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Measurement of overtone frequencies of a toy piano and perception of its pitch

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ABSTRACT

The pitches produced by toy pianos are sometimes perceived to be inaccurate by listeners, some of whom report perceiving the perfect fifth above the nominal note of the pressed key. To investigate these assertions, the overtone frequencies of a toy piano were measured in a frequency range of no more than the eighth harmonic and below 5 kHz, which are considered to be important to the pitch perception of the human auditory system. Time-frequency and time-intensity analyses of the overtones revealed periodic variation in frequency and amplitude, which might be caused by two close vibrational modes. The pitch of a toy piano was found to be tuned to the frequencies of the overtones corresponding the third and fifth harmonics. No fundamental frequency component was observed. The overtone of 1.5 times the missing fundamental frequency appeared above G4. In addition, the overtone of 0.5 times the missing fundamental frequency appeared above G5. The sounds consisting of the prominent overtones corresponding to the 1.5th and 3rd harmonics can be perceived as the perfect fifth above the nominal note. The pitch of the toy piano was perceived as inaccurate in part because the frequencies of the overtones corresponding to the third and fifth harmonics deviated by –4 to +24 cents and +3 to +33 cents from equal temperament, respectively.

INTRODUCTION

The pitches produced by toy pianos are sometimes perceived to be inaccurate by listeners. In particular, some Internet users have noted that some toy pianos have slightly inaccurate tuning. In addition, some people perceive duplex pitch corresponding to frequencies of the nominal note and a perfect fifth above the nominal note in certain registers.

We conducted an informal listening test using a toy piano, the sound source of which consisted of steel bars. Ten listeners with experience playing musical instruments (including six absolute-pitch listeners) evaluated the pitches produced by the toy piano compared to the pitches produced by a grand piano. The nominal pitch was perceived by four listeners including one absolute-pitch listener. The pitch of the perfect fifth above the nominal note was perceived by four listeners including three absolute-pitch listeners. The other two listeners experienced duplex pitch perception, which depends on the listener's attention. The pitch of the toy piano was formed by the pitch of the missing fundamental, since no fundamental component exists and the frequencies of the overtones are not harmonic. This might cause inaccurate and duplex pitch perception.

Although designed primarily for use by children, toy pianos are sometimes played by professional musicians. In this study, the frequencies and amplitude of the overtones of a toy piano below 5 kHz and below the eighth harmonic were measured, which are considered to contribute the pitch of the missing fundamental. In addition, the relationship between theoretical transverse vibration and the frequencies of the overtone and perceived pitch of the toy piano were considered.

SOUND MECHANISMS

The sound source of the toy piano measured in this study consists of steel bars inserted into a steel beam. The bars are approximately 3 mm in diameter. From precise measurements with an electronic vernier micrometer, the cross section of the bar was found to be not a perfect circle but rather a distorted circle, the diameter of which ranged from 2.85 to 3.01 mm.

The base of the bars and the beam are shown in Fig. 1. The base of the bars are scraped to reduce the diameter by approximately 1 mm over a length of 10 to 15 mm, but the the shape and thickness of the scraped sections of the bars are slightly irregular among the bars. Delicate pitch tuning for each tone can be achieved by the degree of scraping. A note is sounded by pressing a key that causes a plastic hammer to strike the bar 30 mm from the clamped end. This mechanism is simple, and often the hammer strikes the bar twice when a key is pressed.

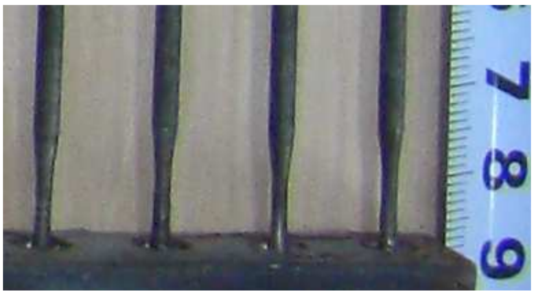

Figure 1: Base of bars.

ANALYSIS

Audio recording

An audio recording was made on the floor of a musical instrument store on an IC recorder (Sharp ICR-PS380RM). The recorder was manually held 15 cm above the keyboard, with the microphone directed toward the key that was pressed. The recording settings were linear PCM, 44.1 kHz sampling, 16-bit quantization, and stereo. Keys C4 to F6 in the chromatic scale were each recorded twice. The following analysis used only the left-channel signal.

Frequency analysis of the C4 tone

The waveforms of the two recordings of the C4 tone are shown in Fig. 2. Figure 3 shows the power spectra that were calculated from the waveforms by applying a Hanning window to the initial 6145 to 8192 samples from the absolute maximum amplitude of the waveform. The vertical axis of Fig. 3 shows the relative spectral level, where the 16-bit full-scale amplitude of a pure tone is taken as 0 dB. As show in Fig. 3, overtones exist up to around 20 kHz.

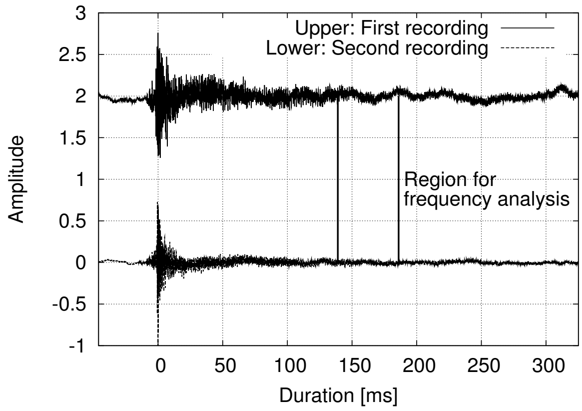

Figure 2: Waveforms of two recordings for the C4 tone.

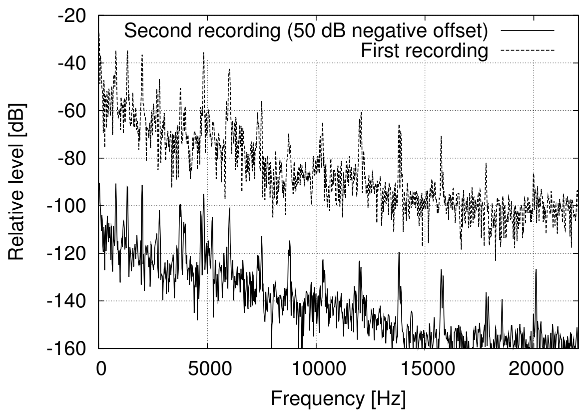

Figure 3: Power spectra of the C4 tone from two recordings.

In this paper, the overtones with frequencies below 5 kHz and below the eighth harmonic frequency are investigated, since other components do not contribute to the perception of the missing fundamental frequency [Ritsma, R. J. (1962)] [Ritsma, R. J. (1963), Moore, B. C. J. et al. (1985)]. For example, the components below the frequency of the eighth harmonics of C4 (262 Hz), were analyzed, that is, below 2096 Hz. The relative levels of the overtones below 2096 Hz of the two spectra in Fig. 3 are nearly the same. The same tendency was observed for other tones recorded twice. These results indicate that the sounds played on the toy piano were sufficiently reproducible.

To estimate the effects of background noise, the maximum power spectrum of the noise obtained from 2048-point half-overlapped FFTs was acquired in 1-s intervals, as shown in Fig. 4. This figure indicates that the components above –50 dB and below 600 Hz and those above –60 dB at other frequencies were the overtones of the toy piano sound.

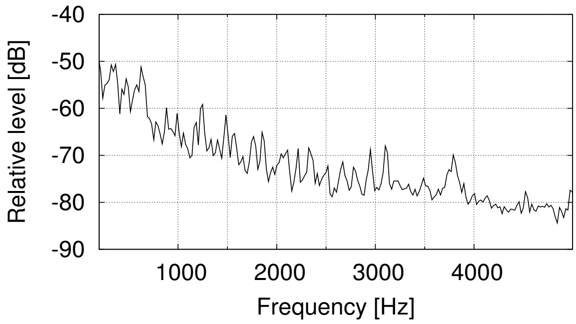

Figure 4: Maximum spectrum of background noise in 1-s intervals.

Spectrogram view of overtones

Figure 5 shows a spectrogram of the chromatic scale from C4 to F6 played on the toy piano sound. The prominent overtones of the notes below F4 approximately correspond to the third, fifth and eighth harmonics. Overtones of 1.5 times the missing fundamental frequency appeared above G4. In addition, the overtones of 0.5 times the missing fundamental appeared above G5.

Time-frequency and time-intensity analyses

To clarify the temporal variation of the frequency and intensity of the overtones, the 8192-point FFT was calculated for the 2048-point Hanning-windowed waveform padded by 6144-point zeros at the end. The starting point of the FFT was the sample that exhibited the absolute maximum amplitude. FFT was calculated iteratively by shifting the waveform by 1024 points. Amplitude and frequency of the overtones were calculated by 3-point interpolation of the nearest points of the spectral peak.

Figure 6 shows amplitude envelopes for each overtone. Figure 7 shows the deviation from the mean frequency for each overtone. These figures show that the amplitude and frequency of each overtone fluctuate in opposite phase. These fluctuations are considered to be a beat with period of 100 to 150 ms, that is, 7 to 10 Hz. Such fluctuations were also observed for other notes. The beat might be caused by two vibrational modes that are close in frequency. These two vibrational modes are the result of the slightly distorted shape of the bars, which deviate from a perfect circle.

Frequency analysis of overtones.

To examine the accuracy of the tuning of the toy piano, the frequency deviation of each overtone from the harmonic frequencies of the fundamental frequency corresponding to the nominal note was precisely measured by calculating the 8192-point FFT with Hanning windowing and 3-point interpolation. Table 1 shows the deviation from the correct harmonic frequencies, that is, 0.5, 1.5, 3, 5 and 8 times the nominal fundamental frequency, in cents for the 13 dominant notes.

As shown in the table, the frequency deviation ratios exhibit no common trend among the overtones for all notes. In other words, practical frequency ratios of the overtones to the fundamental frequency depend on not only the inherent properties of the steel bar but also the tuning work of scraping the base of the bar, as shown in Fig. 1.

The mean deviation of the third harmonic, which is the most dominant contributor to the pitch of the missing fundamental

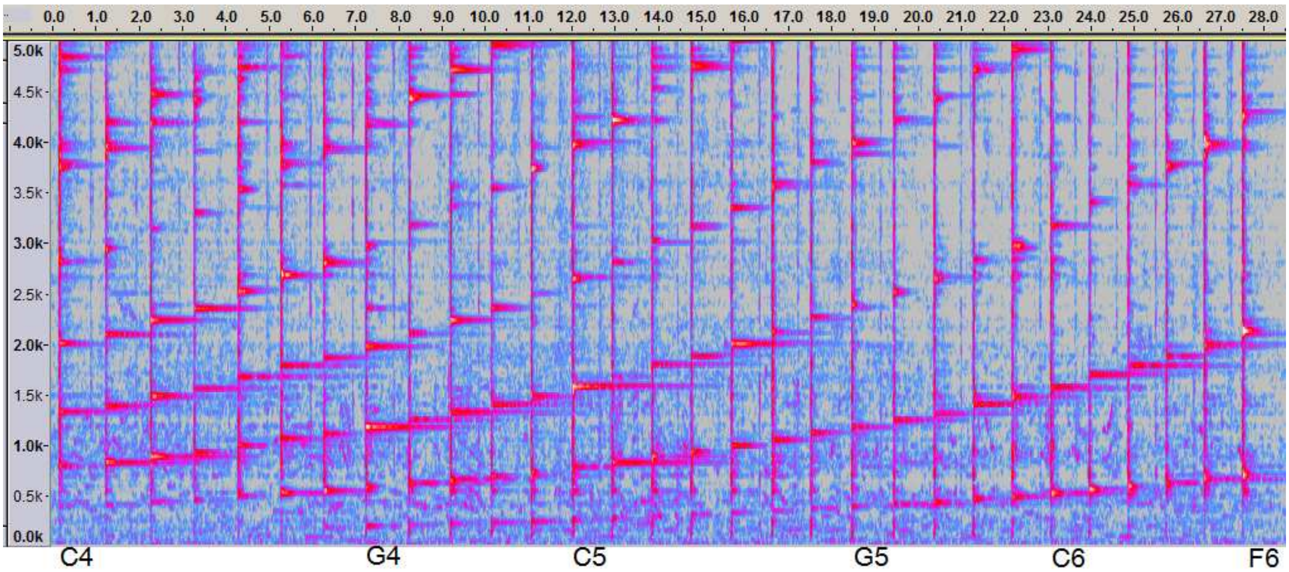

Figure 5: Spectrogram of chromatic scale played on toy piano.

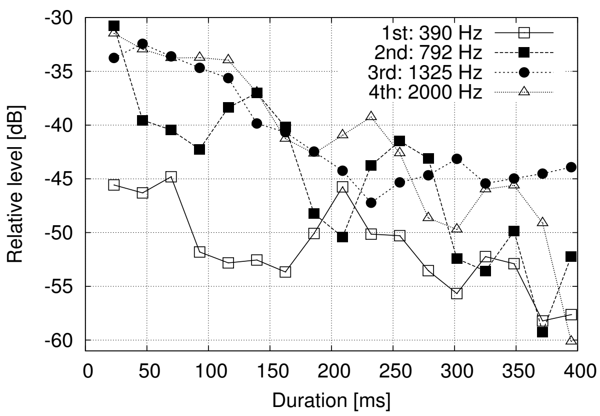

Figure 6: Amplitude envelopes for each overtone.

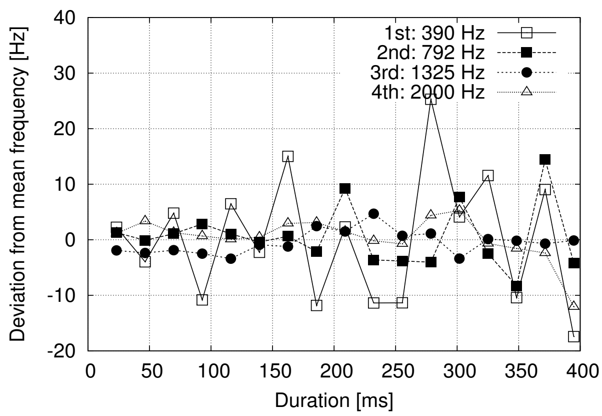

Figure 7: Deviation from mean frequency for each overtone.

frequency [Moore, B. C. J. et al. (1985)], was +10 cents. Accordingly, it may be better to consider the reference pitch of A4 as 442.5 Hz, not as 440 Hz. The mean deviation of the fifth harmonic was 8 cents higher than that of the third. The mean deviation of the fifth harmonic is slightly higher than the inharmonicity of the piano tone [Járveláinen, H., et al. (2000)]. However, the pitch deviation of the third harmonics ranged from –3 to +21 cents and those of the fifth ranged from +3 to +33 cents. The frequency difference limen for a pure tone is approximately 5 cents in the most sensitive frequency region around 1 kHz [Moore, B. C. J. (1973)]. Although the frequency difference limen for pitch in a melody can be large in comparison with a pure tone, Table 1 indicates the possibility that the missing fundamental frequency of the toy piano may be perceived to be incorrect.

Intensity analysis of overtones

The intensity of the overtones was measured in the course of the frequency measurements described in the previous section. Figure 8 shows the relative amplitude in dB for each overtone as a function of the nominal note . The results may be affected to some extent by the directionality of the microphone used for the recording and the radiation pattern of the toy piano; however, Fig. 8 shows that the intensity of the overtones corresponding to the 1.5th and 3rd harmonics were relatively strong for the notes from F4 to F5. In addition, the intensity of the overtones corresponding to the 0.5th harmonic become relatively strong for notes above G5.

DISCUSSION

Dominant overtones and pitch perception

As shown in Fig. 8, the intensity of the overtones corresponding to the 1.5th and 3rd harmonics were relatively strong for the notes from F4 to F5. If a listener perceives these overtones as the first and the second harmonics, the pitch would then be perceived as the perfect fifth above the nominal note; which in fact agrees with the perception of some listeners.

In addition, the intensity of the overtones corresponding to the 0.5th harmonic become relatively strong for notes above G5. It appears that a new low partial tone gradually appears, such as a Shepard tone [Shepard, R. N. (1964)], when the upward chromatic scale is played on a toy piano.

Transverse vibration of bar

The sound source of the toy piano is steel bars inserted into a steel beam. The boundary conditions of the bar are clamped and free. Although the shape of the cross section of the bar was not measured, the shape is assumed to be an ellipse that

Table 1: Deviation from correct harmonic frequency in cents. Asterisk (*) denotes the amplitude of an overtone below maximum noise level. N.F. denotes that no overtone was found. A dash (—) denotes frequencies out of the frequency range (above 5 kHz).

	Nominal note and its frequency in equal temperament.				
order	C4 (262)	E4 (330)	F4 (349)	G4 (392)	A4 (440)
0.5	N.F.	N.F.	N.F.	7.4	9.03
1.5	*–16.7	–5.2	2.2	0.43	–53.6
3	18.7	16.7	22.3	–0.2	4.7
5	19.3	23.6	35.0	8.4	22.1
8	–77.7	–82.2	–71.6	–92.4	–84.0
	C5 (523)	E5 (659)	F5 (698)	G5 (784)	A5 (880)
0.5	16.0	–0.4	–10.0	–4.4	–30.6
1.5	–9.4	1.3	–2.4	3.1	–21.8
3	3.1	16.7	19.8	23.9	–3.9
5	17.3	22.6	27.4	23.9	5.4
8	–94.2	—	—	—	—
	C6 (1047)	E6 (1319)	F6 (1397)	Average	S.D.
0.5	–9.6	5.7	2.9	–1.4	13.2
1.5	0.7	12.9	24.1	–4.0	19.2
3	15.2	5.7	17.7	12.3	9.2
5	—	—	—	20.5	8.6
8	—	—	—	–83.7	8.6

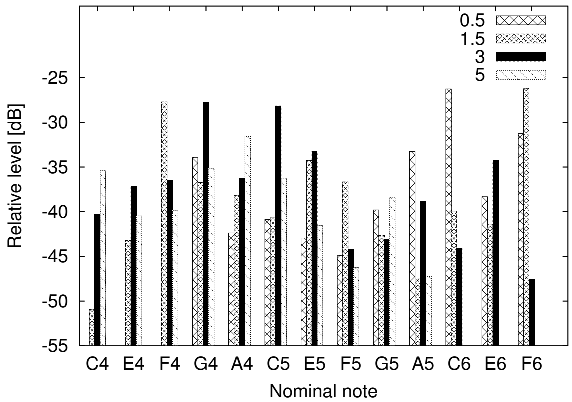

Figure 8: Relative amplitude of each overtone as a function of nominal note.

is defined by the measured diameters of 3.00 and 2.86 mm at angles of 90 degrees difference. The shape of the cross section is also assumed to be constant from one end to the other, while the base of the actual bar is scraped to remove approximately 1 mm of material over the section. Under these assumptions, the frequency of this type of steel bar is given by the following equation:

$$f_i = \frac{1}{4\pi}\left(\frac{\lambda_i}{l}\right)^2 \sqrt{\frac{Ea^2}{\rho}}, \tag{1}$$

where E is Young's modulus $E = 2\text{E}{+}11$ N/m^2, L is the length of the bar, ρ is density $\rho = 7.87\text{E}{+}3$ kg/m^3, a is the major or minor radius of the ellipse, and λ_i is a constant given by $\cos(\lambda_i)\cosh(\lambda_i) + 1 = 0$.

Table 2 shows the calculated and measured frequencies of the overtones. Two theoretical frequencies are calculated for the transverse waves for the major and minor axes, because the two simultaneous vibration modes at close frequencies are considered to be the cause of the beat found in the time-frequency and time-intensity analysis in the previous section.

Table 2: Calculated and measured transverse vibration frequencies.

C4 ($L = 0.289$ m, $a = 1.43\text{E–}3$ or $1.50\text{E–}3$ m)				
		Calculated		
Order i	λ_i	Minor [Hz]	Major [Hz]	Measured [Hz]
4	10.996	842.1	871.1	793.4
5	14.137	1391.9	1439.9	1322.8
6	17.279	2079.3	2151.1	2001.2
C5 ($L = 0.213$ m)				
2	4.694	282.5	292.2	264.1
3	7.855	791.1	818.4	780.6
4	10.996	1550.2	1603.7	1572.6
5	14.137	2562.4	2650.7	2642.6
6	17.279	3827.9	3959.9	3964.3
C6 ($L = 0.155$ m)				
2	4.694	533.47	551.86	520.4
3	7.855	1493.9	1545.4	1570.4
4	10.996	2927.5	3028.4	3167.3

These results suggest that the theoretical transverse frequencies are roughly consistent with the experimental results. However, the calculated frequency spacing of the overtones is smaller than the measured value. This discrepancy in the frequencies may be caused by the scraping of the base of the bars.

The difference between the two calculated frequencies, that is, the beat frequency, is generally higher than the actual beat frequency measured in the time-frequency and time-intensity analyses. This may be caused by the scraping of the base of the bars or the non-uniform diameter of the bars. Detailed measurements of the shape of the base and the diameters of the bars are required in order to simulate and predict the frequencies of the overtones.

In Fig. 8, the order of the dominant overtone decreases as the fundamental frequency increases. This trend is described by the relationship between the striking position of the hammer and the antinodal positions of the bars. Figure 9 schematically shows the transverse vibrations that dominate over each C4, C5, and C6 tone. Figure 9 reveals that striking close to the first antinodal point of the bar gives rise to low-order vibrational modes in the short bars. Furthermore, the lower frequency overtones appear as the frequency of the note increases, that is, as the length of the bar decreases (Fig. 5).

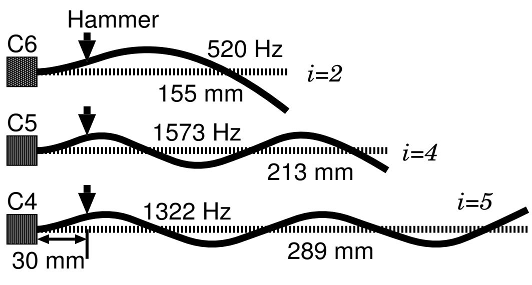

Figure 9: Schematic of transverse vibrations observed for C4, C5 and C6 tone.

SUMMARY

Time-frequency and time-intensity analyses of the overtones revealed periodic variation in frequency and amplitude. This might be caused by two close vibrational modes. The pitch of a toy piano was found to be tuned by the frequencies of the overtones corresponding to the third and fifth harmonics. The overtone of 1.5 times the missing fundamental frequency appeared above G4. In addition, the overtones of 0.5 times the missing fundamental appeared above G5. The sounds consisting of the prominent overtones corresponding to the 1.5th and 3rd harmonic can be perceived as the perfect fifth above the nominal note. The pitch of the toy piano is perceived to be inaccurate in part because the frequencies of the overtones corresponding to the third and fifth harmonics deviated by –4 to +24 cents and +3 to +33 cents from equal temperament, respectively.

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