1], [2]. There have been some works studying simplified measurement of VI and applying VI method to actual structures [3-5].

The vibration energy flow in curved shell consists of the flexural wave component and the extensional wave component. The former occurs from out-plane vibration, the latter occurs from in-plane vibration. Several studies have been made on the measurement of in-plane vibration by VI method and the vibration transmission in shell structure [6-8]. The main measurement object for VI is the flexural wave, because the extensional wave does not directly affect the noise generation. However, our research group reported that these waves propagate with energy interconversion in curved shell [9], [10]. So the extensional wave would complicate the vibration transmission paths.

The purpose of the present research is to elucidate characteristics of vibration transmission paths in L-shaped shell with curvature to one direction shown in Fig. 1 by using finite element method (FEM) analysis. Attention was paid to the extensional wave by VI method and energy interconversion between the flexural wave and extensional wave.

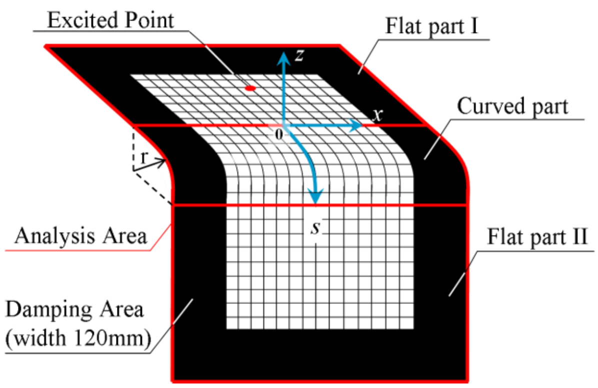


Figure 1: FEManalysis model for L-shaped shell

2. Analysis Method

2.1 Fomulation of Time-averaged Flexural VI and Extensional VI

VI consists of flexural component and extensional component. Flexural component propagates in structure by out-plane motion and extensional component propagates by in-plane motion. So each component of VI is separately defined.

VI by flexural component is defined as the sum of each works done by the shear force Q_x, the bending moment M_x and the twisting moment on cross-section of thin plate for unit width of a plate and unit time. The unit of VI is W/m. The *x*-direction component of flexural VI is represented as Eq. (1),

$$VI_{Fx} = \left\langle Q_x \cdot \dot{w} + M_x \cdot \dot{\theta}_x + M_{xs} \cdot \dot{\theta}_s \right\rangle_t \qquad (1)$$

where $\dot{w}$ is the out-plane velocity, $\dot{\theta}_x$ and $\dot{\theta}_s$ are respectively *x* and *s*-direction twist angular velocities, $< >_t$ in the equation shows time averaged quantity, and the subscript *F* denotes the flexural wave. The flexural VI in the curved plate shown in Fig. 1 is formulated on the basis of the elastic theory on thin cylindrical shell with curvature radius *r*. The VI is expressed as summation of the flexural VI in flat plate and that in curved plate by the following equation,

$$VI_{Fx} = VI_{Fxfp} + VI_{Fxc} \qquad (2)$$

$$VI_{Fxfp} = B\left\langle \frac{\partial}{\partial x}\left(\nabla^2 w\right)\frac{\partial w}{\partial t} - \left(\frac{\partial^2 w}{\partial x^2} + \nu\frac{\partial^2 w}{\partial s^2}\right)\frac{\partial^2 w}{\partial x \partial t} - (1-\nu)\frac{\partial^2 w}{\partial x \partial s}\frac{\partial^2 w}{\partial s \partial t}\right\rangle_t \qquad (3)$$

$$VI_{Fxc} = B\left\langle -\frac{\nu}{r}\frac{\partial^2 v}{\partial x \partial s}\frac{\partial w}{\partial t} + \frac{\nu}{r}\frac{\partial v}{\partial s}\frac{\partial^2 w}{\partial x \partial t} + (1-\nu)\frac{1}{r}\frac{\partial^2 w}{\partial x \partial s}\frac{\partial v}{\partial t}\right\rangle_t \qquad (4)$$

where *w* is out-plane displacement, *v* is in-plane displacement, *B* is bending stiffness, ∇^2 is two-dimensional Laplacian and ν is the Poisson's ratio. The subscript *fp* denotes flat plate and *c* denotes curved part in Eq. (2). The *s*-direction of flexural VI can be obtained by exchanging *x* and *s* in Eq. (2) and (3).

The vibration energy transported by the extensional wave is defined as the sum of each work done by the shear force N_{xs} and the extensional force N_x on cross-section of thin plate for unit width of a plate and unit time. The *x*-direction of extensional VI is expressed as Eq. (5).

$$VI_{Ex} = \left\langle N_x \cdot \dot{u} + N_{xs} \cdot \dot{v} \right\rangle_t \qquad (5)$$

where $\dot{u}$ and $\dot{v}$ are the in-plane direction of velocity, the subscript *E* denotes the extensional wave. Eqation (5) is also based on elastic theory on thin cylindrical shell with curvature radius *r*. The extensional VI in curved plate is given by the following equation.

$$VI_{Ex} = -\frac{Eh}{1-\nu^2}\left\langle \left(\frac{\partial u}{\partial x} + \nu\frac{\partial v}{\partial s}\right)\frac{\partial u}{\partial t} + \frac{1-\nu}{2}\left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial s}\right)\frac{\partial v}{\partial t} + \nu\frac{w}{r}\frac{\partial u}{\partial t}\right\rangle_t \qquad (6)$$

The *s*-direction of extensional VI is expressed by changing *x* to *s* and *u* to *v* in Eq. (6). The extensional VI of thin flat plate is also obtained by setting *r* to infinity in Eq. (6).

In order to obtain a flexural VI vector, vibration data at 12 finite-difference points shown in Fig. 3 are required because Eq. (3) includes up to third-order space-derivative terms. In Fig. 3, δ denotes difference interval. To calcurate of the extensional VI value, vibration data at 4 finite-difference points are required because linear space-delivative term at a maximum in Eq. (6). The former is called 12-point method the latter 4-point method.

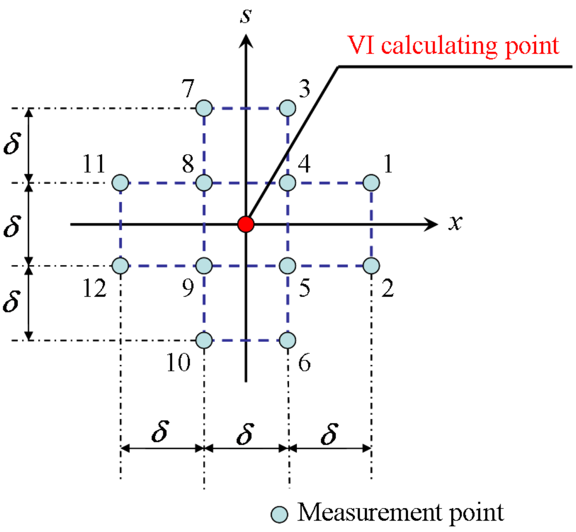


Figure 2: Acceleration measurement points for 12-point method and 4-point method

2.2 Analysis model and method

This research employed L-shaped shell models with five different curvature radii as FEM analysis models, as shown in Fig. 1. Element division is quadrilateral element. Table 1 lists curvature radius and element size in FEM analysis models. As shown in Fig. 1, the direction of minimum curvature is axial (*x*) direction and of maximum curvature is circumferential (*s*) direction. The out-plane direction is *z*. The length of circumferential direction is 900 mm and axial direction is 600 mm in FEM analysis models. The excited position is set just at 200 mm away from the entrance of the curved part for all models. The FEM analysis was performed using MSC. Visual NASTRAN for Windows. We employed the frequency analysis. Table 2 shows setting conditions of the FEM analysis.

Single frequency excitation was made with amplitude of 1 N in the out-plane direction at the excited point of L-shaped shell. The excitation frequency was varied from 10 to 3000 Hz.

Time averaged flexural VI and extansional VI were calcurated through 12-point method and 4-point method with the acceleration data in each nodal point obtained by the FEM analysis. Analysis frequency band was 500 Hz, 1000 Hz and 2000 Hz octave band components.

Time averaged flexural component ES and extensional component ES were calculated by Energy Source (ES) identification method [12] to consider energy transformation in the curved part. In arbitrary frequency and time domein, variation of total vibration energy in each segment of the wall *ES* is the energy source per unit time and unit area and is sum of the term accumulation of vibration energy ES_t and outflow of vibration energy ES_{xs} in each segment of the wall as Eq. (7). The unit of ES is W/m^2.

$$ES(f,t) = \frac{DE}{Dt} = \underbrace{\frac{\partial E}{\partial t}}_{unsteady} + \underbrace{\frac{\partial (VI_x)}{\partial x} + \frac{\partial (VI_s)}{\partial s}}_{convection}$$
$$= \underbrace{ES_t}_{unsteady} + \underbrace{ES_{xs}}_{convection} \quad (7)$$

The subscript f is frequency, t is time, E is the total vibration energy per unit area and VI_x and VI_s is respectively x and s direction VI. Time averaged ES in frequency domain is represended only by convection term expressed as Eq. (8) by time-averaging Eq. (7).

$$ES(f) = ES_{xs}(f) = \frac{\partial (VI_x)}{\partial x} + \frac{\partial (VI_s)}{\partial s} \quad (8)$$

So positive ES value represents energy source and negative value represents energy sink. If ES=0, this means energy passing.

Table 1: FEM analysis models

	Curvature radius [mm]	Element size [mm]
Model 1	95.5	5x5
Model 2	63.7	
Model 3	47.7	2.5x2.5
Model 4	31.8	
Model 5	19.1	

Table 2: Setting conditions of FEM analysis

Young's module of plate	E=2.06x10^{11} Pa*
Poisson's ratio of plate	ν=0.3*
Mass density of plate	ρ=7.68x10^3 kg/m^3*
Structure damping	0.01
Plate thickness	h=1.6 mm
Absorb element (Viscous damping)	Viscous damping coefficient C=5 Ns/m
Constrain condition	Edge of both sides: full constraint The others : free

*Source: (National Astronomical Observatory of Japan 2010 [13])

3. Results and discussion

3.1 Time averaged VI and ES in L-shaped shell

Figure 3 shows both components of time averaged VI and ES distributions in 1000 Hz one-third octave band. The curvature radius was set to 63.7 mm (Model 2). The analysis area is the whole L-shaped model in Fig. 1. Figure 3 however shows one half side area (x=-300~0 mm, s=-200~150 mm) with Flat part I (s=-200~0 mm), Curved part (s=0~100 mm) and Flat part II (s=100~50 mm). The color bar represents ES value.

Flexural VI radiates out from the excited point in Flat part I. In the case that the flexural VI flows to Curved part nearly normal to the axial direction, the vibration energy decreases at the entrance of Curved part (s$\cong$0). The flexural VI propagates in the circumferential direction in Curved part, too.

The direction angle of the flexural VI vector is defined as the angle between flexural VI vector and x-direction, as shown in the following equation,

$$\theta_{C-in} = \tan^{-1}\left(VI_{Fs} \middle/ VI_{Fx} \right) \quad (9)$$

Figure 4 shows flexural VI vector in 1000 Hz one-third octave band and the distribution of the angle calcurated by Eq. (9) as color map. In the case of θ_{C-in}<200 deg., most of the vibration energy do not enter Curved part but flows in the axial direction near the entrance of the boundary between Flat part I and Curved part.

As the flexural VI flows in Curved part, extensional VI appears at the entrance of Curved part as shown in Fig. 3(b). This is because flexural VI causes the in-plane excitation. Extensional component ES represents energy source but flexural component ES represents energy sink where the flexural VI enters Curved part in almost normal direction. The extensional VI in Curved part tends to propagate in the axial derection. However, the extensional VI propagates from Curved part to Flat part I at some area of Curved part. The extensional component of ES represents energy sink in such area, but the flexural component ES represents energy source there. The energy sink in the lower part of Flat part I in Fig. 3(a) is caused by the damping material.

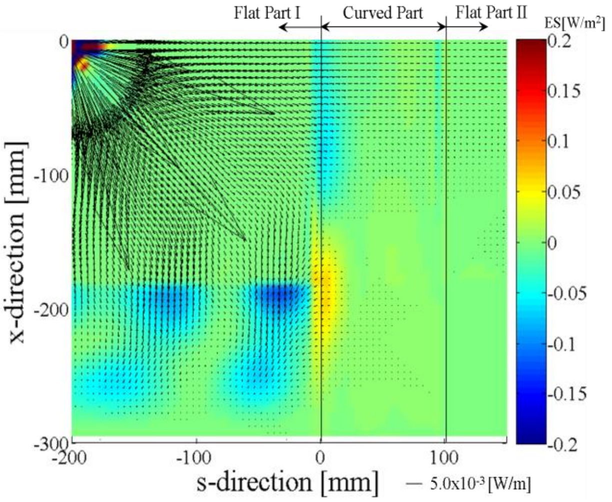
Figure 5 shows the total value of time-averaged VI and ES in 1000 Hz one-third octave band. The total value of ES is nearly 0 in Curved part with the exception at the boundary between Flat part I and Curved part, so the total vibration energy is conserved. Therefore, the flexural wave is transformed to the extensional wave in Curved part and vice versa. There exists some error to calculate time-averaged VI near the boundary because 12-point method or 4-point method needs some accelation data in Curved part to calculate VI in Flat part near boundary and vice versa. So the time-averaged ES value is not 0 at the boudary between Flat part and Curved part.

The extensional wave is converted to the flexural wave from Curved part to Flat part I. The occurrence of this conversion is probably the reason why most of flexural wave do not enter Curved part in the case of θ_{C-in}<200 deg.

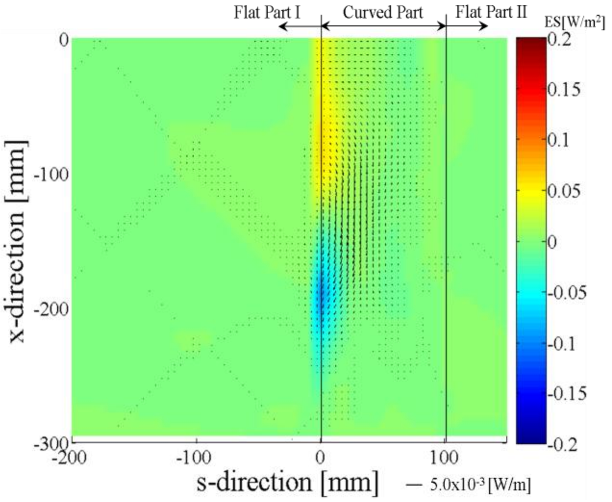
Figure 6 shows time-averaged VI and ES in 2000 Hz one-third octave band in Model 2. In this frequency band, the tendency of both components of the vibration energy flow is the same as that at 1000 Hz.

Figure 7 shows time-averaged VI and ES at 1000 Hz in Model 5. Flexural wave forms curved energy flow in Flat part I. It was reported that VI formula includes extra term if VI method applied to the vibration field where multiple waves with equal frequency exist [14]. The formative factor of curved energy flow in Flat part I might be existence of multiple waves caused by reflection at the boundary between Flat part I and Curved part due to greater effect of curvature with smaller curvature radius. Additionaly, extensional wave is transformed to flexural wave at the boundary between Curved part and Flat part I. This transformation also causes the existence of multiple waves in Flat part I. The curved energy flow in Flat part I appears in Model 4 and Model 5.

As can be seen in Fig. 7, the area of energy source and energy sink are somewhat different from that in Model 2. We confirmed that the total energy is conserved in Curved part of Model 5. Therefore, the energy transformation occurs between both comoponents of VI in Model 5.



(a) Flexural component



(b) Extensional component

Figure 3: Time-averaged VI and ES
(r=63.7 mm, 1000 Hz)

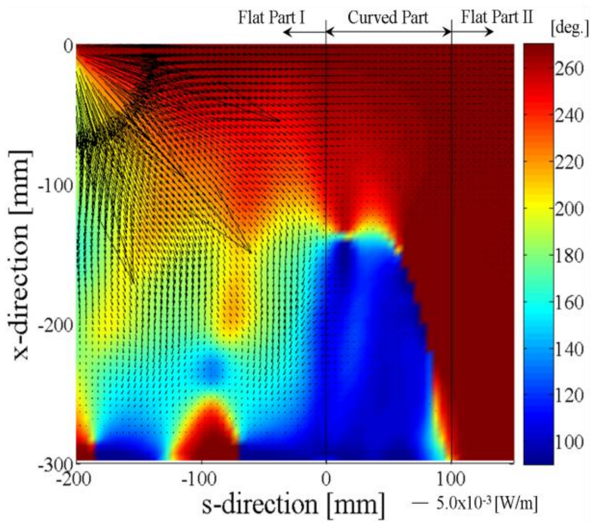


Figure 4: The angle between Flexural VI and *x*-direction
(r=63.7 mm, 1000 Hz)

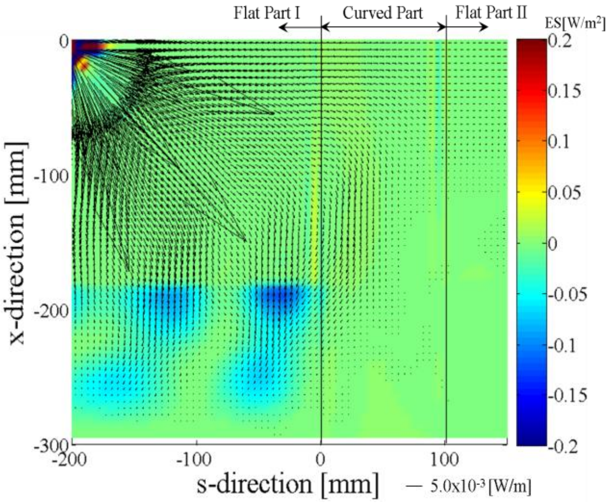
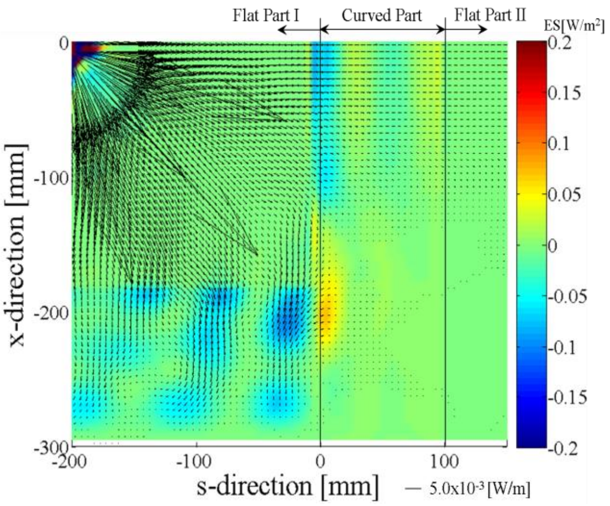
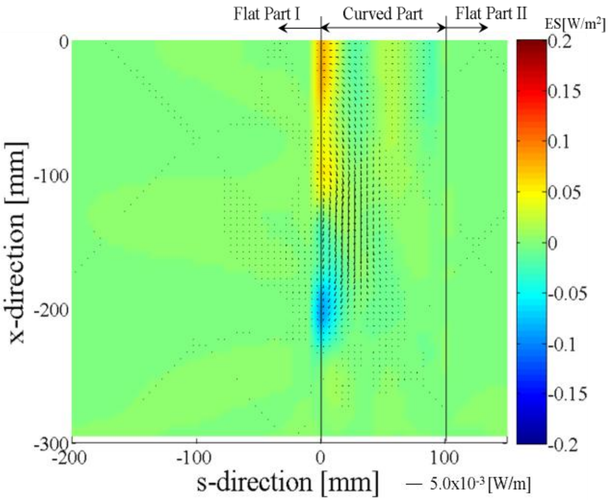


Figure 5: The total value of VI and ES
(r=63.7 mm, 1000 Hz)

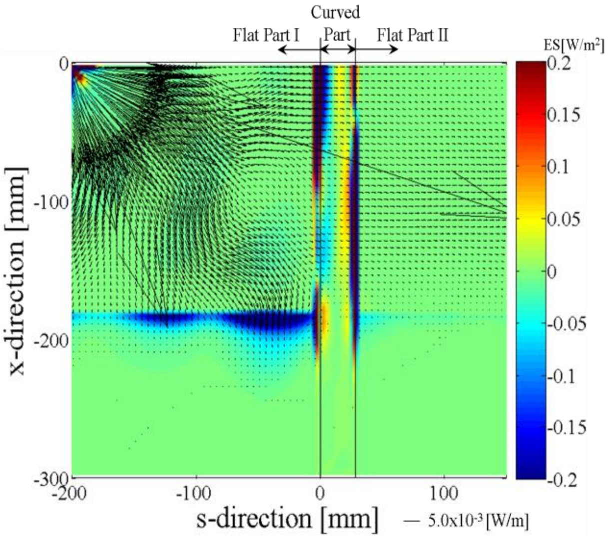


(a) Flexural component

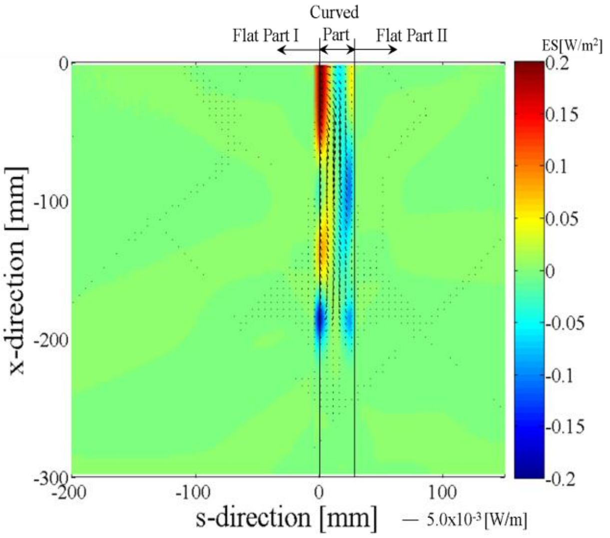


(b) Extensional component

Figure 6: Time-averaged VI and ES
(r=63.7 mm, 2000 Hz)



(a) Flexural component



(b) Extensional component

Figure7: Time-averaged VI and ES
(r=19.1 mm, 1000 Hz)

3.2 Effects of curvature on vibration energy flow in L-shaped shell

The extensional wave obviously appears in Curved part. The energy conversion between the flexural component and extensional component works out at any frequency bands in any analysis models.

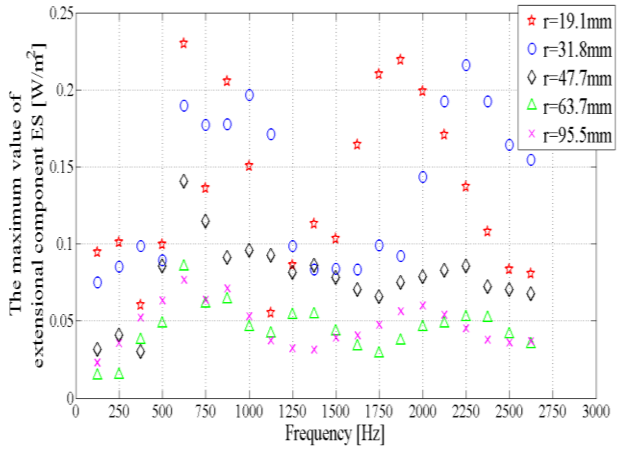
However, the effect of curvature radius on the vibration transmission is not elucidated. We pay attention to the maximum and minimum values of extensional component ES in Curved part where the energy transformation is observed. ES near the boundary between Flat part and Curved part includes error due to calculation based on finite difference method, as mentioned in Section 3.1. So, the area with the error in ES is excluded in extracting the maximum and minimum values of extensional component ES in Curved part.

Figure 8 shows frequencey characteristics of the maximum and minimum values of extensional component ES in Curved part. The maximum value of extensional component ES is larger for smaller carvature radius as shown in Fig. 8(a). This result suggests that more vibration energy of flexural wave is transformed to extensional wave for smaller curvature radius. In Fig. 8(b), the minimum value of extensional component ES is relatively large for r=47.7, 31.8 and 19.1 mm (Model 3, 4 and 5, respectively). However the trend is not the same as in Fig. 8(a).

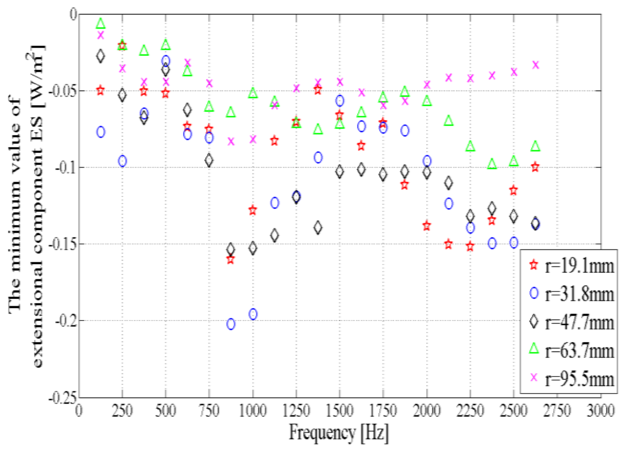
Figure 9 shows the extensional VI and ES in Curved part of Models 4 and 5. In some area of Curved part, the extensional VI propagates from Curved part to Flat part II, because the length of circumferential direction of Curved part is shorter than the other models. Such flow pattern obviously appears in Curved part of Models 4 and 5. Additionaly, the extensional component ES value represents enrgy sink near the boundary between Curved part and Flat part II, where the extensional wave is transformed into the flexural wave.

Figure 10 shows the minimum value of extensional component ES in Curved part of Models 3, 4 and 5 near the boundary between Curved part and Flat part II. The minimum value of extensional component ES in such area seems to be larger for smaller curvature radius. Figure 9(b) shows that the vibration energy by the extensional wave in Curved part of Model 5 transforms to the flexural wave in a few areas, especially near both edges of Curved part. This lowers each minimum value of extensional component ES.

The above mentioned discussion shows that effects of extensional wave propagating in Curve part on vibration energy flow is lager for smaller curvature radius.

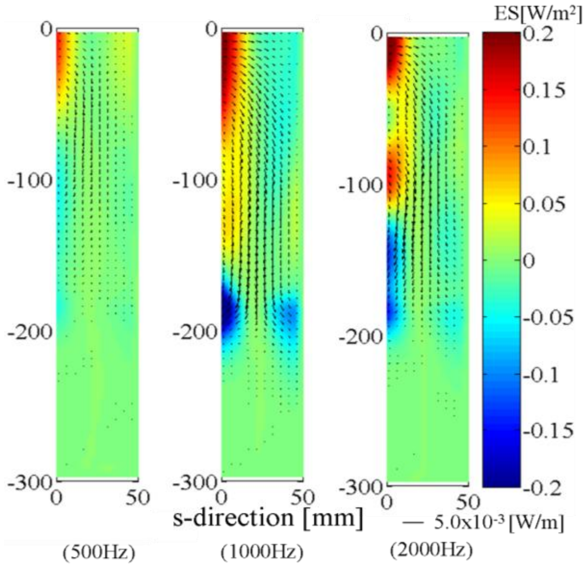


(a) The maximum value of extensional component ES in the case that flexural wave transforms to extensional wave near the boundary between Flat part I and Curved part

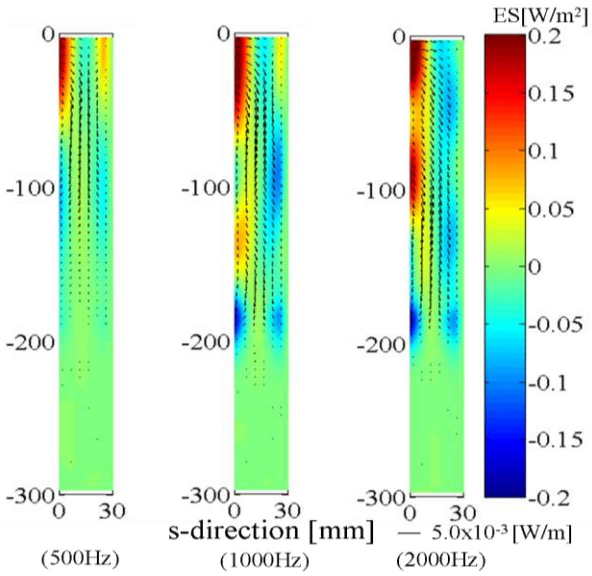


(b) The minimum value of extensional component ES in the case that extensional wave transforms to flexural wave near the boundary between Flat part I and Curved part

Figure 8: Frequency characteristics of the maximum and minimum value of extensional component ES



(a) Extensional component for different frequencies (Model 4, r=31.8 mm)



(b) Extensional component for different frequencies (Model 5, r=19.1 mm)

Figure 9: Time-averaged extensional VI and ES in Curved part

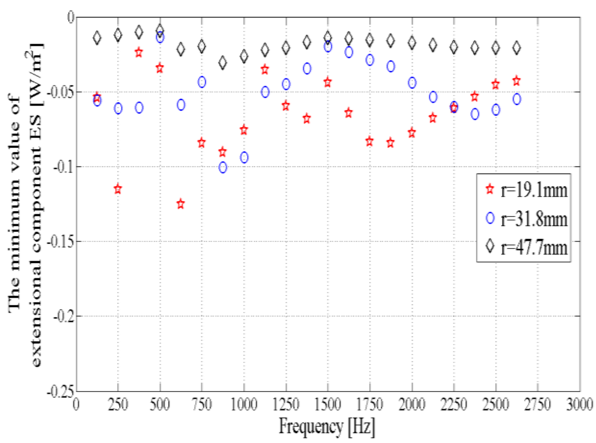


Figure 10: Frequency characteristics of the minimum value of extensional component ES in the case that extensional wave transforms to flexural wave near the boundary between Curved part and Flat part II in Models 3, 4 and 5

4. Conclusions

Characteristics of vibration transmission paths in L-shape shell structures were studied by FEM analysis. Attention was paid to the extensional wave by VI method in Curved part of the shell. The main conclusions are as follows.

(1) Extensional wave propagating through shell sturucture obviously appeared in Curved part.

(2) The total ES value is nearly 0, so the total vibratiton energy is conserved between the flexural component and extensional component in Curved part. Both components of the vibration energy transforms to each other in curved shell.

(3) The results suggest that the vibration energy conversion is more enhanced for smaller curvature radius due to the occurrence of the extensional wave in Curved part.

5. References

[1] Noiseux, D. U., “Measurement of Power Flow in Uniform Beams and Plates”, *J. Acoust. Soc. Am*, **47-1**, pp.238-247, (1970)

[2] Pavic, G., “Measurement of Structure Borne Wave Intensity”, *J. Acoust. Soc. Am*, **49**, pp.221-230, (1976)

[3] Kojima, N., Chang, J. Y., Fujimura, Y., “The Measurement of Transient Vibration Intensity Using Three Channel Method”, *SAE 1993 Transactions* (Journal of English), SAE Paper 931273, pp.83-92, (1993)

[4] Chang, J.Y., Zhou, H., Kojima, N., ‘‘The Measurement of Envelope Vibration Intensity to Identify the Combustion Impact Transmission in Diesel Engine’’, *Proceeding of International Congression Noise Control Engineering ’94*, Vol.3, pp.1701-1704, (1994)

[5] Kojima, N., Zhou, H., Fujibayasi, S., Hirayu, T., ‘‘Measurement of Transient Vibrational Power Flow in Car Door Panel Using Intensity Technique’’, *SAE1997 Transactions (Journal of English)*, SAE Paper 972035, pp.1289-1295, (1997)

[6] Pavic, G., “Vibrational Energy Flow in Elastic Circular Cylidrical Shells”, *J. Acoust. Soc. Am*, **142-2**, pp.293–310, (1990)

[7] Kojima, N., Zhou, H., Endo, M., Saito, M., ‘‘Measurement of Vibration Intensity in U-shaped shell’’, *Inter Noise’96*, pp.1379-1382, (1996)

[8] Morikawa, R., Nakamura, K., Ueha, S., “Measuring in the structural intensity by sending in-plane vibration”, *J. Acoust. Soc. Jpn (E)*, **18**, pp.201–203, (1997)

[9] Kojima, N., Mikami, M., Kumura, S., “Effects of Extensional Motion on Flexural Vibration Energy Flow in Shell”, *Inter Noise2001*, pp.899–902, (2001)

[10] Ogushi, T., Manabu, Y., Masato, M., Kojima, N., “Flexural component and extensional component of vibrational energy in shell structure”, *4th Joint Meeting of ASA/ASJ*, *J. Acoust. Soc. Am*, Vol.120, Issue 5, pp.3289-3289, (2006)

[11] Honda, S., Takada, T., Nagamatsu, A., ‘‘Fundamental Study on Vibration Intensity’’, *Transaction of the Japan Society of Mechanical Engineering*.60-574, C, pp.1941-1948, (in Japanese), (1994)

[12] Ebita, Y., Mikami, M., Kojima, N., Ahn, B., ‘‘Measurement and Analysis of Vibration Energy Flow on Compressor Casings’’, *Inter Noise’2003*, pp.4250-4256, (2003)

[13] National Astronomical Observatory of Japan, ‘‘*Chronological Scientific Tables 2010*’’, Maruzen Ltd., (in Japanese), (2010)

[14] Nishijima, M., Take, H., Mikami, M., Kojima, N., ‘‘Transmission Characteristics of Vibration Energy by Flexural Wave in Flat Plates with Vibration Mode’’, *13th Asia Pacific Vibration Conference*, No.117, (2009)