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Quantitative evaluation of three-dimensional shape of auditorium using elliptic Fourier descriptors

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ABSTRACT

Roughness of ceilings and walls in auditorium is one of the most important points of acoustic design, because the roughness of wall surface is highly related to prevention of acoustic glare and quality of sound. The roughness is now often evaluated by the surface diffusivity index, SDI, which is visual inspection by using photographs or drawings, which is not objective evaluation, proposed by Haan and Fricke[1, 2, 3]. In this paper, we propose an objective evaluation method of three-dimensional roughness of wall surface. This method is realized by elliptic Fourier descriptors and extracts cycle length and amplitude of complex surface shapes, and offers characteristics of the three-dimensional shapes as spatial frequency characteristics.

INTRODUCTION

Auditorium shapes have not yet been designed theoretically and have never been fully analyzed numerically. Because shape analysis is too difficult, thus, volume, surface area, depth, width and so on have been often substituted for it[4]. Therefore, we are trying a series of studies to pick out useful information for design through the analysis of a room shape which makes sound field. Especially, this study is paid attention to periodicity of wall roughness concerned with the propagation of sound reflection.

At First, we proposed how to pick out the periodicity included in a closed contour shape which can be digitized by elliptic Fourier descriptors and how to express it as spatial frequency characteristics[6]. Second, we proposed theoretical method to make the room shape for acoustical ray tracing method[7, 8]. A room shape is created by low-pass filtering in response to sound frequency for ray tracing simulation. In this time, this paper is discussed the method to pick out the periodical characteristics included in three-dimensional shape.

SELECTION OF ANALYSIS METHOD

The method to pick out three-dimensional periodical roughness is mainly conceived two method. One is the method to express a shape with spherical harmonics. Other is the method to expand the idea of elliptic Fourier descriptors.

Spherical harmonics is defined with polar coordinates system (vertical and horizontal angle and length). Therefore, dark side of the shape from origin point can not be expressed by spherical harmonics. Additionally, because of a point expressed by angle and length, the periodical length and amplitude varies with length from the origin to a point. This factor causes complex calculation procedure.

Elliptic Fourier descriptor is mostly used for a closed line contour. It needs for us to create a new idea suiting it to three-dimensional surface area. In this time, the purpose of this paper is therefore clarifying the method to evaluate a three-dimensional room shape quantitatively using elliptic Fourier descriptors.

Analysis method for three-dimensional shape

I. Sliceing in contours from a solid shape

The direction of normal vector for slicing plane to pick out the contour line(s) is your opinion. For simplification of problem, however, the dirxection of each vector are parallel with either one coordinate of x, y or z shown in Fig. 1. The analysis precision is influenced on the interval of slicing planes like sampling theorem. In this analysis, interval is set to 0.01 m (1 cm). In some cases, multi contours are sliced in one plane. These contours are individually analyzed.

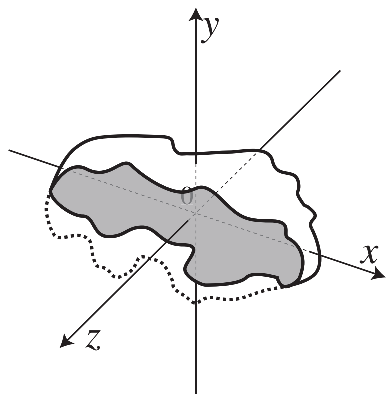

Figure 1: A three-dimensional object is sliced by a i-th xy-plane. Filled Gray area is a section and surrond line is a contour.

II. Expression with elliptic Fourier descriptors

Sliced Contours are digitized using elliptic Fourier descriptors. For instance, shown in Fig.2, the j-th contour which is sliced by the i-th xy plane is digitized. x-coordinate and y-coordinate are expressed as functions of length ℓ along the contour. These functions can be described by Fourier series expansion, therfore,

$$x_{ij}(\ell) = \sum_{n=0}^{\infty} Cx_{n,ij} \exp\left(-\mathrm{j}\frac{2\pi n\ell}{L_{ij}}\right), \tag{1}$$

$$y_{ij}(\ell) = \sum_{n=0}^{\infty} Cy_{n,ij} \exp\left(-\mathrm{j}\frac{2\pi n\ell}{L_{ij}}\right), \tag{2}$$

where Cx and Cy are complex amplitude, L is length of the contour.

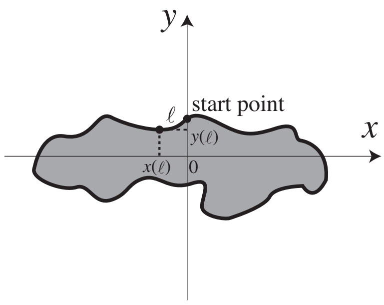

Figure 2: A sciced contour by i-th xy-plane. x-coodinate and y-coordinate are digitized with length along the contour from discretionary start point.

III. Spacial frequency of the n-th harmonic

For predicting a periodic size of roughness from equation (1) and (2), it is calculated from the wavelength of one cycle in n-th harmonic. spatial frequency of n-th harmonic fs_n is calculated as follow:

$$fs_n = \begin{cases} n/L_n & (n=1) \\ n/L_{n-1} & (n>1) \end{cases}, \tag{3}$$

where L_n is along distance of the reconstructed contour with up to the n-th harmonic.

IV. Amplitude of the n-th harmonic

Amplitude is calculated to each coordinate. Amplitude for x-coordinate is Ax, for y-coordinate is Ay, z-coordinate is Az, for example Ax of n-th harmonic is calculated as follow:

$$Ax_{n,ij} = \sqrt{Cx_{n,ij} \cdot Cx^{*}_{n,ij}} \tag{4}$$

V. summing up data

Spatial frequency and Amplitude can be calculated with procedure III and IV. Next, we will try to sum up these amplitudes in each coordinate.

combining octave band

Each contour line length L is varied , sampling width is varied, too. Therefore, amplitude data of a contour is combined in 1/6 octave band. The combined data is expressed as $x_{ij}(f_s)$, $y_{ij}(f_s)$ and $z_{ij}(f_s)$ in each coordinate.

summing up contours

All data is summed up with following equations in each coordinate. In this summation, the data lower than f_0 is excepted. Therefore, N_{fs} is number of amplitudes at each octave band frequency f_s.

$$\begin{aligned} x(f_s) &= \tfrac{1}{Nx_{fs}} \textstyle\sum_i \sum_j x_{ij}(f_s) \\ y(f_s) &= \tfrac{1}{Ny_{fs}} \textstyle\sum_i \sum_j y_{ij}(f_s) \\ z(f_s) &= \tfrac{1}{Nz_{fs}} \textstyle\sum_i \sum_j z_{ij}(f_s) \end{aligned} \tag{5}$$

x-, y- and z-coordinate spatial frequency characteristics are calculated by this procedure.

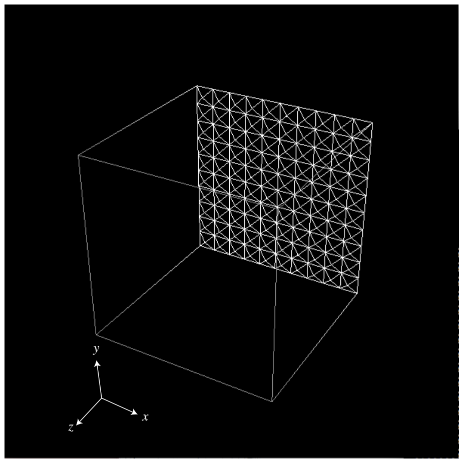

Figure 3: This wireframe is a object for verification test. It is 20 m on a side and has 2 m squared wedge diffuser which height is 50 cm. Diffuser is located on one wall only.

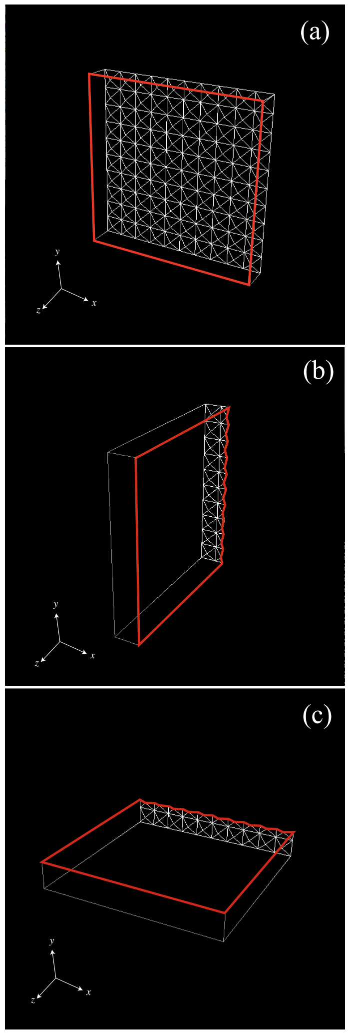

Figure 4: These figures are drawn the sliced contours shown red lines. Wireframes are resting for easy understanding. (a) is sliced by xy-plane, (b) is sliced by yz-plane and (c) is sliced by zx-plane.

VERIFICATION TEST

condition

A object for verification test is a cube 20 m on a side. It is installed wedge shaped diffuser on one plane, shown in Fig. 3. The height of wadge shaped diffuser is 0.25 m and The width of it is 2 m. Show in Fig. 4 is the instance cases of contours which are drawn with red line. Fig .4 (a) is scliced by *xy*-plane, (b) is done by *yz*-plane and (c) is done by *zx*-plane. Contours are picked out at 1 cm intervals of *xy*-, *yz*- or *zx*-plane, consequently 5997 contours are picked out.

Result

First, let us show Fig. 5. OverAll trend of characteristics is steadily decreasing. This characteristics are common to almost all objects. From this figure, we can see that x and y is mostly equal but z is not, this expression however is not easy to understand the character. Therfore, we tried to convert this result to easily understandable expression. Equation 5 is converted into the ratio of amplitude to wavelength (*RAW*), which is calculated by following equation.

$$\begin{aligned} RAW_x(f_s) &= 2f_s \cdot x(f_s) \\ RAW_y(f_s) &= 2f_s \cdot y(f_s) \\ RAW_z(f_s) &= 2f_s \cdot z(f_s) \end{aligned} \quad (6)$$

To focus attention on Fig. 6, the characiteristics of z is differ from that of x and y. This fact let us notice that some object is located at z-direction. The mainly different point from x and y is a peak at 0.5 (1/m). 0.5 (1/m) means that this object has periodical figure size of 2 m (inverse number of 0.5) at z-direction.

From this test, we can see that the periodical characteristics from three-dimensional object in each coordinate can be picked out using this method.

DISCUSSION

The purpose of this report is clarifying the method to express the three-dimensional room shape numerically. If we can do it, quantitative evaluation of rooms will become possible.

First, we chose the expansion method of elliptic Fourier descriptors. Because, it leads us easy to understand the periodical length of roughness and its amplitude of it which is included in a object. But there is no conclusive proof that elliptic Fourier descriptors has much advantage over spherical harmonics.

Second, the analysis procedure for three-dimensional shape using elliptic Fourier descriptor was mentioned. A object was sliced with flat planes vertical to each coordinate and picked out contour lines. contour lines are described with elliptic Fourier descriptors Eq. 1 and Eq. 2, since spatical frequency and amplitude was calculated Eq. 3 and Eq. 4.

Next, the many contours characteristics data was gathered and simplified for easy understanding. For gathering many methods can be thought, but Eq. 5 is likely easiest. Additionally, Eq. 6 proposed us more reasonable view, it tell us the distinctive spatial frequency easily. This method can be shown the characteristics in each coordinate like Fig. 5 and 6.

The sequence of prosedure give us the “spatial frequency characiteristics” like “frequency characiteristics” of sound. This paper will becomes the first step to reveal the numerical analysis of three-dimensional room shape characteristics. This method needs more verification with several shape objects.

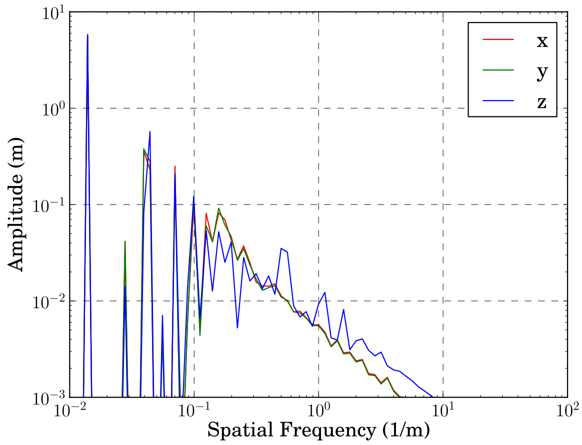

Figure 5: Spatial frequency characteristics of the object shown in Fig. 3. The longitudinal coordinate is Amplitude.

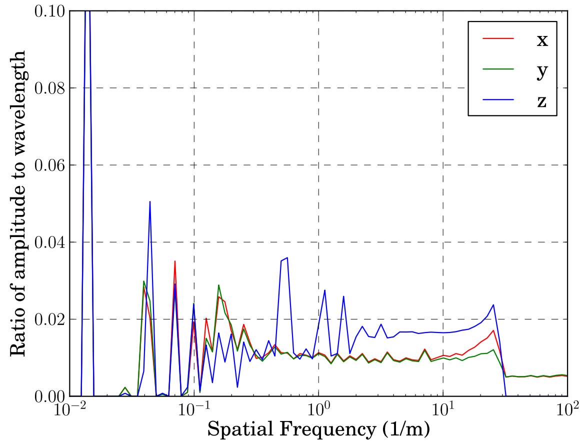

Figure 6: Spatial frequency characteristics of the object shown in Fig. 3. The longitudinal coordinate is Ratio of Amplitude to wavelength.

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