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# A numerical investigation of the sound intensity field in rooms by using diffusion theory and particle tracing

**Chiara Visentin (1), Vincent Valeau (2), Nicola Prodi (1), Judicaël Picaut (3)**

(1) Dipartimento di Ingegneria, Università di Ferrara, Italia. chiara.visentin@unife.it, nicola.prodi@unife.it
(2) Institut PPRIME, CNRS - Université de Poitiers - ENSMA, UPR 3346, France. vincent.valeau@lea.univ-poitiers.fr
(3) Laboratoire Central des Ponts et Chaussées, Bouguenais, France. judicael.picaut@lcpc.fr

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## ABSTRACT

This work focuses on the calculation of net intensity vectors in rooms, by using two different methods: a geometrical method, based on particle tracing, and the room-acoustic diffusion theory. The classical assumption for diffuse sound fields is that the net flow of reverberant energy at any location in room, i.e., the reverberant intensity vector, is null. The reverberant field in rooms with homogeneous dimensions and uniform absorption coefficients is usually considered as diffuse. This study focuses first on the spatial structure of the intensity vector field in such rooms, showing that, although the energy density variation is weak, an organized structure of energy flows can be observed throughout the room. In a second part, the net intensity field in more complex rooms, such as, for example, long rooms, will be investigated in the same way, for both diffuse and specular reflections, with the aim of providing numerical estimations of the sound intensity field and of the room-acoustic diffusion coefficient.

## INTRODUCTION

Over the last years a model for the prediction of the sound field inside room, based on a diffusion process, has been developed [1, 2]. Applying this method, both the stationary sound field and the temporal sound decay can be computed within single enclosures (with different geometrical characteristics) or coupled rooms [3, 4]: the obtained results are in good agreement with both experimental data and ray-tracing simulations. Moreover, although the diffusion model has been developed for rooms with diffuse reflecting boundaries, few successful attempts to apply the method to rooms with specular boundaries have been made [5] by comparing the results with ray-tracing simulations. At the basis of the model lays the assumption that the energy flow inside the room is proportional to the sound energy-density gradient: the proportionality constant is the diffusion coefficient, obtained as a function of the geometrical characteristics of the room.

The aim of this work is to investigate this basic equation of the diffusion theory (in the stationary state), comparing the model with a particle-tracing method. The two simulation tools derive from similar assumptions (e.g., modelization of the sound field through a cloud of sound particles) but the particle-tracing algorithm allows to calculate the sound intensity within the room, regardless of the energy density, and to simulate enclosures with mixed diffuse and specular reflections.

In the following sections, the basic equations of the diffusion theory for diffusely reflective rooms are summarized; then, the particle-tracing method and its implementation inside a numerical tool are detailed. Following this, the sound field in a proportionate room (where the sound field is supposed to be diffuse) is investigated, regarding both energy density and sound intensity. In the last section, a disproportionate (long) room is analyzed, with diffuse and specular reflecting boundaries: then a numerical estimation of the diffusion coefficient for both configurations is also provided.

## DIFFUSION MODEL

The diffusion model [1, 2] is based on the analogy between the diffusion of gas particles through spherical scattering objects (as originally studied in [6]) and the sound propagation inside an enclosed space with diffusively reflective walls, and allows to describe the sound energy density distribution inside rooms.

Following this analogy and assuming that in diffusion phenomena the rate of change of the involved quantities varies slowly as a function of time, the local energy-density flow $\mathbf{I}(\mathbf{r},t)$ can be approximated as the gradient of the reverberant energy density $w(\mathbf{r},t)$, as:

$$\mathbf{I}(\mathbf{r},t) = -D_{th}\nabla w(\mathbf{r},t), \quad (1)$$

where the variables $\mathbf{r}$ and $t$ denote respectively the position and the time. This energy flow can be interpreted as the sound intensity.

Starting from Equation (1) the sound energy density can be obtained as the solution of the following diffusion equation:

$$\frac{\partial w(\mathbf{r},t)}{\partial t} - D_{th}\nabla^2 w(\mathbf{r}) = F(\mathbf{r},t), \quad (2)$$

where $F(\mathbf{r},t)$ is a source-related term. $D_{th}$ is the theoretical diffusion coefficient that accounts for the geometrical characteristics of the room; its analytical expression is taken directly from the theory of diffusion for particles in a scattering medium:

$$D_{th} = \frac{\lambda c}{3} = \frac{4V}{S}\frac{c}{3}, \quad (3)$$

where $\lambda = 4V/S$ is the mean free path of the room, $c$ is the speed of sound, $S$ and $V$ are respectively the surface and the volume of the room.

In the context of this work only stationary sound sources are considered, and the stationary form of the diffusion equation is then solved (i.e., Equation (1) without the time-dependency term). It has been pointed out [2] that the solution of the diffusion equation is not valid in the near vicinity of the source, as diffusion is not yet established in this area. To overcome this problem, the Green function of the stationary diffusion equation can be investigated:

$$G(r) = \frac{W\exp(-\sqrt{\sigma/D_{th}r})}{4\pi D_{th}r}, \quad (4)$$

where $\sigma$ accounts for absorption at room boundaries and $r$ is the source-receiver distance.

Close to the source, a $1/r$ singularity dominates the solution of the diffusion equation in terms of energy density while, theoretically, in this region, the direct field should be dominant close to the source, with a $1/r^2$ decrease. As a consequence, for the case of rooms with proportionate dimensions, the following 'corrected' reverberant energy density is calculated :

$$w_c(r) = w(r) - \frac{W}{4\pi D_{th}r}, \quad (5)$$

in order to remove the non-physical $1/r$ singularity close to the source.

The net stationary acoustic intensity vector due to the reverberant field can then be estimated by using Equation (1) with $w_c(r)$ instead of $w(r)$. This operation has been shown to be necessary in order to perform estimations of the reverberant intensity vector that are coherent with the particle-tracing simulations, in the case of proportionate rooms.

## THE PARTICLE-TRACING SOFTWARE: SPPS

A numerical model for simulating the sound field within enclosures has been proposed in [7]. The developed algorithm, that is the kernel of the SPPS code, stems from the typical assumptions of geometrical acoustics and is based on the concept of *sound particles* originally suggested by Stephenson [8, 9].

The particle-tracing method models the sound field of a room thorough a multitude of elementary sound particles, propagating along straight lines at the speed of sound, without any mutual interaction and carrying an infinitesimal amount of energy. The sound energy density in the room can therefore be assimilated to the density of particles at each position,weighted by the energy of each particle. Since the model derives from geometrical acoustics, it can not deal with the undulating nature of the sound waves, thus the sound particle approach is applicable only in the high frequency range, where some physical characteristics of sound waves can be neglected. In this range, the notion of frequency is introduced via acoustic properties of the room surfaces.

The simulation principle of SPPS code is based on the tracking of sound particles, emitted from a source with sound power $W$, through the enclosure. Each particle, that carries an initial energy $e_{init} = W/N$ (where $N$ is the number of particles emitted from the source), propagates into the room until the collision with a wall or an obstacle, from which the particle energy can be absorbed or reflected (as a function of the absorption and scattering coefficients of the wall).

For all the simulations shown in this paper, an *energetic* approach has been employed, where at each collision the particle energy is weighted according to the surface absorption coefficient, while diffusion and reflection phenomena are modeled by successive drawing of random numbers, with a simulation procedure that can be assimilated to a Monte Carlo method.

Since SPPS is a numerical tool, in order to describe the sound propagation inside the room spatial and temporal discretizations are required. Temporal discretization is carried out calculating the position of each particle within the room every multiple of a constant time step $\Delta t$. Spatial discretization is performed both discretizing the propagation domain into a finite number of tetrahedral elements and working with punctual receiver of finite dimensions (as it is necessary to calculate the number of particles crossing the receiver position, a finite spherical volume $V_{rec}$ for the receiver has to be defined).

For each frequency band, the energy $E_{rec}(n)$ that arrives at the receiver at time step $n$ is equal to the sum of the amount of energy carried by each particle that crosses the receiver volume during the time step:

$$E_{rec}(n) = \sum_{i}^{N_0} \frac{W}{N}\varepsilon_i \Delta t_i, \quad (6)$$

where $N_0$ is the number of particles that pass through the receiver volume and $\Delta t_i$ is the amount of time that each particle spends within the receiver volume; this quantity can be also expressed as a function of the distance $l_i = c\Delta t_i$ covered by the particle within the receiver volume.

The energy density (in J/m$^3$) inside the receiver is therefore obtained as:

$$w_{rec}(n) = \frac{E_{rec}(n)}{V_{rec}} = \frac{W}{N}\frac{1}{V_{rec}}\sum_{i}^{N_0}\varepsilon_i\frac{l_i}{c}. \quad (7)$$

The particle tracing method allows also to calculate the net sound intensity vector at each receiver, defined as the sum of the amount of energy density carried by the particles that travel with velocity $\mathbf{v}_i$ (with norm $c$) through the receiver volume:

$$\mathbf{I}_{rec}(n) = \frac{W}{N}\frac{1}{V_{rec}}\sum_{i}^{N_0}\varepsilon_i l_i \frac{\mathbf{v}_i}{c}. \quad (8)$$

The vectors $\mathbf{I}_{rec}(n)$ can then be summed for all time steps, leading to the stationary net intensity vector.

## NUMERICAL RESULTS FOR ROOMS WITH HOMOGENEOUS DIMENSIONS

In this section numerical results are presented for rooms with proportionate dimensions, uniform absorption and having diffusely reflecting boundaries. Accordingly to the statistical theory, in this type of enclosures the reverberant sound field is expected to be diffuse, i.e., characterized by the presence of plane waves with random phase and uniform amplitude that arrive at each point of the room with equal probability from all directions. Therefore, according to this theory, inside the enclosure there should be no net flow of reverberant energy vector and the total net sound intensity at any internal point should be equal only to the direct intensity vector (which norm is $W/4\pi r^2$) [10].

A cubic room with dimensions $10 \times 10 \times 10$ m$^3$ (Figure 1) has been investigated, with an omni-directional sound source located in its center, radiating a constant sound power level $L_W = 100$ dB; the absorption coefficient of all the surfaces is $\alpha = 0.1$, while the scattering coefficient is $s = 1$ (diffuse reflecting boundaries). According to statistical theory the reverberant radius of the room is $r_c = 1.15$ m. Line 1 and Line 2 in Figure 1 are the lines along which sound pressure and intensity levels are calculated.

Particle-tracing simulations have been carried out with $N = 5 \cdot 10^6$ particles (as particles reflection is depicted through random drawing, the best results can be obtained when the number of drawing is high, thus when $N$ is high), a time step $\Delta t = 0.002$ s and a total simulation time $T = 3$ s.

The numerical simulations for the diffusion model have been performed using a software based on the finite element method (FEM), where a cubic domain and a spherical subdomain of radius $r_s = 0.1$ m (centered in the source position) have been defined and meshed with 36503 quadratic Lagrange-type elements. As pointed out in [2], in order to solve the diffusion equation, the size of the meshing elements is no longer related to the minimum wavelength involved in the analysis but depends only on the mean free path of the room (elements maximum size should be much smaller than one mean free path).

The energy density gradient has been estimated for both simulation tools applying central finite differences, with a $4^{th}$ order accuracy.

All the acoustic quantities have been evaluated over a quarter of the horizontal plane that contains the source (grey zone in Figure 1), for a regular grid of receiver points (grid step: 0.5 m).

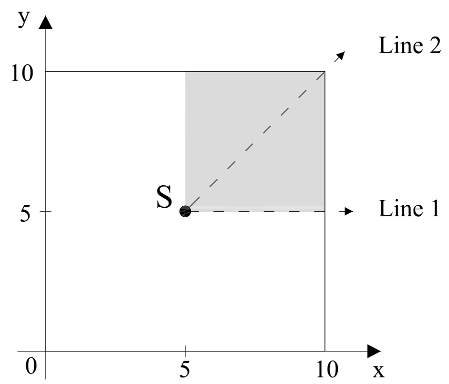


Figure 1: Upper view of the $10 \times 10 \times 10$ m$^3$ cubic-room case study with the source position S and the two dashed lines along which sound pressure and intensity levels are evaluated.

### Energy density and sound intensity

From the numerical solution for $w(\mathbf{r})$ the sound pressure level (SPL) at each position can be estimated as:

$$\mathrm{SPL}(\mathbf{r}) = 10\log\left(\frac{w(\mathbf{r})\rho c^2}{p_{ref}^2}\right), \quad (9)$$

where $p_{ref} = 2\cdot 10^{-5}$ Pa and $\rho = 1.204$ kg/m$^3$ (air density).

In order to compare the results obtained with the two simulation tools, only the reverberant part of the energy density is taken into account, evaluated as:

- the local total acoustic energy density minus $w_{dir}(r) = W/4\pi c r^2$ for particle-tracing simulations,
- the corrected energy density $w_c(r)$ for diffusion model calculations.

In Figure 2 the predicted reverberant SPL is compared with that given by Sabine's statistical theory showing that all the models lead to similar results (with relative differences lower than 0.2 dB). The SPL calculated from the solution of the diffusion equation, $w(r)$, is also plotted for comparison: it demonstrates the need for removing the $1/r$ singularity and working with the corrected energy density, $w_c(r)$ (Equation (5)). It is also worth noticing that while the statistical theory predicts a constant reverberant field, both the simulation models show a slight decrease of the SPL value in the positions closest to the boundaries, that should mean, a non-null net flow of reverberant energy within the room (as $\mathbf{grad}(w)$ is not null).

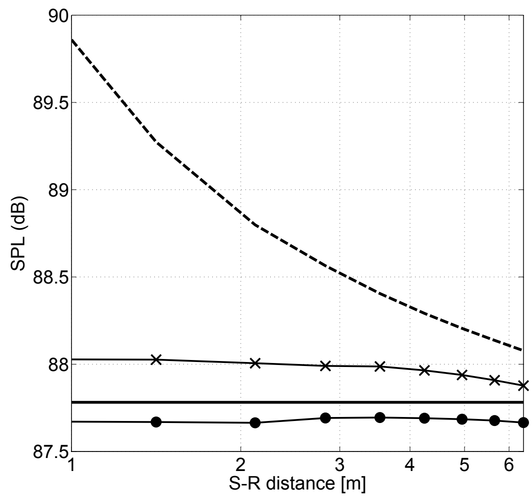


Figure 2: Sound pressure level (SPL) of the reverberant field inside a $10\times10\times10$ m$^3$ cubic room with $\alpha = 0.1$ and diffuse reflecting boundaries. The results are plotted along Line 2: particle-tracing simulation ($\times$), Sabine's statistical theory (solid line); (- -) and ($\bullet$) indicate respectively the SPL calculated with the diffusion model from $w(\mathbf{r})$ and $w_c(\mathbf{r})$.

The total sound intensity level (SIL) is calculated by using :

$$\mathrm{SIL}(\mathbf{r}) = 10\log\left(\frac{I(\mathbf{r})}{I_{ref}}\right), \quad (10)$$

where $I(\mathbf{r})$ is the norm of the calculated intensity vector and $I_{ref} = 10^{-12}$ W/m$^2$.

The predicted total sound intensity level along Line 1 and Line 2 is plotted in Figure 3 together with the SIL decay of the direct field. According to statistical theory, the estimated SIL should be equal to the SIL of the direct field. However, Figure 3 indicates that some differences arise. Both numerical methods show that the total SIL tends to be higher (for Line 2) or lower (Line 1) than the SIL of the direct field in the vicinity of the room boundaries. The difference reaches about 2 dB in the considered case. This may be due to the fact that close to the walls, the norm of the net intensity vector of the reverberant field reaches significant amounts compared to the norm of the direct intensity vector. This observation can be better explained by considering the reverberant intensity vector field.

### Intensity vector patterns

In Figure 4 the spatial distribution on the horizontal plane of reverberant intensity vectors is displayed, as obtained from both particle-tracing and diffuse theory simulations: vectors define a well oriented path from the sound source toward the edges of the room, from which they are attracted. Both models provide the same oriented pattern, in spite of some discrepancies between the vectors magnitude.

In the corner of the room, close to the edge, the reverberant intensity vectors are oriented in the same direction than the direct field, so that the total SIL is higher than the direct field SIL (about 2 dB more), as shown in Figure 3b. Conversely, close to the wall the reverberant intensity vectors are oriented in the opposite direction to the direct field intensity vector; as a consequence, the total SIL in this area is lower than the direct field SIL (Figure 3a).

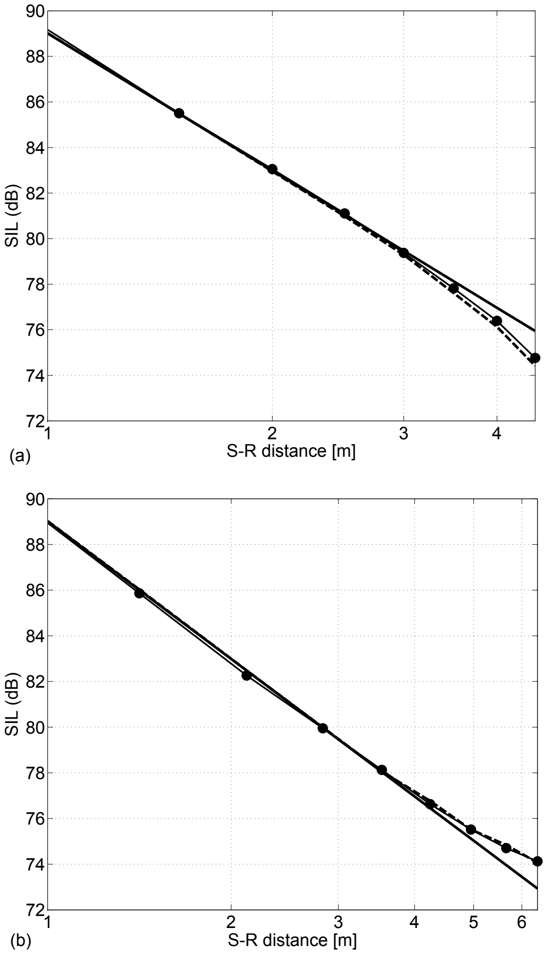


Figure 3: Total sound intensity level (SIL) as a function of the source-receiver (S-R) distance, including both direct and reverberant field, inside a $10 \times 10 \times 10$ m$^3$ cubic room with $\alpha = 0.1$. Results plotted along Line 1 (a) and Line 2 (b): particle-tracing simulation (- -), diffusion theory (•), theoretical value of the direct field (solid line)

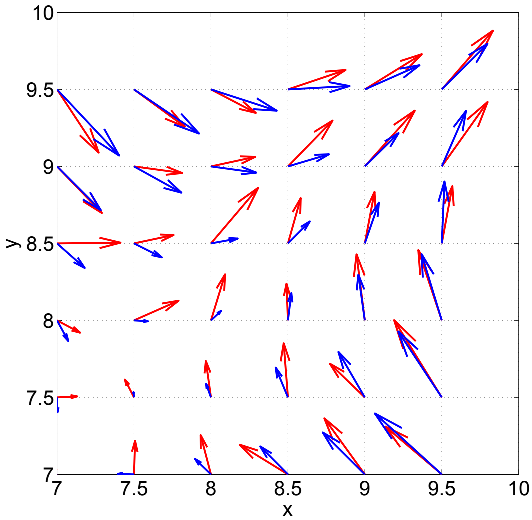


Figure 4: Reverberant intensity vector distribution over the horizontal plane in a cubic room $10 \times 10 \times 10$ m$^3$ with $\alpha = 0.1$ and diffuse reflecting boundaries. Particle-tracing simulation (red arrows) and diffusion theory (blue arrows).

Finally, a numerical simulation of a sphere with radius $R = 5$ m has been carried out employing the diffusion theory model, in the aim of demonstrating the effect of edges on the reverberant intensity vectors distribution. The sound source is located in the center of the sphere and the absorption coefficient is $\alpha = 0.1$; within the FEM solver the domain has been meshed with 62614 quadratic Lagrange-type elements. The acoustic quantities have been evaluated over the horizontal plane that contains the sound source: because of symmetry, the results would be the same on each plane of the domain containing the sphere center. In Figure 5 the total SIL is represented over a line from the source to the boundary, showing that in such a room the reverberant energy density flow is null all over the enclosure: the total sound intensity at any internal point is equal to its direct component. This confirms that the edges of the room tend to 'attract' the reverberant intensity vectors.

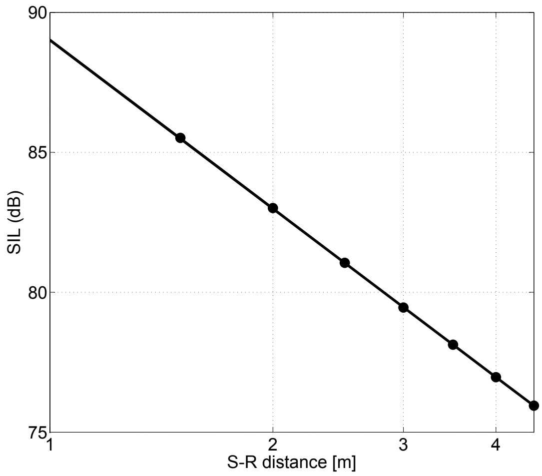


Figure 5: Total sound intensity level (SIL), including both direct and reverberant field, as a function of the source distance inside a sphere with radius $R = 5$ m and $\alpha = 0.1$: diffusion theory (•), theoretical value of the direct field (solid line)

## NUMERICAL RESULTS FOR LONG ROOMS

In this section some results for the energy flow inside a disproportionate room are presented: the examined enclosure is a long room with dimensions $4 \times 4 \times 40$ m$^3$ and a uniform absorption coefficient $\alpha = 0.1$. The omni-directional sound source is located two meters from one of the extremities of the room, in the center of the cross-section and emits a constant sound power level $L_W = 100$ dB.

Particle-tracing simulations have been carried out with $N = 5 \cdot 10^6$ particles, a time step $\Delta t = 0.002$ s and a total time $T = 2$ s; simulations with diffuse ($s = 1$) or specular ($s = 0$) reflecting boundaries have been performed. For the numerical solution of the diffusion equation with a FEM-based software the domain has been meshed with 6114 quadratic Lagrange-type elements.

The energy density gradient has been estimated for both simulation tools applying central finite differences, with a $4^{th}$ order accuracy.

Given the geometrical characteristics of the room it is reasonable to assume that the energy density varies mainly along the length of the room, i.e., it is quite constant over the cross-section. So, all the acoustic quantities have been investigated along the longest dimension of the room, from the source to the end wall. Moreover, inside a long room, except for positions very close to the sound source where the sound field is dominant, the total sound field is everywhere coincident with its reverberant part.

### Energy density and sound intensity

As pointed out for example in [11], because of the dissimilar dimensions, in a long room the sound field is not diffuse but the SPL decreases continuously along the length (as shown in Figure 6).

Close to the end wall of the room (about 10 m from the surface), the decay experiments a change and the SPL decreases with a lower slope. The comparison between particle-tracing (with diffuse boundaries) and diffusion results shows how the latter method can not properly model this wall effect, resulting in an increasing difference between the two SPL curves that, near the end wall, becomes almost equal to 4 dB.

Concerning the effects of the surface reflection law on the sound energy density, it appears from the SPL curves, that the total amount of energy within the long room with diffuse reflecting boundaries is lower than with geometrical reflecting boundaries. This is the consequence of the bigger probability that, under diffuse reflections, the sound particles have to hit a wall and then lose consequently a part of their energy [11].

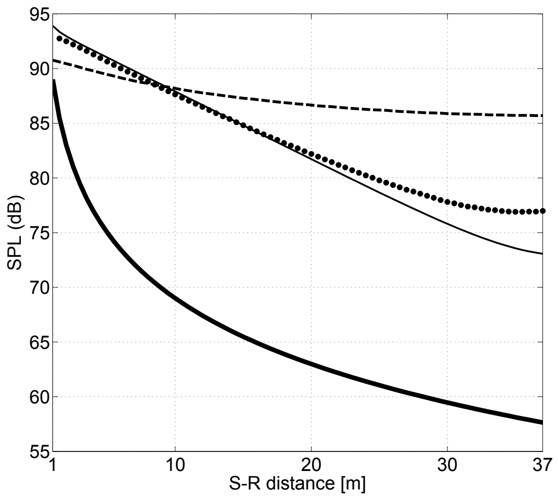


Figure 6: Sound pressure level (SPL) of the sound field inside a $4\times4\times40$ m$^3$ long room with $\alpha = 0.1$, as a function of the source-receiver distance (S-R distance): diffusion theory (thin solid line), particle-tracing simulations for the room with diffuse (●) and specular (- -) boundaries, direct field (thick solid line).

In Figure 7 the sound intensity level is represented, showing again a discrepancy between results predicted by the two simulation tools for the long room with diffuse reflecting boundaries. Moreover, it can be observed that the sound intensity is always higher in long room with specular reflecting boundaries, even if, according to Equation (1), the higher slope of the SPL decay curve should provide a higher SIL (as this involves a higher value of the norm of $\mathbf{grad}(w)$). This apparent contradiction can be explained by investigating the local diffusion coefficient inside the room.

### Numerical estimation of the diffusion coefficient in long rooms

The theoretical diffusion coefficient of an enclosure can be calculated by using Equation (3), for a room with perfectly diffuse boundaries. It is supposed to be constant over the whole room.

Starting from the output data of the particle-tracing model, a numerical local estimation $D_{est}(r)$ of the diffusion coefficient within the long room can be carried out, for diffuse or specular

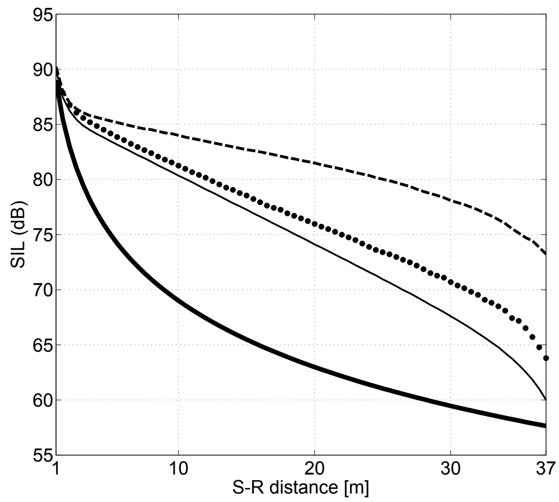


Figure 7: Sound intensity level (SIL) of the sound field inside a $4\times4\times40$ m$^3$ long room with $\alpha = 0.1$: diffusion theory (thin solid line), particle-tracing simulations for the room with diffuse (●) and specular (- -) boundaries, direct field (thick solid line).

reflections. As diffusion concerns only the reverberant part of the energy density, $D_{est}(r)$ has been computed as:

$$D_{est}(r) = \frac{\|\mathbf{I}_{rev}(r)\|}{\|\nabla w_{rev}(r)\|}, \quad (11)$$

where $\mathbf{I}_{rev}$ and $w_{rev}$ are the simulated intensity and energy density after subtracting the contribution of the direct sound field.

The ratio between the estimated and the theoretical diffusion coefficient value is represented in Figure 8 for the two types of reflection laws: a clear increasing trend of the curves can be observed, that probably explains the erroneous prediction of the energy density with the diffusion model (based on the constant value of $D_{th}$). The slope of the curve for the room with specular reflecting boundaries is almost 10 times higher than with diffuse reflecting boundaries. The high diffusion coefficient for long rooms with specular reflecting boundaries is in agreement with the results of reference [5]; this can explain the higher SIL for these rooms, following Equation (1). In both cases, in the region near the end wall, the curve slope increases rapidly, which may probably be a consequence of reflection effects on the end wall.

## CONCLUSIONS

In this paper the validity of the room-acoustic diffusion theory has been investigated, by comparison with a particle-tracing method. Both proportionate and disproportionate (long) rooms have been simulated, in order to study the spatial distribution of energy density and sound intensity inside enclosures.

Within a proportionate room with uniform absorption, according to statistical theory, a diffuse reverberant field should be expected, with a constant energy density in each internal point of the room and, as a consequence, a null reverberant sound intensity vector. Instead, with both simulation tools, a slight variation of the reverberant energy density inside the room has been found, together with an organized path of the reverberant intensity vectors that, in the region close to the boundaries leads to local deviations of the total SIL compared to the direct one. This effect is a consequence of the presence of room edges, thus can not be observed inside a spherical room. Inside proportionate rooms, only a part of the solution provided by the diffusion model (the part that not experiments a $1/r$ singularity near the source) should be taken into account in order to obtain estimation of the stationary reverberant intensity vectors coherent with

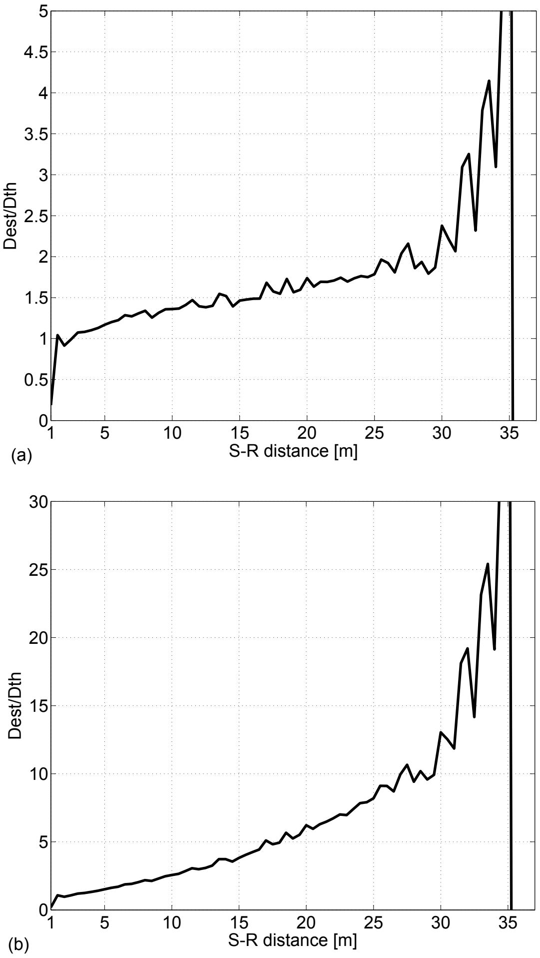


Figure 8: Ratio beteween the estimated and the theoretical diffusion coefficient inside a $4 \times 4 \times 40$ m$^3$ long room with $\alpha = 0.1$ and diffuse (a) or specular reflecting boundaries (b).

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that obtained with particle-tracing simulations.

Within the long room, simulations have been carried out for both specular and diffuse reflecting boundaries, showing how sound energy density and intensity are strongly affected by the reflection law of the walls: inside long room with specular boundaries the two acoustical quantities are always higher than inside a long room with diffuse boundaries. An estimation of the real value of the diffusion coefficient has been then provided, starting from particle-tracing results: in long rooms this coefficient is no longer a constant (as theorized by the diffusion model) but increases with the distance from the source, with a slope which is a function of the enclosure scattering coefficient.

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