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Sound field reproduction by using a scatterer

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ABSTRACT

This study proposes a method to reproduce a sound field that we desire in a selected control region by using an array of loudspeakers, both temporally and spatially. The desired sound field means a sound field that we can have in concert halls and stadiums, or that we want to have for special effects in movies and computer games. If a sound field that is identical to the desired field is generated by using loudspeakers, one who is in the field will have the same feeling as that in the desired field. In other words, this study aims to mimic the desired field in a control region so as to make listeners feel as if they were in the desired field. The proposed method uses a scatterer on the surface of which microphones are mounted to measure the surface pressure. Then, by using the measured pressure, the input signals to loudspeakers are obtained that makes the reproduced field identical to the desired field in the control region. In other words, if we put the scatterer in any sound field, and measure the surface pressure, then we can reproduce the sound field by loudspeakers. This method is based on the fact that the pressure on the surface of a scatterer uniquely determines the incident sound field which is generated if the scatterer is not placed. The use of the scatterer enables us to reproduce sound fields at all frequencies. This paper proves the fact, and explains and verifies the proposed method with simple examples.

1. INTRODUCTION

If we can generate the sound field that is identical to that of a concert hall in our rooms by using loudspeakers, then the listener will have the same feeling as that in the concert hall because the sound field is physically identical. We do not need to consider human characteristics of hearing sound to provide an actual feeling of three-dimensional sound. This control of sound field can be expressed as ‘sound field reproduction’. In other words, we aim to mimic a sound field that exists in other places by using an array of loudspeakers in a finite control region so as to make listeners feel as if they were in the sound field. The desired field can be any sound field that we can have in our lives, i.e. concert halls and stadiums, or that we want to have for special effects in movies and computer games. The control region can be defined as a finite region of interest where listeners are situated.

Attempts to reproduce a sound field began in late 20th century. In 1970’s, Gerzon invented the Ambisonics [1]. In 1990’s, Berkhout proposed WFS (Wave Field Synthesis) [2], Kirkeby and Nelson proposed a method to use the least-square filter [3], and Ise proposed BSC (Boundary surface control) to use inverse filtering for the boundary pressure [4]. In addition, Ambisonics was extended to HOA (High order Ambisonics) [5-7] that uses high order of coefficients in spherical harmonics expansion. Choi proposed a method to generate a plane wave by focusing acoustic energy at a point in wavenumber domain [8], and Chang and Kim extended the method to three-dimenasional case by using spherical harmonics expansion, and provided a solution to generate a plane wave without matrix inversion [9].

Reproduction of sound field includes two problems: encoding problem and decoding problem. The former is to record the desired sound field. This does not mean that we have to sample the entire sound field, but we can obtain some information from which we can predict the sound field. For example, HOA uses the coefficients of the spherical harmonics expansion as the sound field information [5-7], and BSC uses sound pressure on the boundary surface [4]. On the other hand, the latter is to obtain the input signals which are fed into the loudspeakers. In order to generate the identical sound field to the desired field, the relation between the input signals and the sound field or the information of the sound field generated by loudspeakers are needed to be obtained.

However, Ise showed that there are some frequencies at which the interior sound field cannot be obtained by boundary pressure [4]. Open sphere microphone [10] array has also the same problem: at some frequencies, the coefficients of spherical harmonics expansion cannot be obtained. This restriction is overcome by using a spherical microphone array that is mounted on a surface of a rigid sphere that has been proposed by Meyer and Elko. However, these works are based on the spherical harmonics expansion, so that the region of interest is restricted to spherical shape. In addition, these methods based on the spherical harmonics assume free field condition, and then the effect of the reverberation is not theoretically explained.

This paper shows that the surface pressure on an arbitrary shaped scatterer uniquely determines sound field in a region of interest based on the Kirchhoff-Helmholtz integral equation. This enables us to reproduce a sound field by matching surface pressure on the scatterer at all frequency range regardless of the shape of the scatterer. This method is verified by simple two-dimensional examples, and the relation between reproducible region and the sampling spacing on the scatterer is discussed.

2. PROBLEM DEFINITION

2.1 The desired field

Let us consider a space which is bounded by a surface as illustrated in Figure 1. The boundary condition is regarded to be arbitrary: rigid, absorptive, open, and so on. This means that there can be reflections from the boundary, or not. The region of interest is defined as a finite source-free region in the space. The region is denoted as V_d, and its boundary surface is denoted as S_d. Sound sources are regarded to be located at arbitrary positions outside the region of interest. Then, sound waves from the sound sources come into the region of interest directly, and reflections from the boundary come into the region. The sound field in the region of interest is defined as the desired field. It is composed of incident waves from outside, so that it is denoted as $P_{in}(\vec{r},\omega)$ in frequency domain.

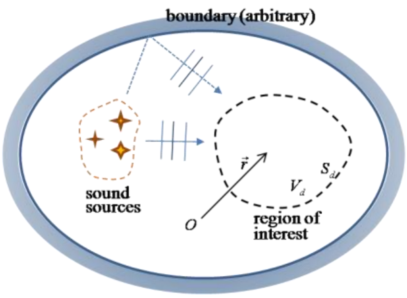

Figure 1. The desired sound field: the region of interest is defined as a finite source-free region surrounded sound sources. The region is denoted as V_d, and its boundary surface is denoted as S_d.

2.2 The reproduced field

Let us define the sound field that we generate by using loudspeakers as the reproduced field (Figure 3(left)). We denote the field as $\tilde{P}_{in}(\vec{r},\omega)$ because it has only incident waves from the loudspeakers. Then, it is expressed by the superposition of sound fields that are generated by loudspeakers. If we denote the incident field generated by the l-th loudspeaker as $\tilde{P}_{in}^{(l)}(\vec{r},\omega)$, it is regarded as the multiplication of the input signal $q^{(l)}(\omega)$ and the transfer function between the input signal and the incident sound field $h_{in}^{(l)}(\vec{r},\omega)$ as follows:

$$\tilde{P}_{in}(\vec{r},\omega)=\sum_{l=1}^{L}\tilde{P}_{in}^{(l)}(\vec{r},\omega)=\sum_{l=1}^{L}q^{(l)}(\omega)h_{in}^{(l)}(\vec{r},\omega), \quad (1)$$

where L is the number of loudspeakers (Figure 3 (right)).

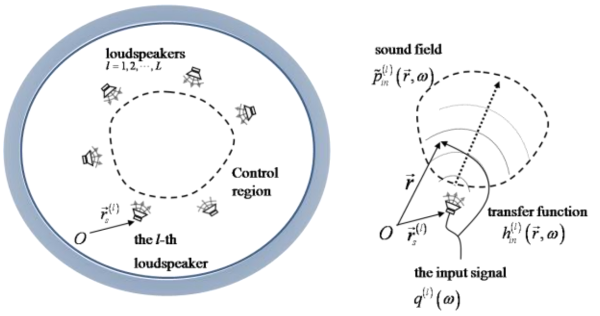

Figure 2. The reproduced field generated by all loudspeakers (left) and the incident sound field by the l-th loudspeaker (right)

3. SOLUTION METHOD

3.1 The changed sound field due to a scatterer

If we locate a scatterer inside of the desired field as illustrated in Figure 3, the sound field is changed by the scattering effect due to the scatterer. The scattered field is normally defined as the difference between the incident field and the changed field, and the changed sound field is normally called as total field because it is the sum of the incident and the scattered field. That is,

$$P_{tot}(\vec{r},\omega)=P_{in}(\vec{r},\omega)+P_{sc}(\vec{r},\omega), \quad (2)$$

where $P_{tot}(\vec{r},\omega)$ and $P_{sc}(\vec{r},\omega)$ are the total and the scattered field, respectively.

The scatterer is defined as an arbitrary shaped object that disturbs the incident sound waves. Let us denote its surface as S_{sc}. On the assumption of local reaction, the surface impedance on the scatterer is

$$Z_s(\vec{r}_s,\omega)=\left.\frac{P(\vec{r}_s,\omega)}{v_n(\vec{r}_s,\omega)}\right|_{S_{sc}}. \quad (3)$$

where $\vec{r}_s$ indicates an arbitrary position on the surface of the scatterer, and $v_n(\vec{r}_s,\omega)$ is surface-normal velocity. If the surface of the scatterer is rigid, then the impedance $Z_s(\vec{r},\omega)$ goes to infinite.

It is noteworthy that the pressure on the surface of the scatterer is regarded to be known by measurement. The surface pressure is the total field on the surface of the scatterer, and it is denoted by $P_{tot}(\vec{r},\omega)\big|_{S_{sc}}$. On the other hand, the sound field that we want to reproduce is the incident sound field in the region of interest, $P_{in}(\vec{r},\omega)\big|_{V_d}$.

Similarly, if we locate a scatterer inside of the reproduced field as illustrated in Figure 4, the sound field is changed. The total fields generated by the loudspeakers have the following relations according to the principle of superposition (Figure 4 (right)):

$$\tilde{P}_{tot}(\vec{r},\omega)=\sum_{l=1}^{L}\tilde{P}_{tot}^{(l)}(\vec{r},\omega)=\sum_{l=1}^{L}q^{(l)}(\omega)h_{tot}^{(l)}(\vec{r},\omega). \quad (4)$$

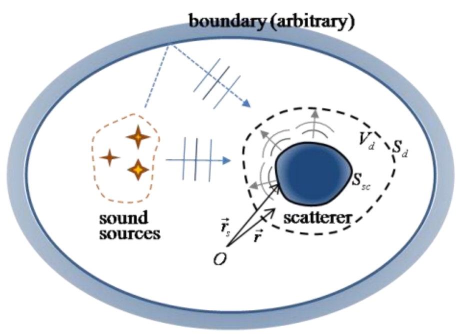

Figure 3. The changed desired field due to a scatterer: the scatterer is located inside of the region of interest, and the changed sound field is called as the total field.

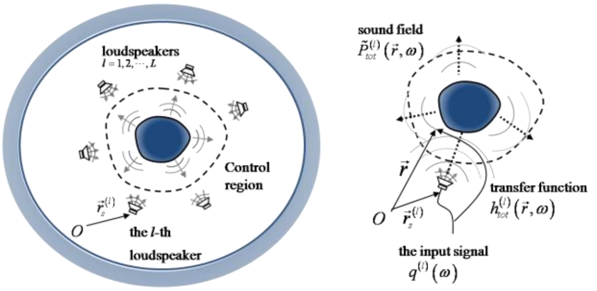

Figure 4. The changed reproduced field due to a scatterer: the changed field is the superposition of the total fields generated by loudspeakers.

3.2 Surface pressure matching on the scatterer

Practically, we cannot sample the pressure on the entire area but at the discrete points on the surface of the scatterer. Let us denote the number of the sampling points as M, and the positions as $\vec{r}_m$ where m is the index of the points, 1 to M (Figure 5). Then, we have the following relation at the sampling points as:

$$\tilde{\mathbf{P}}_{tot}(\omega) = \mathbf{H}_{tot}(\omega)\mathbf{Q}(\omega), \tag{5}$$

where

$$\tilde{\mathbf{P}}_{tot}(\omega) = \left[\tilde{P}_{tot}(\vec{r}_1,\omega) \quad \tilde{P}_{tot}(\vec{r}_2,\omega) \quad \cdots \quad \tilde{P}_{tot}(\vec{r}_M,\omega)\right]^T,$$

$$\mathbf{H}_{tot}(\omega) = \begin{bmatrix} h_{tot}^{(1)}(\vec{r}_1,\omega) & h_{tot}^{(2)}(\vec{r}_1,\omega) & \cdots & h_{tot}^{(L)}(\vec{r}_1,\omega) \\ h_{tot}^{(1)}(\vec{r}_2,\omega) & \ddots & & \vdots \\ \vdots & & & \\ h_{tot}^{(1)}(\vec{r}_M,\omega) & \cdots & & h_{tot}^{(L)}(\vec{r}_M,\omega) \end{bmatrix},$$

$$\mathbf{Q}(\omega) = \left[q^{(1)}(\omega) \quad q^{(2)}(\omega) \quad \cdots \quad q^{(L)}(\omega)\right]^T. \tag{6}$$

In the same manner, we express the surface pressure on the scatterer in the desired field at the sampling points $\mathbf{P}_{tot}(\omega)$ by

$$\mathbf{P}_{tot}(\omega) = \left[p_{tot}(\vec{r}_1,\omega) \quad p_{tot}(\vec{r}_2,\omega) \quad \cdots \quad p_{tot}(\vec{r}_M,\omega)\right]^T. \tag{7}$$

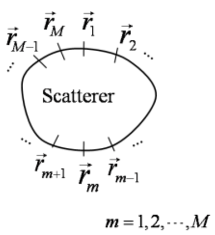

Figure 5. The sampling points on the surface of the scatterer: the number of sampling points is denoted as M, and the positions as $\vec{r}_m$ where m is the index of the points, 1 to M.

As proved in Appendix, the surface pressure on the scatterer determines the incident field. Then, if the surface pressure in the desired field is equal to that in the reproduced field, then the incident field of the desired field is equal to that of the reproduced field. That is,

$$P_{tot}(\vec{r}_s,\omega) = \tilde{P}_{tot}(\vec{r}_s,\omega) \quad \Leftrightarrow \quad P_{in}(\vec{r},\omega) = \tilde{P}_{in}(\vec{r},\omega). \tag{8}$$

Therefore, if we control the input signals into loudspeakers for the surface pressure on the scatterer in the reproduced field to be equal to that in the desired field, then we can reproduce the incident field by using loudspeakers in the control region.

The input signal $\mathbf{Q}(\omega)$ is obtained by the matrix inversion:

$$\mathbf{Q}(\omega) = \mathbf{H}_{tot}(\omega)^{+}\mathbf{P}_{tot}(\omega), \tag{9}$$

where $+$ indicates Moore-Penrose pseudo-inverse, and $\tilde{p}_{in}(\vec{r},\omega)$ is expected to be equal to $p_{in}(\vec{r},\omega)$ in the control region by feeding the input signals. It is noteworthy that the transfer function matrix $\mathbf{H}_{tot}(\omega)$ remains same if the system is not changed, then it is a known value. This means that we can obtain the input signal $\mathbf{Q}(\omega)$ from the measured surface pressure $\mathbf{P}_{tot}(\omega)$.

4. SIMPLE EXAMPLES

4.1 Simulation setup

Let us verify the proposed method in two-dimensional simple cases. For simplicity, we make several assumptions as follows: the scatterer is regarded as a disk whose radius is denoted as a, and then the size of the scatterer is simply expressed by only the radius as illustrated in Figure 6. The control region is regarded as a circle whose radius is a_c. The center is regarded to locate at the origin. The surface property of the scatterer is assumed to be uniform on the entire surface of the scatterer, and then it is expressed by the surface impedance $Z_s(\omega)$. The microphones are assumed to be equally spaced on the surface of the scatterer, and then the positions of microphones are determined as

$$\vec{r}_m = (a\cos\phi_m, a\sin\phi_m),\ m = 1,2,\cdots,M \tag{10}$$

where $\phi_m = 2\pi m/M$ (Figure 6). The number of microphones M is regarded to be same as that of loudspeakers L, so that we always have the unique solution. The loudspeakers are planewave sources, so that the position of the loudspeakers is determined by only the angle $\phi^{(l)}$, and let us regard $\phi^{(l)} = \phi_m$. Then, the propagating direction of the plane wave generated by the l-th loudspeaker is expressed by $\phi^{(l)} + \pi$ (Figure 6). Let us define the magnitude of each planewave as the input signal $q^{(l)}(\omega)$, and then the plane wave generated by the l-th loudspeaker is expressed as

$$\tilde{P}_{in}^{(l)}(r,\phi,\omega) = q^{(l)}(\omega)e^{ikr\cos\left(\phi-\phi^{(l)}-\pi\right)}. \tag{11}$$

The desired field is regarded as a plane wave field that propagates to ϕ_0, which is determined by $\phi_0 = \left(\phi^{(1)} + \phi^{(L)}\right)/2 + \pi$ to avoid the propagating directions of the plane waves of all loudspeakers. Regarding the magnitude as 1, and the sound field is expressed as

$$P_{in}(r,\phi,\omega) = e^{ikr\cos(\phi-\phi_0)}. \tag{12}$$

The surface pressure on the surface of a disk can be obtained by using the Fourier-Bessel expansion. That is, Eqn.(12) is expressed as

$$P_{in}(r,\phi,\omega) = e^{ikr\cos(\phi-\phi_0)} = \sum_{n=-N}^{N} i^n e^{-in\phi_0} J_n(kr) e^{in\phi}, \tag{13}$$

where N is the maximum order of coefficient that has to be determined for the normalized truncation error[7] to be smaller than a criterion on the boundary of the control region, which is defined as

$$\bar{e}^2(ka_c)=\frac{\int_0^{2\pi}\left|\sum_{n=-\infty}^{\infty} i^n e^{-in\phi_0} J_n(ka_c)e^{in\phi}-\sum_{n=-N}^{N} i^n e^{-in\phi_0} J_n(ka_c)e^{in\phi}\right|^2 d\phi}{\int_0^{2\pi}\left|\sum_{n=-\infty}^{\infty} i^n e^{-in\phi_0} J_n(ka_c)e^{in\phi}\right|^2 d\phi} \quad (14)$$

On the other hand, the scattered field by the disk is expressed as

$$P_{sc}(r,\phi,\omega)=\sum_{n=-N}^{N} A_n(\omega)H_n^{(1)}(kr)e^{in\phi}, \quad (15)$$

where $A_n(\omega)$ is a complex coefficient, and $H_n^{(1)}$ is the n-th order of the first kind Hankel function. Then, the sound pressure and the normal velocity on the surface of the disk are

$$P_{tot}(r=a,\phi)=\sum_{n=-N}^{N}\left[i^n e^{-in\phi_0} J_n(ka)+A_n(\omega)H_n^{(1)}(ka)\right]e^{in\phi}, \quad (16)$$

and

$$\frac{\partial P_{tot}(r,\phi)}{\partial r}\Big|_{r=a}=\sum_{n=-N}^{N}\left[ki^n e^{-in\phi_0} J_n'(ka)+kA_n(\omega)H_n^{(1)\prime}(ka)\right]e^{in\phi}. \quad (17)$$

On the assumption that the surface impedance is uniform on the surface, Eqn.(3) is rewritten as

$$\sum_{n=-N}^{N}\left[ki^n e^{-in\phi_0} J_n'(ka)+kA_n(\omega)H_n^{(1)\prime}(ka)\right]e^{in\phi} = \frac{i\omega\rho_0}{Z_s(\omega)}\sum_{n=-N}^{N}\left[i^n e^{-in\phi_0} J_n(ka)+A_n(\omega)H_n^{(1)}(ka)\right]e^{in\phi}, \quad (18)$$

and then we obtain the coefficient as

$$A_n=-\left[kH_n^{(1)\prime}(ka)-\frac{i\omega\rho_0}{Z_s(\omega)}H_n^{(1)}(ka)\right]^{-1}\left[kJ_n'(ka)-\frac{i\omega\rho_0}{Z_s(\omega)}J_n(ka)\right]i^n e^{-in\phi_0}. \quad (19)$$

In the same way, we obtain the transfer function between the input signal $q^{(l)}(\omega)$ and the surface pressure on the surface as

$$h_{tot}^{(l)}(r=a,\phi)=\sum_{n=-N}^{N}\left[i^n e^{-in\phi_0} J_n(ka)+A_n^{(l)}(\omega)H_n^{(1)}(ka)\right]e^{in\phi} \quad (20)$$

where

$$A_n^{(l)}=-\left[kH_n^{(1)\prime}(ka)-\frac{i\omega\rho_0}{Z_s(\omega)}H_n^{(1)}(ka)\right]^{-1} \times\left[kJ_n'(ka)-\frac{i\omega\rho_0}{Z_s(\omega)}J_n(ka)\right]i^n e^{-in\left[\phi^{(l)}+\pi\right]}. \quad (21)$$

The surface pressure and the transfer function at the sampling points, $\mathbf{P}_{tot}(\omega)$ and $\mathbf{H}_{tot}(\omega)$, are obtained as follows:

$$\mathbf{P}_{tot}(\omega)=\left[P_{tot}(r=a,\phi_1,\omega)\ \ P_{tot}(r=a,\phi_2,\omega)\ \cdots\ P_{tot}(r=a,\phi_M,\omega)\right]^T, \quad (22)$$

$$\mathbf{H}_{tot}(\omega)=\begin{bmatrix} h_{tot}^{(1)}(r=a,\phi_1,\omega) & \cdots & h_{tot}^{(L)}(r=a,\phi_1,\omega)\\ \vdots & \ddots & \vdots \\ h_{tot}^{(1)}(r=a,\phi_M,\omega) & \cdots & h_{tot}^{(L)}(r=a,\phi_M,\omega)\end{bmatrix}. \quad (23)$$

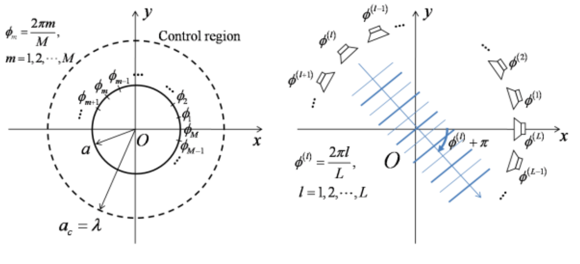

Figure 6. The control region, the sampling points (left), and the loudspeakers (right): the control region is regarded as a circle whose radius a_c. The M microphones are assumed to be equally spaced on the surface of the scatterer, and the L loudspeakers are equally spaced. The propagating direction of the plane wave by the l-th loudspeaker is $\phi^{(l)}+\pi$.

4.2 The solution and the reproduction result

We obtain the unique solution $\mathbf{Q}(\omega)$ by using Eqn.(9) because $\mathbf{H}_{tot}(\omega)$ is a square matrix. Figure 7 illustrates the magnitude and phase of the pressure on the surface of the disk if the surface impedance $Z_s(\omega)$ is 10^3 [pa/ms^{-1}], the radia of the control region and disk a are λ and $\lambda/2$, respectively, and $L=M=17$. The sign 'x' indicates the sampling points. Figure 8 illustrates the magnitude (left) and phase (right) of the reproduced field. The inner circle indicates the scatterer, and the outer circle is the control region. The propagating direction of the desired plane wave ϕ_0 is $18\pi/17$, and it is illustrated by arrows. In the control region, we can see that the plane wave field in the desired field is generated in the reproduced field. The normalized field error between the desired and the reproduced fields is 0.0011 which is defined as

$$\bar{e}_f^{\,2}=\frac{\int_0^{a_c}\int_0^{2\pi}\left|P_{in}(r,\phi)-\tilde{P}_{in}(r,\phi)\right|^2 d\phi\, rdr}{\int_0^{a_c}\int_0^{2\pi}\left|P_{in}(r,\phi)\right|^2 d\phi\, rdr}. \quad (24)$$

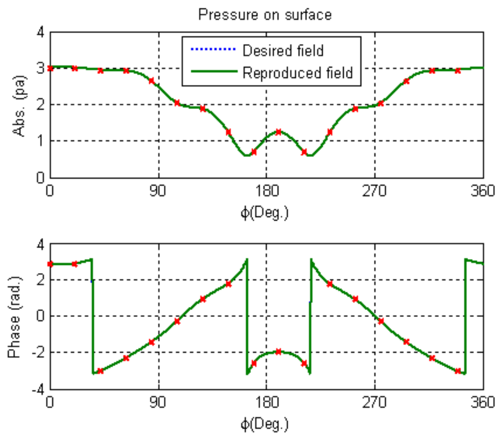

Figure 7. The pressure on the surface of the scatterer: the surface impedance $Z_s(\omega)$ is 10^3 [pa/ms^{-1}], the radius of the disk a is $\lambda/2$, and $L=M=17$. The sign 'x' indicates the sampling points.

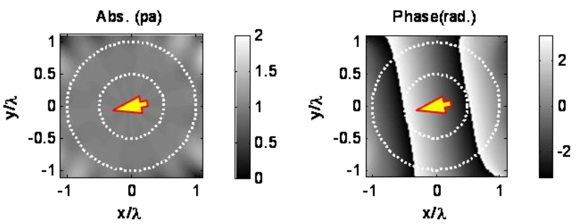

Figure 8. The magnitude (left) and phase (right) of the reproduced field: the inner circle indicates the scatterer, and the outer circle is the control region. The propagating direction of the desired plane wave ϕ_0 is $18\pi/17$, and it is illustrated by arrows. In the control region, we can see that the plane wave field in the desired field is generated in the reproduced field.

4.3 The effect of the spacing of control points

In order to use this method practically, we need to determine the spacing of the control points. The spacing depends on the size of the scatterer with respect to the wavelength and the number of the sampling points. However, the size of the scatterer cannot be defined unless the shape of the scatterer is defined. Then, it is difficult to find out a condition of the spacing for an arbitrary scatterer. Therefore, let us study the effect of the spacing for a simple case, a disk scatterer in two-dimensional case, because the size of the disk is defined by only the radius with respect to wavelength.

Figure 9 represents the normalized field error in a circular region whose radius is a_s (top) and $2a_s$ (bottom) with respect to the radius and the number of the sampling points. In other words, the normalized field errors are defined as Eqn.(24) if the radii of the control region are a_s and $2a_s$, respectively. Let us define the errors as the first and the second kind of error. The solid line is the contour line of the error, and the dotted line is the contour line of the sampling spacing. The error increases as the radius increases or as the number of the sampling points decreases because the spacing increases. The first kind of error is less than -10dB if the spacing is less than about half-wavelength, and the second kind less than -10dB if the spacing is less than quarter-wavelength. Therefore, as a rule of thumb, the spacing of control points has to be less than half-wavelength to reproduce the region as large as the scatterer, and the spacing less than quater-wavelength to reproduced the region as twice large as the scatterer. In addition, the smaller spacing we have, the larger region can be reproduced.

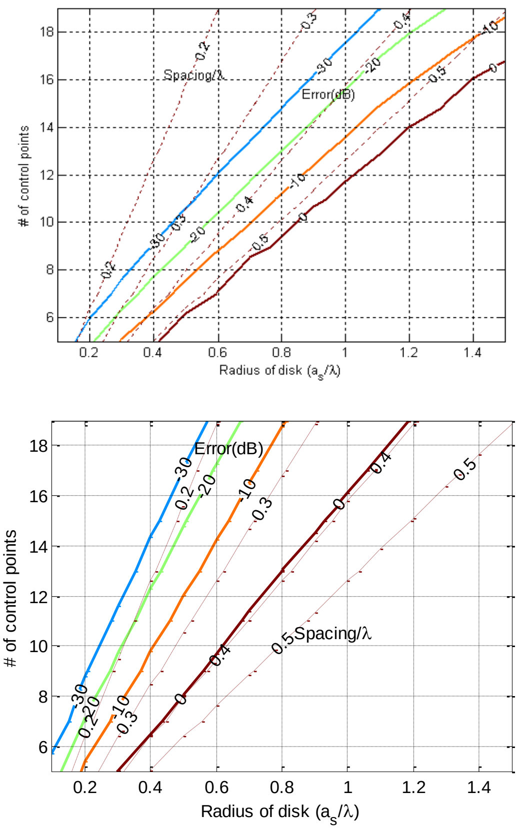

Figure 9. The normalized field error in the circular region whose radius is a_s (top) and $2a_s$ (bottom) with respect to the radius and the number of the sampling points: the solid line is the contour line of the error, and the dotted line is the contour line of the sampling spacing.

5. CONCLUSION

This study proposed a sound field reproduction method by using a scatterer. It is showed that a sound field can be reproduced by matching the surface pressure on the scatter based on the fact that the surface pressure on the scatterer uniquely determines the incident sound field without some exceptional frequencies. With simple examples in the two-dimensional case, the feasibility was verified.

In addition, the effect of the spacing of control points was studied. It is showed that the spacing of control points has to be less than half-wavelength to reproduce the region as large as the scatterer, and the spacing less than quater-wavelength to reproduced the region as twice large as the scatterer. It is noteworthy that this method is not limited by the shape of the scatterer and the kind of wave fronts.

ACKNOWLEDGEMENT

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APPENDIX

The region of interest is divided to two regions by the surface of the scatterer, S_{sc}, as illustrated in Figure A1. Firstly, let us show that the surface pressure of the scatterer in the total field determines the sound field inside the scatterer. Sound field inside the surface of scatterer S_{sc} can be obtained by using the boundary pressure and velocity based on the Kirchhoff-Helmholtz integral equation as follows:

$$P_{in}(\vec{r},\omega)=\int_{S_{ss}+S_{out}}\left[P_{in}(\vec{r}_0,\omega)\frac{\partial G(\vec{r}|\vec{r}_0,\omega)}{\partial n_0}-G(\vec{r}|\vec{r}_0,\omega)\frac{\partial P_{in}(\vec{r}_0,\omega)}{\partial n_0}\right]dS_0 , \quad \text{(A1)}$$

where S_{out} is the outer boundary surface, and S_{ss} is the surface surrounding sound sources as illustrated in Figure A1. Then, the region of integration is the entire region except the region of sound sources, and denoted as V_I. In addition, $\partial/\partial n_0$ is the derivative with respect to the inward normal of the surface of the zone of interest (Figure A1), and $G(\vec{r}|\vec{r}_0,\omega)$ is the Green's function, which satisfies the inhomogeneous wave equation as follows:

$$\nabla^2 G(\vec{r}|\vec{r}_0,\omega)+k^2 G(\vec{r}|\vec{r}_0,\omega)=-\delta(\vec{r}-\vec{r}_0) , \quad \text{(A2)}$$

where k is the wavenumber.

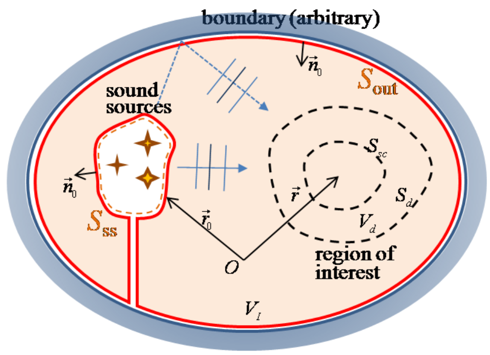

Figure A1. The region of integration of Eqn.(A1): the region (V_I) is the entire region except the region of sound sources, and bounded by S_{ss} and S_{out}.

On the other hand, in the total field, the Kirchhoff-Helmholtz integral equation is obtained as follows in the region of integration illustrated in Figure A2:

$$0=\int_{S_{ss}+S_{out}+S_{sc}}\left[P_{tot}(\vec{r}_0,\omega)\frac{\partial G(\vec{r}|\vec{r}_0,\omega)}{\partial n_0}-G(\vec{r}|\vec{r}_0,\omega)\frac{\partial P_{tot}(\vec{r}_0,\omega)}{\partial n_0}\right]dS_0 \quad , \qquad \text{(A3)}$$

where the region of integration is the entire region except the regions of sound sources and the scatterer, and denoted as V_{II}, and $\vec{r}_0$ indicates the position on its surface, S_{out}, S_{ss}, and S_{sc}.

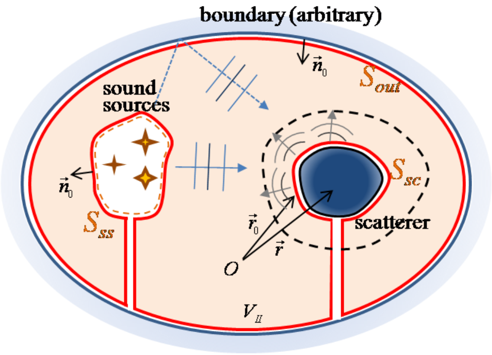

Figure A2. The region of integration of Eqn.(A3): the region (V_{II}) is the entire region except the regions of sound sources and the scatterer, and bounded by S_{out}, S_{ss}, and S_{sc}.

Then, we can rewrite Eqn. (A3) as

$$-\int_{S_{sc}}\left[P_{tot}(\vec{r}_0,\omega)\frac{\partial G(\vec{r}|\vec{r}_0,\omega)}{\partial n_0}-G(\vec{r}|\vec{r}_0,\omega)\frac{\partial P_{tot}(\vec{r}_0,\omega)}{\partial n_0}\right]dS_0$$
$$=\int_{S_{in}+S_{out}}\left[P_{tot}(\vec{r}_0,\omega)\frac{\partial G(\vec{r}|\vec{r}_0,\omega)}{\partial n_0}-G(\vec{r}|\vec{r}_0,\omega)\frac{\partial P_{tot}(\vec{r}_0,\omega)}{\partial n_0}\right]dS_0. \qquad \text{(A4)}$$

It is noteworthy that the total field on S_{ss}, $P_{tot}(\vec{r}_0,\omega)\big|_{S_{ss}}$, can be regarded to be equal to the incident field on S_{ss}, $P_{in}(\vec{r}_0,\omega)\big|_{S_{ss}}$ because the distance to sound sources is zero so that magnitude of the incident field is much bigger than that of the scattered field:

$$P_{tot}(\vec{r}_0,\omega)\big|_{S_{in}}=P_{in}(\vec{r}_0,\omega)\big|_{S_{in}}+P_{sc}(\vec{r}_0,\omega)\big|_{S_{in}} \quad . \qquad \text{(A5)}$$
$$\cong P_{in}(\vec{r}_0,\omega)\big|_{S_{in}}$$

In addition, let us assume that the distance between the scatterer and the outer boundary S_{out} is enough long that the magnitude of the incident field is also much bigger than that of the scattered field on S_{out}. Then, we have

$$P_{tot}(\vec{r}_0,\omega)\big|_{S_{out}}=P_{in}(\vec{r}_0,\omega)\big|_{S_{out}}+P_{sc}(\vec{r}_0,\omega)\big|_{S_{out}} \quad . \qquad \text{(A6)}$$
$$\cong P_{in}(\vec{r}_0,\omega)\big|_{S_{out}}$$

Therefore, Eqn. (A4) is written as

$$-\int_{S_{sc}}\left[P_{tot}(\vec{r}_0,\omega)\frac{\partial G(\vec{r}|\vec{r}_0,\omega)}{\partial n_0}-G(\vec{r}|\vec{r}_0,\omega)\frac{\partial P_{tot}(\vec{r}_0,\omega)}{\partial n_0}\right]dS_0$$
$$\cong\int_{S_{ss}+S_{out}}\left[P_{in}(\vec{r}_0,\omega)\frac{\partial G(\vec{r}|\vec{r}_0,\omega)}{\partial n_0}-G(\vec{r}|\vec{r}_0,\omega)\frac{\partial P_{in}(\vec{r}_0,\omega)}{\partial n_0}\right]dS_0. \qquad \text{(A7)}$$

The right-hand-side term is equal to Eqn.(A1), and then Eqn.(A1) is rewritten as

$$P_{in}(\vec{r},\omega)$$
$$=\int_{S_{ss}+S_{out}}\left[P_{in}(\vec{r}_0,\omega)\frac{\partial G(\vec{r}|\vec{r}_0,\omega)}{\partial n_0}-G(\vec{r}|\vec{r}_0,\omega)\frac{\partial P_{in}(\vec{r}_0,\omega)}{\partial n_0}\right]dS_0 \qquad \text{(A8)}$$
$$\cong-\int_{S_{sc}}\left[P_{tot}(\vec{r}_0,\omega)\frac{\partial G(\vec{r}|\vec{r}_0,\omega)}{\partial n_0}-G(\vec{r}|\vec{r}_0,\omega)\frac{\partial P_{tot}(\vec{r}_0,\omega)}{\partial n_0}\right]dS_0.$$

On the surface of the scatterer, the pressure and velocity has the relation of Eqn. (3). According to Euler's equation, sound velocity is expressed as follows:

$$v_n(\vec{r}_s,\omega)\big|_{S_{sc}}=\frac{1}{j\omega\rho_0}\frac{\partial P_{tot}(\vec{r}_s,\omega)}{\partial n}\bigg|_{S_{sc}} . \qquad \text{(A9)}$$

By using the relation between the pressure and velocity on the surface of scatterer (Eqn.(2)), we obtain

$$\frac{\partial P_{tot}(\vec{r}_s,\omega)}{\partial n}\bigg|_{S_{sc}}=j\omega\rho_0 v_n(\vec{r}_s,\omega)\big|_{S_{sc}}$$
$$=\frac{j\omega\rho_0}{Z_s(\vec{r}_s,\omega)}P_{tot}(\vec{r}_s,\omega)\big|_{S_{sc}} . \qquad \text{(A10)}$$

Then, by substituting Eqn.(A10) for the normal derivative of pressure in Eqn. (A8), we have

$$P_{in}(\vec{r},\omega)=-\int_{S_{sc}}\left[P_{tot}(\vec{r}_0,\omega)\frac{\partial G(\vec{r}|\vec{r}_0,\omega)}{\partial n_0}-G(\vec{r}|\vec{r}_0,\omega)\frac{\partial P_{tot}(\vec{r}_0,\omega)}{\partial n_0}\right]dS_0$$
$$=-\int_{S_{sc}}\left[P_{tot}(\vec{r}_0,\omega)\frac{\partial G(\vec{r}|\vec{r}_0,\omega)}{\partial n_0}-\frac{j\omega\rho_0}{Z_s(\vec{r}_0,\omega)}G(\vec{r}|\vec{r}_0,\omega)P_{tot}(\vec{r}_0,\omega)\right]dS_0$$
$$=-\int_{S_{sc}}\left[P_{tot}(\vec{r}_0,\omega)\left\{\frac{\partial G(\vec{r}|\vec{r}_0,\omega)}{\partial n_0}-\frac{j\omega\rho_0}{Z_s(\vec{r}_0,\omega)}G(\vec{r}|\vec{r}_0,\omega)\right\}\right]dS_0, \qquad \text{(A11)}$$

where $\vec{r}_s$ is substituted by $\vec{r}_0$ because they indicate an arbitrary position on S_{sc}. This equation implies that the surface pressure on the scatterer $P_{tot}(\vec{r}_0,\omega)\big|_{S_{sc}}$ uniquely determines the incident field $P_{in}(\vec{r},\omega)$ in the region of interest. It is noteworthy that this equation is equivalent to the spherical harmonics expansion if the scatterer is a rigid sphere. In other words, the rigid sphere case is a special case of Eqn.(A11).

If the assumption in Eqn.(8) is not satisfied, then Eqn.(A11) contains error. That is, if the scattered field on the outer boundary S_{out} is not negligible, then Eqn.(A11) is rewritten as

$$P(\vec{r},\omega)$$
$$=-\int_{S_{sc}}\left[P_{tot}(\vec{r}_0,\omega)\left\{\frac{\partial G(\vec{r}|\vec{r}_0,\omega)}{\partial n_0}-\frac{j\omega\rho_0}{Z_s(\vec{r}_0,\omega)}G(\vec{r}|\vec{r}_0,\omega)\right\}\right]dS_0$$
$$-\int_{S_{out}}\left[P_{sc}(\vec{r}_0,\omega)\frac{\partial G(\vec{r}|\vec{r}_0,\omega)}{\partial n_0}-G(\vec{r}|\vec{r}_0,\omega)\frac{\partial P_{sc}(\vec{r}_0,\omega)}{\partial n_0}\right]dS_0. \qquad \text{(A12)}$$

The second term in the right-hand-side represents the error in Eqn. (A11). The error is determined by the pressure and velocity of the scattered field on the outer boundary S_{out}. This means that the scatterer has to be located far from the outer boundary enough that the existence of the scatterer does not affect the sound field on the outer boundary. This limits the use of the scatterer, including the rigid sphere, in small spaces, or near the outer boundary even in large spaces compared with the size of the scatterer.

On the other hand, let us denote an arbitrary position on the boundary surface of the region of interest as $\vec{r}_0$ as illustrated in Figure A3. By the Kirchhoff-Helmholtz integral equation, incident sound field inside the surface S_{sc} can be expressed as

$$P_{in}(\vec{r},\omega)=\int_{S_d}\left[P_{in}(\vec{r}_0,\omega)\frac{\partial G(\vec{r}|\vec{r}_0,\omega)}{\partial n_0}-G(\vec{r}|\vec{r}_0,\omega)\frac{\partial P_{in}(\vec{r}_0,\omega)}{\partial n_0}\right]dS_0 \cdot \quad \text{(A13)}$$

The surface pressure on S_{sc} can be also expressed by the Kirchhoff-Helmholtz integral equation as follows:

$$\frac{1}{2}P_{in}(\vec{r},\omega)=\int_{S_d}\left[P_{in}(\vec{r}_0,\omega)\frac{\partial G(\vec{r}|\vec{r}_0,\omega)}{\partial n_0}-G(\vec{r}|\vec{r}_0,\omega)\frac{\partial P_{in}(\vec{r}_0,\omega)}{\partial n_0}\right]dS_0 , \quad \text{(A14)}$$

where $\vec{r}$ indicates an arbitrary position on the boundary. This equation gives the relation between sound pressure and velocity on the surface S_{sc}. In addition, it is noteworthy that this incident sound field inside the surface $P_{in}(\vec{r},\omega)$ is obtained by Eqn. (A11) as well, and then we have the following relation:

$$P_{in}(\vec{r},\omega)=-\int_{S_{sc}}\left[P_{tot}(\vec{r}_0,\omega)\left\{\frac{\partial G(\vec{r}|\vec{r}_0,\omega)}{\partial n_0}-\frac{j\omega\rho_0}{Z_s(\vec{r}_0,\omega)}G(\vec{r}|\vec{r}_0,\omega)\right\}\right]dS_0$$
$$=\int_{S_d}\left[P_{in}(\vec{r}_0,\omega)\frac{\partial G(\vec{r}|\vec{r}_0,\omega)}{\partial n_0}-G(\vec{r}|\vec{r}_0,\omega)\frac{\partial P_{in}(\vec{r}_0,\omega)}{\partial n_0}\right]dS_0. \quad \text{(A15)}$$

We have two unknowns, $P_{in}(\vec{r}_0)\big|_{S_d}$ and $\partial P_{in}(\vec{r}_0)/\partial n_0\big|_{S_d}$, and two equations, Eqns.(A14) and (A15), and then we can obtain the two unknowns. This can be carried out using a discretization of the integral.

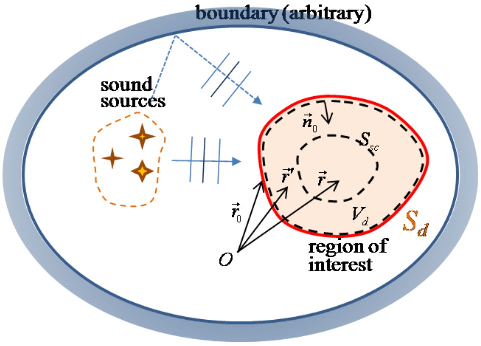

Figure A3. The region of integration of Eqns. (A13): the region of interest is divided to two regions: the region inside the scatterer and the region outside the scatterer.

REFERENCES

1 P. B. Fellgett, "Ambisonic reproduction of directionality in surround-sound systems," Nature, Vol. 252, pp.534-538, December 13 (1974)
2 A. J. Berkhout, D. de Vries, and P. Vogel, "Acoustic control by wave field synthesis," J. Acoust. Soc. Am., Vol. 93, No. 5 (1993)
3 Ole Kirkeby and Philip A. Nelson, "Reproduction of plane wave sound fields," J. Acoust. Soc. Am. Vol . 94, No. 5, pp. 2992-3000 (1993)
4 S. Ise, "A principle of sound field control based on the Kirchhoff-Helmholtz integral equation and the theory of inverse systems," Acoustica, vol. 85, pp. 78-87 (1999)
5 Rozenn Nicol and Marc Emerit, "3D-Sound Reproduction over an Extensive Listening Area: A Hybrid Method Derived from Holophony and Ambisonic," Audio Engineering Society, 16th International Conference, pp. 436-453 (1999)
6 Darren B. Ward and Thushara D. Abhayapala, "Reproduction of a Plane-Wave Sound Field Using an Array of Loudspeakers," IEEE TRANSACTIONS ON SPEECH AND AUDIO PROCESSING, Vol. 9, No. 6 (2001)
7 M. A. Poletti, "Three-dimensional surround sound systems based on spherical harmonics," J. Audio Eng. Soc., Vol. 53, No. 11, pp.1004-1025 (2005)
8 J.-W. Choi, *Spatial manipulation and implementation of sound*, Doctorial thesis, KAIST, pp.20-99 (2005)
9 J.-H. Chang and Y.-H. Kim, "A planewave generation method by wavenumber domain point focusing," J. Acoust. Soc. Am (in press, 2010)
10 T. D. Abhayapala and D. B. Ward, "Theory and design of high order sound field microphones using spherical microphone array," proceedings of ICASSP 2002, pp.1949-1952
11 J. Meyer and G. Elko, "A highly scalable spherical microphone array based on an orthonormal decomposition of the sound field," proceedings of ICASSP 2002, Vol. II, pp.1781-1784