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Quantifying Elasticity and Viscosity of Urethane Rubber Using Shearwave Dispersion Ultrasound Vibrometry (SDUV) and the Embedded Sphere Method

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ABSTRACT

Diastolic dysfunction is the inability of the left ventricle (LV) to supply sufficient stroke volumes under physiological conditions and is often accompanied by LV myocardial stiffening. Our group has been investigating the use of *Shearwave Dispersion Ultrasound Vibrometry (SDUV)*, a noninvasive ultrasound based method for quantifying viscoelasticity of the soft tissues. The primary motive of this study is the design and testing of a viscoelastic material suitable for validation of the Lamb wave model in the heart. Here, we report the results of quantifying viscoelasticity of urethane rubber samples using SDUV and our embedded sphere method. A urethane plate was embedded in gelatin inside a plastic container and mounted on a stand inside a water tank. A mechanical actuator was used to induce harmonic waves in the frequency range 40 – 500 Hz. A pulse-echo transducer was used to detect the motion at multiple points away from the excitation point. Linear regression of the phase data provided estimates of shear wave speed at each frequency (shear wave dispersion). An antisymmetric Lamb wave model was fitted to the dispersion data to estimate elasticity and viscosity of the material. ABAQUS finite element model (FEM) of a viscoelastic plate submerged in water was used to study the appropriateness of the Lamb wave dispersion equations. An embedded sphere method was used as an independent measurement of the viscoelasticity of the urethane rubber. The FEM dispersion data were in excellent agreement with the theoretical predictions. Elasticity and viscosity of the urethane rubber were 39.8±1.3 kPa and 5.0±0.4 Pa·s, using SDUV, and 42.5±2.8 kPa and 5.2±0.4 Pa·s, using the embedded sphere method.

INTRODUCTION

Close to 50% of patients presenting with heart failure have preserved systole and thus are caused by diastolic dysfunction [1]. Diastolic dysfunction is the inability of the heart, namely the left ventricle, to supply sufficient stroke volumes under physiological conditions and is often accompanied by stiffening of the left ventricular (LV) free-wall myocardium. Quantifying material properties of the myocardium noninvasively would be higly beneficial in clinical settings.

Our group has previously reported the use of Shearwave Disperseion Ultrasound Vibrometry (SDUV) to quantify viscoelasticity of the liver *in vivo* [2]. The SDUV method uses focused ultrasound radiation force to excite shearwaves in the region of interest and a pulse echo transducer to track the motion [3]. The ultrasound radiation force is focused in the middle of the liver and the induced wave motion is well approximated by shear wave propagation in an infinite medium. Due to the focal length of the ultrasound radiation force and geometry of the heart wall, exciting pure shear waves in the myocardial wall can prove to be difficult. Following Kanai et al. [4], the material deformation of the LV free-wall myocardium due to perpendicular excitation force is represented by an anti-symmetric Lamb wave.

The goal of this study is the design and testing of a viscoelastic material for the purpose of validating the Lamb wave SDUV method in the heart. Here, we report the results of quantifying viscoelasticity of urethane rubber samples using Lamb wave SDUV technique and our embedded sphere method [5].

METHODS

Principle of Lamb Wave SDUV

A mechanical actuator (shaker) is used to excite monochromatic shear waves in the region of interest. A pulse echo (detect) transducer is used to track the motion of the medium at multiple points by transmitting ultrasound pulses at the region of interest at a pulse repetition rate of a few kHz, as shown in Figure 1. Cross-spectral correlation of the echos is used to calculate tissue displacement as a function of time [6]. A specialized Kalman filter is applied to the displacement versus time data to extract the motion at the excitation frequency of the push transducer and estimate the shear wave amplitude and phase [7]. Due to the nature of excitation, the medium displacement close to the excitation point is mostly parallel to the excitation axis, characteristic of a cylindrical shear wave.

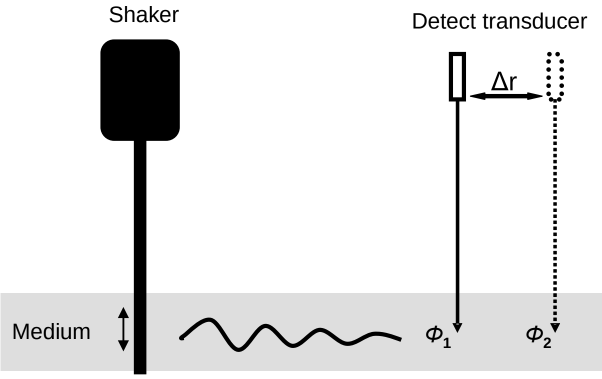

Fig. 1. Principle of Lamb Wave SDUV: a mechanical actuator (shaker) is the source of force that induces harmonic shear wave propagation in medium of interest; the motion is measured by a pulse echo (detect) transducer.

In the far field, the phase delay of such a wave varies linearly with distance [3], so by controlling the frequency of the excitatory harmonic wave, one can estimate the speed of propagation of the shear wave by measuring the change in phase $\Delta\phi = \phi_2 - \phi_1$ over distance from the excitation Δr :

$$c_s = \omega \frac{\Delta r}{\Delta \phi} \quad (1)$$

Cylindrical Anti-Symmetric Lamb Wave Model

A plane anti-symmetric Lamb wave model was proposed by Kanai [4] to represent the motion of the heart septum due to closure of the aortic valve. The frequency response of the myocardium was assumed to obey the Voigt model so that the shear modulus $\mu = \mu_1 + i\omega\mu_2$, where μ_1 and μ_2 are the elastic and viscous moduli. Since the bulk modulus is much larger than the shear modulus, the anti-symmetric Lamb wave model requires the following equation to hold:

$$\begin{aligned} &4k_L^3 \beta \cosh(k_L h)\sinh(\beta h) = \\ &(k_s^2 - 2k_L^2)^2 \sinh(k_L h)\cosh(\beta h)\,, \\ &+ k_s^4 \cosh(k_L h)\cosh(\beta h) \end{aligned} \quad (2)$$

where $k_L = \omega / c_L$ is the Lamb wave number, ω is the angular frequency, c_L is the frequency dependent Lamb wave velocity, $\beta = \sqrt{k_L^2 - k_s^2}$, $k_s = \omega\sqrt{\rho_m/\mu}$ is the shear wave number, ρ_m is the density of the sample and h is the half-thickness of the sample.

It is important to note that the dispersion equations for a plate due to cylindrical and plane waves are identical. Equation (2) is fitted to the Lamb wave dispersion curves (velocity versus frequency) to obtain elasticity and viscosity coefficients μ_1 and μ_2.

Lamb wave SDUV experiment

Urethane rubber samples were prepared by mixing equal amounts by mass of Part A and Part B of Reoflex 20 (Smooth-On, Inc., Easton, PA), and twice the amount of softener (So–Flex Flexibilizer, Smooth-On Inc., Easton, PA). In order to provide scatterers for ultrasound waves, 4% by mass of cellulose powder (Aldrich Chemical Company, Inc., Milwaukee, WI), was added to the mixture. The mixture was poured into an 11 cm x 8 cm x 1.2 cm plastic plate mold and a 5 cm in diameter and 4.5 cm in height cylindrical mold. The urethane was allowed to cure for 24 hours prior to use. The plate was set in gelatin (70% water, 10% glycerol, 10% 300 Bloom gelatin, 10% potassium sorbate preservative, all by volume and all manufactured by Sigma-Aldrich, St. Louis, MO) inside a plastic container and mounted inside a water tank. A window was cut from the bottom and of the container to allow for ultrasound motion detection.

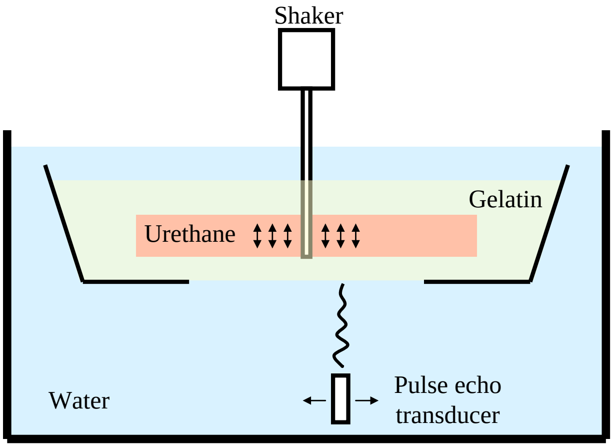

Fig. 2. Experimental set up for the Lamb Wave SDUV approach: a mechanical actuator (shaker) was used to excite shear waves in the urethane rubber sample; pulse echo transducer was used to detect the motion. The gelatin was used as a stabilizer.

A mechanical actuator (V203, Ling Dynamic Systems Limited, Hertfordshire, UK) coupled to a glass rod that was glued through the thickness of the rubber was used to induce harmonic waves (40 – 500 Hz). Motion was measured at each frequency in four orthogonal surface directions using a 5 MHz pulse echo transducer with a pulse repetition rate of 4 kHz. Motion was recorded at 31 points along a line, 0.5 mm apart. Phase data was measured at each point and was used to estimate shear wave speed at each excitation frequency (shear wave dispersion). The antisymmetric Lamb wave model from Equation 2 was fitted to the dispersion data to estimate elasticity and viscosity of the material.

Embedded Sphere Method

Our group has developed an embedded sphere method [5] for estimating material properties of nonbiological material. The method is based on measuring displacement of a solid sphere embedded in the medium of interest due to constant radiation force. A mixture of urethane rubber was poured into a cylindrical mold (as mentioned above) containing a steel sphere in the middle of the sample.

Finite Element Analysis (FEA)

A finite element model of a plate immersed in water was constructed and analysed in ABAQUS 6.8-3. A solid viscoelastic isotropic plate with the dimensions of .5m x .5m x .01m, Young's modulus of 300 kPa, Poisson's ration of .495 and the density of 1080 kg/m^3, had material properties defined in terms of the Prony series where $g_1 = .5$, $\tau_1 = 10^{-6}$. The plate was surrounded by acoustic elements with properties that of water (K = 2.2 GPa, ρ = 1000 kg/m^3). Cylindrical Lamb waves were propagated when the slab was harmonically vibrated through the thickness using a horizontal line source in the frequency range 100 – 500 Hz with excitation displacements of around 10 μm. Horizontal displacements were recorded at every millimeter for 28 millimeters along a line of propagation. The displacement data was analyzed in the 2-D Fourier domian to calculate the propagation velocity. Antisymmetric Lamb wave dispersion equation with the material properties defined by the Prony series (above) was plotted next to the FEA dispersion data for the purpose of comparison.

RESULTS

Figure 3 shows the displacement at 28 different points in time, 1 mm apart, in the middle of an isotropic plate submerged in water. The motion is caused by a harmonic line excitation at 200 Hz through the thickness of the plate. The particle displacement shown in Figure 3 is along the axis parallel to the line of excitation. This data was used to create the k-space of the displacement field by performing a two-dimmentional Discrete Fourier transform with respect to one spatial and one time component.

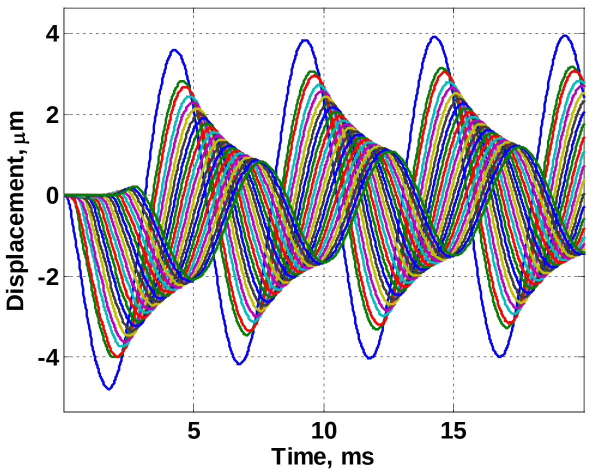

Fig. 3. ABAQUS displacement at 28 different points along the line of cylindrical wave propagation in an isotropic plate submerged in water, 1 mm apart, at the excitation frequency of 200 Hz.

The k-space has dimensions of 1/λ and frequency, so one can calculate the speed of Lamb wave propagation at the given frequency by dividing it by the wave number coordinate of each peak in k-space (Figure 4).

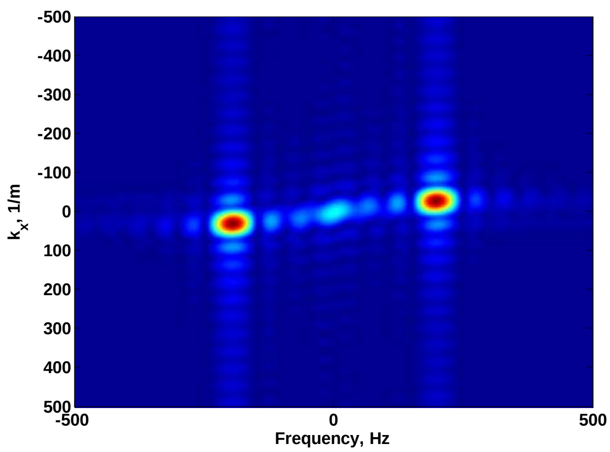

Fig. 4. Two-dimmensional Discrete Fourier Transform of the data presented in Figure 3 yeilds the k-space map that can be used for calculation of the Lamb wave speed at the given excitation frequency.

This method was repeated for multiple frequencies of excitation, ranging from 100 to 500 Hz. The calculated Lamb wave velocities at each frequency form the dispersion curve shown in red in Figure 5. The material properties of the plate used in the FEA simulations were inserted in the analytical expression in Equation 2 in order to produce the theoretical prediction of the Lamb wave dispersion (blue in Figure 5).

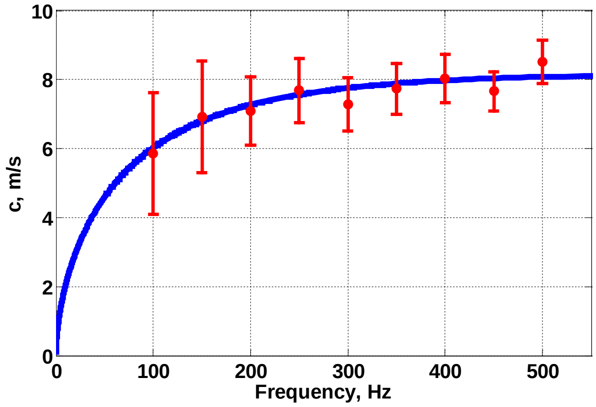

Fig. 5. Lamb wave ABAQUS simulations: Lamb wave dispersion equation in blue is fitted to the finite element method Lamb wave speeds ± one standard deviation shown in red.

The Lamb wave SDUV method was used to measure the Lamb wave propagation speeds at multiple frequencies in four orthogonal directions on the surface of a urethane rubber sample. The experimentally obtained Lamb wave velocities in two of the four directions are shown in blue in Figures 6A and 6B. The Lamb wave model (Equation 2) was fitted to the experimental data (red line) to estimate the elasticity and viscosity of the rubber sample. The estimated values for the two directions are shown above the plots. The summary of the results in four orthogonal directions with averages and standard deviations are shown in Table I.

TABLE I
ELASTICITY AND VISCOSITY RESULTS OF URETHANE RUBBER OBTAINED USING LAMB SDUV AND THE EMBEDDED SPHERE METHODS

Method	Elasticity (μ_1) (kPa)	Viscosity (μ_2) (Pa·s)
Lamb SDUV	39.8 ± 1.3	5.0 ± 0.4
Embedded Sphere	42.5 ± 2.8	5.2 ± 0.4

Table I. Results of measuring elastic and viscous moduli of a urethane rubber sample using the Lamb wave SDUV and the embedded sphere methods.

The embedded sphere method has been previously validated by comparison with mechanical testing [5] and was used for the purpose of validation of the Lamb wave SDUV results. The results of comparison are summarized in Table I.

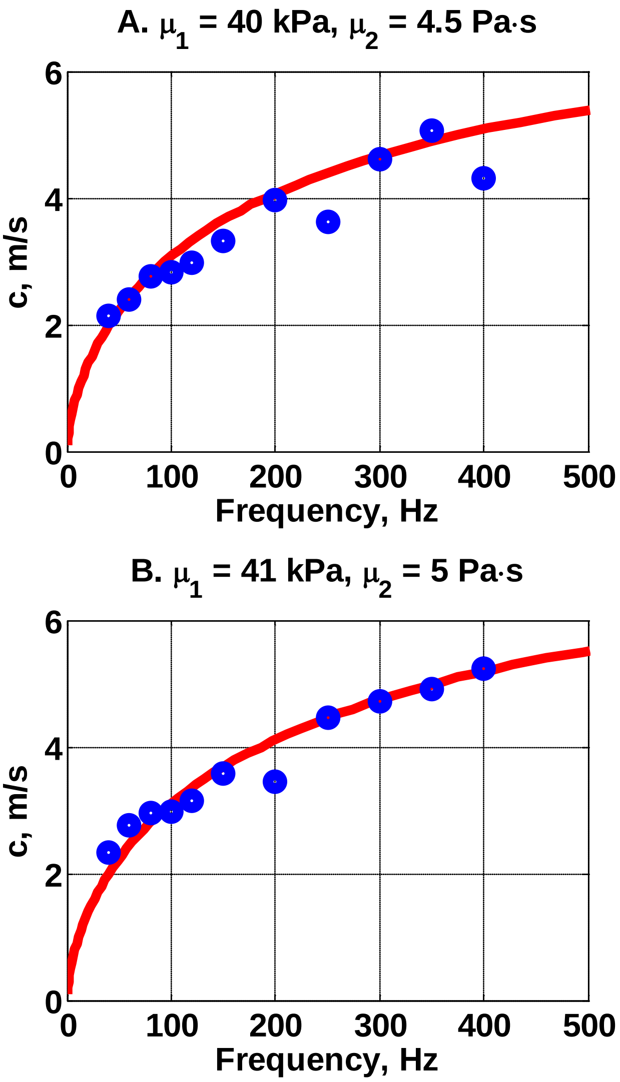

Fig. 6A and 6B. Urethane rubber: experimentally obtained shear wave dispersion curves are shown as blue circles. Lamb wave model (red line) was fitted to the data to obtain the values of elasticity and viscosity μ_1 and μ_2.

DISCUSSION

The finite element model of a viscoelastic isotropic plate submerged in water produced Lamb wave dispersion velocities that were consistent with the theoretical predictions. This is encouraging and will allow us to gain insight into Lamb wave propagation using finite element analysis. We intent to explore the feasibility of using direct inversion to estimate model free material properties and ABAQUS simulations will be a useful tool for this purpose. The elasticity and viscosity of the urethane rubber material obtained using the embedded sphere and the Lamb wave SDUV method are in good agreement. Further studies will be aimed towards validation of the Lamb wave SDUV approach. The elasticity and viscosity of the urethane rubber plate in four orthogonal surface directions are fairly similar. This is to be expected since the urethane rubber plate is a fairly uniform material. The use of the shaker for excitation of shear waves could violate some of our assumptions. We assumed that the excited waves are purely cylindrical throughout the sample, but the glass rod is a spherical object with a finite diameter and is not a perfect line source, so and our results could be influenced by the near-field affect. To our best knowledge, the appropriateness of the Voigt model in urethane rubber has not yet been confirmed. In the future, we intend to explore the feasibility of model free estimates using SDUV.

CONCLUSIONS

The agreement of the FEM and the theoretically predicted Lamb wave dispersion suggests that the mathematical model accurately describes the motion of the medium. The values of elasticity and viscosity measured using the SDUV and embedded sphere methods agree within one standard deviation, suggesting that the SDUV method has the capacity to produce accurate estimates of material properties of viscoelastic rubber plates.

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