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Experimental investigation of particle clumping position on circular plates using acoustic streaming generated by ultrasonic vibrations

Rito Mijarez (1), Jeremy J. Hawkes (2) and Pramukh N Jayassekera(3)

(1) Gerencia de Control e Instrumentación, Instituto de Investigaciones Eléctricas, Calle Reforma 113, Col. Palmira, C.P. 62490 Cuernavaca, Morelos, México
(2) School of Chemical Engineering and Analytical Science, The University of Manchester, PO Box 88, Sackville Street, Manchester, M60 1 QD, UK
(3) DSTL Chemical and Biological Sciences, Bldg 06, Porton Down, Salisbury, Wiltshire, SP4 OJQ, UK

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ABSTRACT

Acoustic streaming induced by ultrasonic vibrations on circular plates in order to control particle clumping position is experimentally investigated. Three-dimensional Finite Element modelling was employed to predict the acoustic streaming fields produced on circular plates of different thickness. Experimental validations were carried out using glass beads on the surface of the plates, a stepped horn and a piezoelectric bolted 40 kHz transducer. The particles on the plate always accumulated at the displacement nodes, and the agreement between frequencies of models and experiments was very high (<2% frequency error). Particle levitation and formation of clumps was attained by positioning a reflector above the circular plates to obtain resonance in the air. Consequently, the circular plate's vibration patterns with patterns of streaming and clump formation were adequately correlated.

INTRODUCTION

Acoustic streaming is the formation of a steady rotational airflow in the acoustic field, which may result from either vibrations of a solid body adjacent to a fluid at rest or from an acoustic standing wave generated in a fluid adjacent to a solid wall. It has been known that acoustic streaming is induced by two factors: spatial attenuation of a wave in a free space and the friction between a medium and a vibrating object [1]. As sound waves propagate, they experience absorption and scattering that leads to attenuation. In general, this attenuation is insignificant in a short distance of propagation. However, the propagation of high intensity sound waves results in a significant decrease of pressure that allows the production of steady bulk air flows. This type of streaming is usually associated with high-viscosity materials; the other type of acoustic streaming, nonetheless, is attributed to the friction between a solid wall and a medium when the former is vibrating in contact with the latter or vice versa [2]. Providing that there is an oscillating tangential relative velocity, it is not important whether the source of a relative motion comes from either acoustic oscillations in the medium or vibrations of the solid. Both cases lead to frictional dissipation within Stokes boundary layer [3]. This acoustic streaming possesses two components: inner and outer streaming. The former is induced within the boundary layer due to the friction between the medium and the wall; subsequently, the inner streaming also induces relatively large scale steady streaming outside the boundary layer.

Faraday (1831) first found that currents of air rise at displacement anti-nodes on plates and descend at the nodes [4]. Nevertheless, the first theoretical analysis of the acoustic streaming phenomenon was performed by Rayleigh (1945). Further developments of the theory were placed on the fundamental role of dissipation of the acoustic energy in the evolution of the gradients in the momentum flux (Schlichting 1955, Nyborg 1958, and Lighthill 1978). Since then, several researchers have investigated acoustic streaming induced by either longitudinal or flexural vibrations. Most of these investigations, however, have focused on acoustic streaming induced by longitudinal vibrations in an enclosed channel such as Kundt tube [5]. The use of flexural vibrations as a source of acoustic streaming permits for a slim profile and low power operation because flexural impedance of an elastic beam is, generally, much smaller than longitudinal impedance. Hyun et al (2002, 2005) have investigated, numerically and experimentally, the steady characteristics of momentum and heat transfer due to acoustic streaming induced by ultrasonic flexural vibrations in an open channel. The numerical simulations employed Computational Fluid Dynamics (CFD) to observe acoustic streaming patterns and velocities, and their results were validated using heat transfer analysis.

In this experimental investigation, particle cumpling position on circular aluminium plates was carried out in an open channel using acoustic streaming generated by ultrasonic vibrations. The use of natural frequencies and mode shapes to produce ultrasonic vibrations in a circular plate was performed. The design process for producing the desired vibration modes and particle movement involved formulae, Global Matrix model (*Disperse©*) and Finite Element (FE) model (*Abaqus©*) analysis. Experiments carried out, using a 40 kHz PZT bolted transducer and crystal beads, corroborate the simulated results on circular plates of different thickness. Particle levitation was attained by positioning a reflector above the circular plate; as a result, the circular plate’s vibration patterns with patterns of streaming and clump formation were adequately correlated.

DESIGN PROCEDURE

Natural frequencies and mode shapes are important parameters in structures subject to vibrations. Mode shapes are functions defined over a structure which describe the relative displacement in any point on the structure as the structure vibrates in a single mode [6]. A mode shape is associated with each natural frequency of a structure, i.e. the frequency at which a linear elastic structure will tend to vibrate once it has been set into motion. If the deflection of a linear vibrating structure in some direction is denoted by $Y(x,t)$, in the k mode, the deflection can be written as [6].

$$Y(x,t) = y'_k(x)y_k(t) \qquad \text{[Eq. 1]}$$

where $y'_k(x)$ is the mode shape, which is a function only of space, and $y_k(t)$ is a function only of time. If the function vibrates in various modes, the total displacement is the sum of the modal displacements.

$$Y(x,t) = \sum_{i}^{N} y'_k(x)y_k(t) \qquad \text{[Eq. 2]}$$

The design procedure to obtain the desired natural frequencies and mode shapes involved the selection of a single mode at a required frequency. A longitudinal mode and a flexural mode were selected to carry out the design and construction of circular plates to produce ultrasonic vibrations to induce acoustic streaming.

In this work, a longitudinal 40 kHz bolted ultrasound transducers and a propetary stepped horn design were employed [7]. Therefore, the selected frequency for driving the horn and the circular plates was 40 kHz. The determination of the geometry of these structures, based on the prescribed frequency and mode shape, entailed the use of formulae, Global matrix modelling (*Disperse*©) and FE modelling (*Abaqus*©), as it is described in the following sections

Formulae, circular plate design

Mode shapes of circular plates can be expressed only for three elemtary boundary conditions, i.e. free, simple supported and campled. The natural frequencies for this type of plates are given by [6]

$$f_{ij} = \frac{\lambda_{ij}^2}{2\pi a^2}\left[\frac{Eh^3}{12\gamma(1-v^2)}\right]^{1/2} \qquad \text{[Eq. 3]}$$

Where a is the radious of the plate; h, is the thickness of the plate; i, is the number of nodal diameters; j, is the number of nodal circles, no couting the boundary; E, is the modulus of elasticity (7.08x10^{10} Nm^{-2}); γ, is the mass per unit area (μh), and μ is the density (2700 kgm^{-3}); v, Poisson´s ratio. The λ function depends on the support condition, and it is determinated by the relationship between the modal indixes i and j, and mode shape; figure 1 depicts a circular plate with simply supported edges, indixes values of i=0, j=0 and j=1.

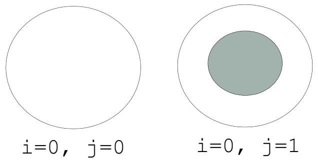


Figure 1. Nodal lines for circular plates with simply supported edges and i=0.

The method adopted was to drive 70 mm diameter aluminium discs at 40 kHz with the aim of producing displacement antinodes at the centre of each disc. Hence, considering a free edge, free centre yielded a $\lambda_{01}^2 = 9.084$, and for a free edge clamped centre it yielded a $\lambda_{00}^2 = 3.752$. The obtained thicknesses of the plates were 21.59 mm for a free disc and 52.27 mm for a disc clamped at the centre.

Disperse©, identification of excited modes

In order to indentify the possible excited modes, a general purpose package, *Disperse*, which use Global Matrix method to solve the wave propagation equation, was used. *Disperse* is a program for tracing dispersion curves, and was employed to model an aluminium rod 70 mm diameter. Figure 2 depicts the dispersion curve, group velocity (*Vgr*) vs. frequency (*f*), of this aluminium structure.

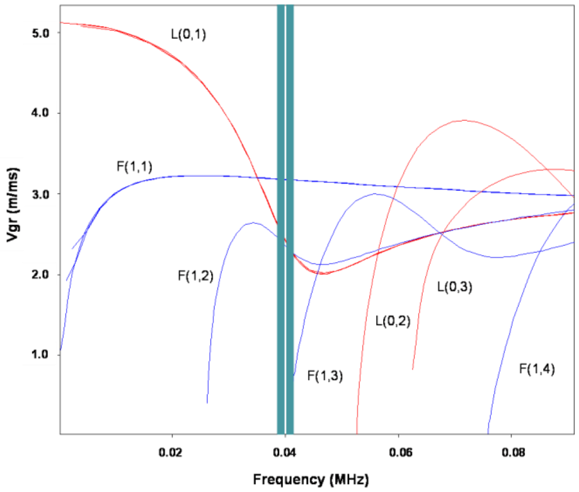


Figure 2. Dispersion curve of an aluminium rod 70mm diameter

For a resonance frequency, f, the thickness of a circular plate, h, is directly proportional to half-wavelegnth ($\lambda/2$) and can be calculated by

$$h = {}^{c}\!/_{2f} \qquad \text{[Eq. 4]}$$

where c is the fundamental longitudinal velocity of the aluminium bar determined by the modulus of elasticity and its density ($\sqrt{E/\rho}$). Considering the circular plate of thickness of 52.27 mm and f=40 kHz, and using hf=c/2, i.e. 52.27mm *X* 40 kHz = 2090 ms^{-1}. The closet value identified according to figure 2 was 2079 ms^{-1} at f= 43.5 kHz, which corresponds to the longitudinal mode L(0,1). The mode shape is illustrated in Figure 3.

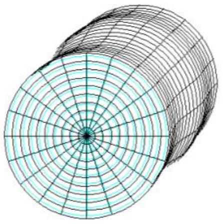


Figure 3. Mode shape of L(0,1) mode at *f*=43.5 kHz and *Vgr*=2079 ms^{-1}.

Employing the same methology, for the circular plate of 21.59 mm thickness and *f*=40 kHz, *hf*=*c*/2, i.e. 21.59mm *X* 40 kHz = 863 ms^{-1}. The closet value was 871 ms^{-1} at *f*= 41.8 kHz, which corresponds to the flexural mode F(1,3). The mode shapes are illustrated in Figure 4.

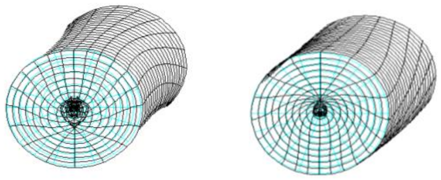


Figure 4. Mode shapes of F(1,3) mode at *f*=41.8 kHz and *Vgr*=871 ms^{-1}

According to *Disperse*, the circular plates of 52.27 mm and 21.59 mm shall excite the axis-symmetric mode L(0,1) and a non axis-symmetric mode F(1,3), respectively, as can be appreciated in figures 3 and 4. However, using this software was not possible to visualise adequately the displacement anti-nodes at the centre of each disc.

Abaqus©, visualisation of displacements anti-nodes

A Finite Element (FE) modelling was carried using the commercial package *Abaqus*©. The dimensions of the aluminium plates were modelled by imposing longitudinal forces, at the centre, on the 70mm base diameter of the plate, where the piezoelectric transducer force would be located, as illustrated in figure 5.

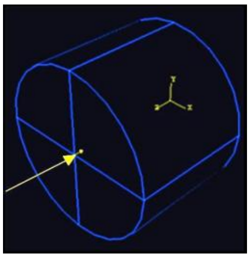
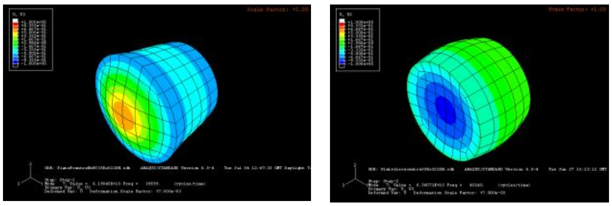


Figure 5. Longitudinal forces applied at the centre, on the 70mm base diameter of the circular plate.

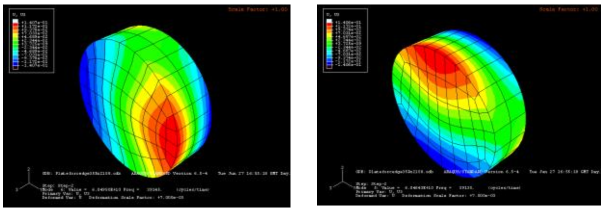
The range of frequencies to operate the model was set from 35 kHz to 45 kHz. Boundary conditions were selected as free, i.e. all the degrees of freedom of displacements and rotations are unconstrained. A sufficiently dense and structured hex mesh was selected to guarantee accuracy in the results. The natural resonance frequencies and corresponding modal shapes were found by performing a modal analysis; figure 6 shows the resonance frequencies achieved close to 40 kHz.



a) b)

Figure 6. Natural resonance frequencies for a circular plate 70 mm diameter, 52.27 mm thickness; a) mode shapes at f=39.59 kHz; b) mode shapes at f=40.14 kHz

The same modeling approach was performed for the aluminium plate dimensions 70 mm diameter, 21.59 mm thickness. The obtained natural resonance frequencies and resulting modal shapes are displayed in figure 7.

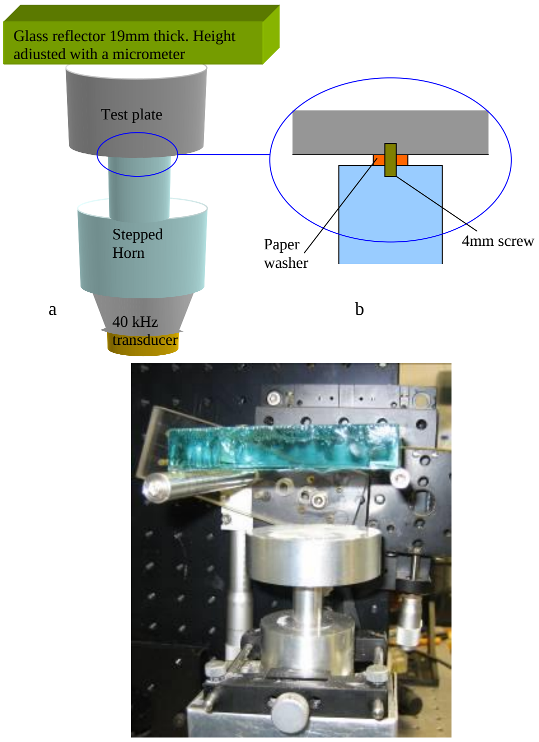


a) b)

Figure 7. Natural resonance frequencies for a circular plate 70 mm diameter, 21.59 mm thickness; a) mode shapes at f=39.13 kHz; b) mode shapes at f=39.14 kHz.

ACOUSTIC STREAMING EXPERIMENT SET-UP AND RESULTS

In order to visualise the acoustic streaming, using ultrasonic 40 kHz vibrations on circular plates, and carry out the clumping of particles, the experiment shown in figure 8 was setup.



c)

Figure 8. a) Assembly of the test plate, drive and reflector; b) attachment of test disc to the stepped horn; c) photograph of assembly and micromanipulators.

The overall construction of the experiment involves the test plates, which were attached at the centre by a grub-screw and paper spacer to a 40 kHz stepped horn. The horn was fitted by another grub-screw to a 40 kHz transducer. The horn was attached at the centre of the plate. The PZT transducer was excited via an ENI 240 L power amplifiers with 40 kHz continuous sinusoidal waveform setting up a range of 250mV to 450 mV in a digital generator. A microphone was employed to find the exact resonance frequency on the plate produced by the horn. Crystal beads (106 μm diameter) were distributed onto the plate. The process was completely recorded using a digital camcorder.

Particle clumping

Primarily, the plate of 52.27 mm thickness was tested. Figure 9 depicts snap shots of the plate once power was applied. The natural resonance frequencies attained were at 40.03 kHz and 40.07 kHz, shown in figure 9a and 9b respectively. These images pictured a defined pattern of accumulated particles, validating the FE modelling results shown in figure 6a and 6b.


a) b)

Figure 9. Natural resonance frequencies attained on the circular plate 70 mm diameter, 52.27 mm thickness; a) particles clumping at *f*=40.03 kHz; b) particles clumping at *f*=40.07 kHz.

Posteriorly, the plate of 21.59 mm thickness was tested. Figure 10 displays the snap shots obtained. The natural resonance frequencies attained were at 39.46 kHz and 39.60 kHz, shown in figure 10a and 10b, respectively. Also, these images depicted a defined pattern of agglomerated particles, validating the FE modelling results shown in figure 7a and 7b.

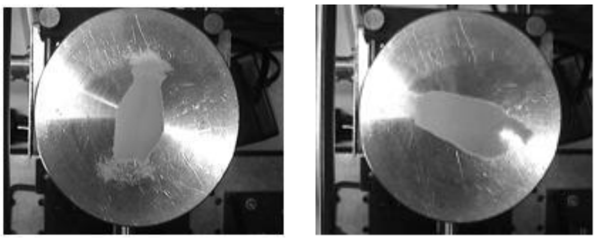

a) b)

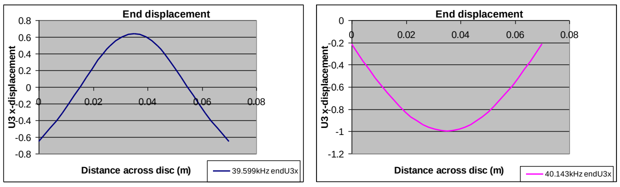
Figure 10. Natural resonance frequencies attained on the circular plate 70 mm diameter, 21.59 mm thickness; a) particles clumping at f=39.46 kHz; b) particles clumping at f=39.60 kHz.

The maximum vibration that induced acoustic streaming was at anti-nodes, pushing away the crystal beads due to air rising, and a minimum at nodes, where particles remained.

Particle levitation

A further experiment was carried out using a reflector, as depicted in figure 8, in order to produce a resonant field in the air above the disc and attain levitation of the crystal beads distributed onto the plate. The reflector was made of a 90x90 mm x 19mm (~$^1/_4$ wavelength at 40 kHz) thick glass plate. The glass plate position was adjusted with a micrometer to get maximum resonance in the air, as determined by particle levitation. A spirit level was used to ensure that plate and reflector were level, since particle levitation positions were strongly affected by gravity and miss alignment.

This experiment concentrated on the 52.26 mm thick plate, which in accord with FE analysis produces two natural resonance frequencies, 39.59 kHz and 40.14 kHz. The obtained displacements anti-nodes across the plate yielded a maxium at the centre, as shown in figure 11.



a) b)

Figure 11. Displacement obtained by FE analysis on the circular plate 52.26 mm thick; a) at 39.59 kHz; b) 40.14 kHz.

The maximum displacement across the plate is related to a high intensity sound, which propagates with a strong directionality, i.e. it does not scatter but propagates like a beam of concentrated immense energy for a short distance of travel [5]. Upon encountering a boundary, such as the glass reflector, the wave is reflected, bringing about a pressure standing wave owing to the interaction with the incident wave. At the gap between the plate and the glass reflector, a half-wavelength resonant pressure standing wave is created at an excitation frequency of approximately 40 kHz. Consequently, the pressure standing wave is strong enough to levitate the crystal beads placed on the top of the plate. Figure 12 and figure 13 show the levitation of crystal beads at 40.03 kHz and 40.07 kHz, respectively.

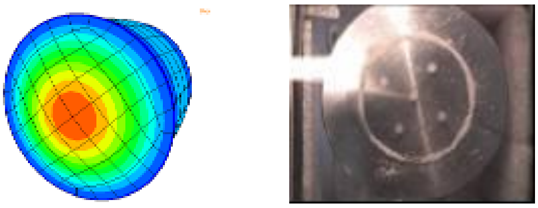

a) b)

Figure 12. Aluminium plate 52.26 mm thick; a) FE model at 39.59 kHz; b) levitation of particles using a glass reflector at 40.03 kHz

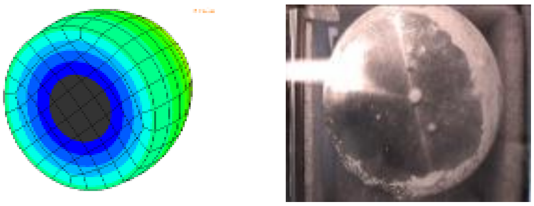

a) b)

Figure 13. Aluminium plate 52.26 mm thick; a) FE model at 40.14 kHz; b) levitation of particles using a glass reflector at 40.07 kHz

Crystal beads in the gap are levitated above the anti-nodes and remain at the midpoint of the gap where pressure is zero. Aditionally, acoustic streaming induces the particles to rotate. There are, in theory, two acoustic streaming patterns symmetrical with respect to the anti-nodes in every half-wavelength interval along the length of the reflector [5]. Because of the symmetry of acoustic streaming patterns, levitated particles should stand still above the anti-nodes; however, these induced patterns are not perfectly symmetrical, causing the levitated particles to rotate due to nonuniform thickness of the reflector and the gap. Moreover, whirl of circular airflow was observed on the particles, entering and leaving the levitated clumps.

CONCLUSIONS

An experimental investigation of acoustic streaming induced by 40 kHz ultrasonic vibrations is presented. The investigation includes not only the accumulation of particles on a circular aluminium plate, but also the levitation of particle clumps by positioning a glass reflector above the plates to accomplish resonace in the air. Natural frequencies and mode shapes were employed to produce ultrasonic vibrations in the plate. The design procedure to determine the geometry of the circular plates to induce acoustic streaming, entailed the use of formulae, Global matrix modelling (*Disperse©)* and FE modelling (*Abaqus©*). Formulae were used to select two different thicknesses, 21.59 mm and 52.27mm, of circular plates with a predefined 70 mm diameter and different boundary conditions. The software *Disperse* provided the capability to identify the possible excited modes according to the thickness of the plates. Thereby, the identified modes were the axis-symmetric L(0,1) and the non axis-symmetric F(1,3), for the 52.27 mm and the 21.59 mm plates, respectivaley. Longitudinal and flexural mode shapes were appreciated; however, it was not possible to visualise satisfactorily the displacements anti-nodes in each plate.Whereas FE analysis, via *Abaqus©*, depicted precise three dimensional patterns on the plates that were validated experimentally. Maximum vibrations that induce acoustic streaming were at anti-nodes, pushing away the particles due to air rising and minimum at nodes, where the particles clumped. The agreement between the natural resonance frequencies obtained by FE modelling and experiments was very high, less than 2% error. Additionaly, using a reflector above the plate of 52.27 mm thickness, levitation of clumps at the anti-nodes was achieved. These results are very motivating taking the authors to perform acoustic streaming employing the FE analysis approach that could lead to control the agglomeration of particles in liquids.

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