

Vibration Modelling of Composite Laminates with Delamination Damage

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ABSTRACT

Composite materials are widely used in aeronautical, marine and automotive industries, because of their excellent mechanical properties, low density and ease of manufacture. However, composite laminates are susceptible to delaminations, which may not be visible externally, but can substantially affect the performance of the structure. The final objective of this research is to develop a Structural Health Monitoring (SHM) system based on vibration monitoring, to detect, locate and assess delamination damage in laminated composite structures. Towards this end, finite element modelling is employed to simulate the dynamic response of composite laminates (beams and plates) with delamination and extract their vibration parameters. According to the reported theoretical models, we built two finite element (FE) models using Ansys, one free and the other constrained. Modal analysis is applied to the free FE model to extract eigen frequencies while transient analysis is used to get natural frequencies from the constrained model where contact elements haven't been ignored as in the former. The natural frequencies extracted from the current numerical simulations are compared with previous theoretical, numerical and experimental results as well as from our experimental modal analysis conducted on composite beams with delaminations.

1 INTRODUCTION

1.1 Composite materials and delamination damage

Composites are categorised by those materials that are a macroscopic combination of two or more of other structural materials. Generally a composite consists of two phases, the reinforcing phase and the matrix phase. Fibre-reinforced Polymer (FRP) is an important member in the composites family which uses fibres like carbon fibres or glass fibres as reinforcement. Composite materials are widely used in aeronautical, naval and automotive industries because of their high strength-to-weight ratio and stiffness-to-weight ratio. However, damage can be caused during manufacture or during service in the form of matrix cracking, fibre breakage and delaminations. Delamination [1], probably the most frequently occurring damage, appears as a debonding of adjoining plies in laminated composites due to the weak shear strength of polymer matrix in the through-the-thickness direction. Therefore, it is very significant to detect the existence of delamination at an early age, and further study the damage size and location if the composite structure is delaminated.

1.2 Vibration monitoring for Structural Health Monitoring

There are a number of methods widely used for damage detection in engineering industries such as ultrasonic techniques, eddy-current technology and radiography. However, these traditional non-destructive inspection (NDI) methods are not applicable for Structural Health Monitoring (SHM) since they are not able to monitor integrity of the structure in real-time and in situ [2]. Vibration-based methods, which use vibration characteristics of the structures, namely their natural

frequencies, mode shapes and damping ratios to detect damage are among the promising SHM techniques since the operation of machines often generates vibration [3]. Compared to mode shapes or damping ratios, natural frequencies have the advantages that measurements can be conducted only at a single location and measured data is less scattered and more reliable [4].

Our goal is to develop a vibration monitoring method for detecting delaminations in composite structures based on the changes in natural frequencies before and after damage.

Two main existing vibration-based detection algorithms for delamination in composites are model-updating [5] and pattern recognition [6]. Model-updating relies on modifying the model to match the measured dynamic response and pattern recognition such as neural network (NN) depends on creating a large database for training system. Both methods are computationally intensive and time consuming. Some researchers, however, tried to capture the analytical relationship between change in frequency and damage characteristics. Using perturbation method, Gudmundson [7] predicted that eigen frequency changes due to geometric change are proportional to the strain energy stored in the structures. Based on this fundamental fact, Kannappan et al. [8] successfully detected the damage existence, size and location in isotropic beams with different crack configurations and extended their research to isotropic plates. In the proposed method, we will follow the same line to establish the theoretical relationship between frequency change and delamination details in composite laminates using energy formulation. Towards this end, our research will include three parts: development of the theory, finite element (FE) models (using ANSYS 12.0) and experimental modal analysis. In this paper, the preliminary study of FE models for intact and delaminated composite beams and

plates as well as vibration testing on carbon/epoxy laminated beams is presented.

2 THEORETICAL VIBRATION MODELS FOR DELAMINATED BEAM

One of the earliest models for vibration analysis of composite beams with delaminations was proposed by Ramkumar et al. [9]. They modeled a beam with one through-width delamination by simply using four Timoshenko beams connected at delamination edges. Natural frequencies and mode shapes were solved by a boundary eigenvalue problem. However, the predicted natural frequencies were consistently lower than the experimental results. Ramkumar et al. suggested that the inclusion of the contact effect may improve the analytical prediction since they assume the sublaminates vibrate freely. Instead of taking this suggestion, Wang et al. [10] improved the analytical solution by including the coupling between flexural and axial vibrations of the delaminated sublaminates. They found that the calculated natural frequencies were closer to experimental results indicating that flexural and axial coupling is significant to study dynamic characteristic. However, their model did not consider contact effect within delaminated segments, and is referred as “free model” by later researchers. Later, Mujumdar and Suryanarayan [11] pointed out that some delamination opening mode shapes predicted by Wang et al. are physically incompatible because of possible overlap between the delaminated sublaminates. To avoid this kind of incompatibility and to keep a linear model, they imposed a constraint between the delaminated parts to have the same flexural deformations. It was also assumed that there is no separation between the layers in the delamination region under all conditions; hence this model was called the “constrained model” in contrast to the “free model” proposed by Wang.

However, free or constrained, neither model can capture the delamination opening and closing behavior observed in the experiments [12]. To the author's best knowledge there is no analytical model, so far, which can be used to obtain accurate predictions of frequency change due to delaminations because of the possible “opening and closing” phenomenon during vibration. But this does not necessarily mean these models have no value for studying the vibration behavior of delaminated composite beams. Actually, current theoretical models are helpful to get a good understanding of vibration behaviour of beams with delamination. Especially in some cases, when the delamination length is small and is located close to the geometric mid-plane, natural frequencies obtained from free models are reported to agree very well with experimental data, because in such cases, there is no significant delamination opening and closing during vibration and hence the nonlinear model approaches a linear one [13].

3 FINITE ELEMENT MODEL USING ANSYS

In the present research, we use the commercial finite element software ANSYS to build FE models for both undamaged and delaminated Carbon Fiber Reinforced (CFR) composite beams and plates to study their vibration behavior. The solid185, a 3D eight-node layered structural solid element with three degrees of freedom at each node is adopted. The layer information is defined by using shell section. For each model, a sensitivity study is first done to observe how the meshing parameters affect the convergence of the numerical results.

According to the theoretical “free model” and “constrained model”, we built two FE models for delaminated beams and plates: free FE model and constrained FE model.

In the free FE model, nodes in the delaminated part are allowed to overlap but left to be separate. Eigen modal analysis is carried out to extract the natural frequencies of flexural bending modes using the Block Lanczos method (torsion and in-plane bending modes are filtered by checking mode shapes).

In the constrained FE model, we add “no separation” contacts between the delaminated segments. “No separation” is one of the many contact behaviours provided by ANSYS in which the target and contact surfaces are tied (sliding is permitted) for the remainder of the analysis once contact is established. This exactly matches the assumption in Mujumdar's “constrained model”. However, since contact introduces nonlinearity to the model, modal analysis cannot be used to extract the natural frequencies since it relies on the eigenvalues of the stiffness matrix. Without the benefit of the eigenvalues, the natural frequencies have to be obtained from an accurate enough dynamic simulation the physical modal testing process. Hence we conduct transient analysis for our constrained FE model.

4 EXPERIMENTAL TESTS

Vibration tests were conducted on eight-layer carbon/epoxy composite beams with artificial delaminations. A total of 10 specimens (five unidirectional beams and five quasi-isotropic ones [45/-45/0/90]s) were manufactured with two samples without delaminations and eight samples having mid-plane delaminations with different sizes and lengthwise locations. The delaminations were introduced by inserting Teflon films as suggested by researchers [9]. **Figure 1** shows the dimensions and delamination details of composite specimens used in the study. A 30 mm long through-width delamination is located at 50mm, 125mm and 200mm away from the fixed end, i.e., at 25%, 50% and 75% of the length of the beam in samples U2, U3 and U4. U1 is the undelaminated sample while, sample U4 has a 90mm long delamination located at 50% of its length. The U samples shown in Figure 1 are the unidirectional specimens; the quasi-isotropic samples also had delaminations of similar size at similar locations. **Table 1** lists the details of beam dimensions and material properties. E_1 , E_2 are the Young's modulus in fibre direction and in the transverse direction, respectively. G_{12} is the in-plane shear modulus.

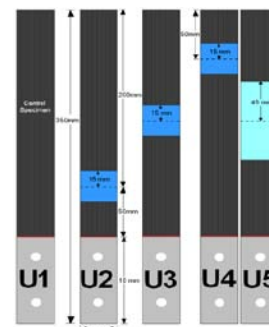


Figure 1 Location and size of simulated delaminations

Table 1 Specimen dimension and material properties

Length (mm)	250
Width (mm)	25
Thickness (mm)	1.88
E_1 (GPa)	65
E_2 (GPa)	8
G_{12} (GPa)	5
Poisson's ratio	0.33
Density (Kg/m^3)	1477

A Pulse system (type 3560-B-040) from Brüel & Kjær (B&K) was used for signal generation and acquisition. One end of the composite beam was clamped for cantilever boundary condition. Marks close to the clamped end of the beams were made in advance. An impact hammer (B&K type 8202) was used to hit the beam ten times at the marked point and the data averaged for each test. Five repeated tests were conducted for each beam. The force was measured using a force transducer (B&K type 8200) with charge amplifier (B&K type 2635). An accelerometer (B&K type 4374) was mounted at the tip and the middle of the beam to measure its dynamic response.

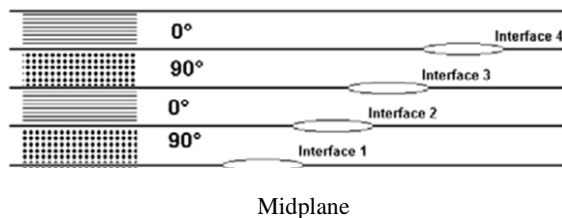
5 RESULTS AND DISCUSSIONS

5.1 Numerical results

Firstly, static analysis on the composite beam and plate FE models was performed for model validation by comparing the numerical results with the theoretical predictions. The comparison shows good agreement between the numerical and analytical values (discrepancies are mostly lower than 0.8%).

5.1.1 Modal analysis

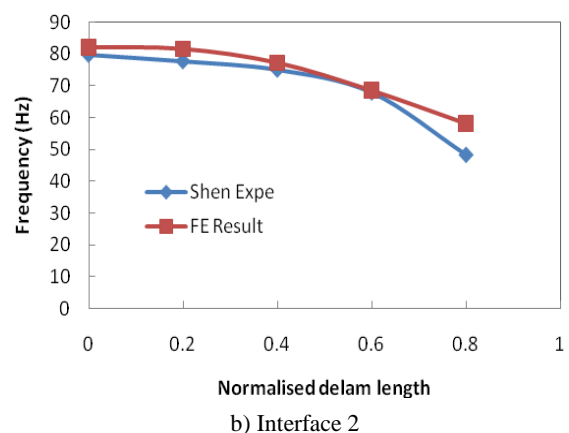
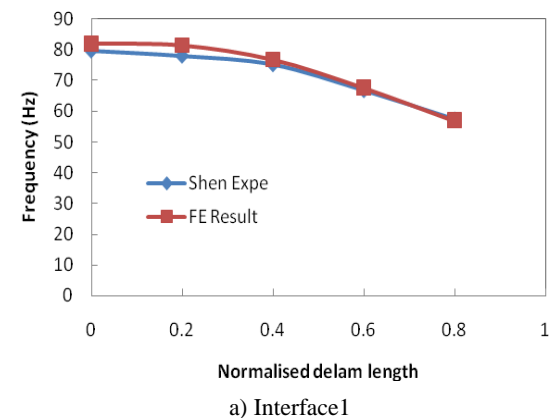
Modal analysis was conducted on ANSYS to extract natural frequencies for the undamaged beam and the delaminated beams (free FE model). The frequencies from the FE modal analysis were compared with reported experimental results from Shen and Grady's work. Shen et al. [12] experimentally studied the vibration behaviour of 8-layer $[(0/90/0/90)_s]$ carbon-epoxy beams with delaminations of four sizes (25.4, 50.8, 76.2 and 101.6 mm) centrally located at four interfaces (Figure 2). Dimensions and material properties tested by Shen et al. are provided in Table 1. Comparison between the present FE results and Shen's experimental results are shown in Figure 3.

**Figure 2** The stacking configuration of Shen's specimens[14]**Table 2** Dimension and material properties of Shen's specimens[12]

Length (mm)	127
Width (mm)	12.7
Thickness (mm)	1.016
E_1 (GPa)	134.55
E_2 (GPa)	10.35
G_{12} (GPa)	5.0025
Poisson's ratio	0.33
Density (Kg/m^3)	1477.05

Source: (Shen and Grady, 1992)

Figure 3 a) to d) respectively show the fundamental frequency against delamination length for delaminations located at interface 1 to 4. As can be seen, the FE results are in good agreement with experimental data for delaminations located at the first three interfaces. But when the delamination is located close to the free surface (interface 4), the discrepancy between our FE model and Shen's experimental data is substantial. This is likely to be due to the "opening and closing" behavior of the delamination (Figure 4) during vibration. The "free" FE model captures the opening behaviour (separation between the sub-laminates) but does not allow for contact pressure between them. The study of models with contact elements is described in section 5.1.2.



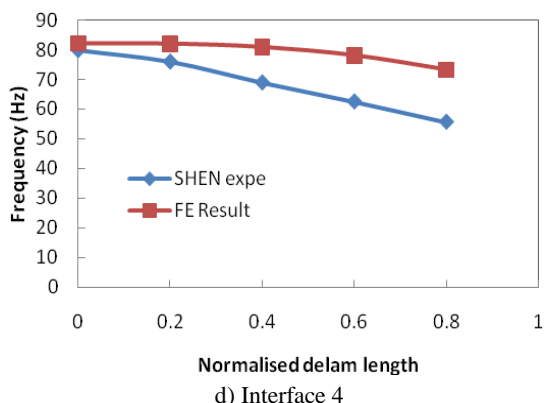
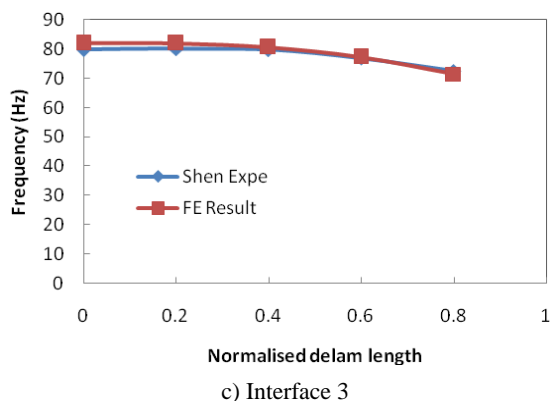


Figure 3 Comparison of present FE results with published test data for cantilever composite beam

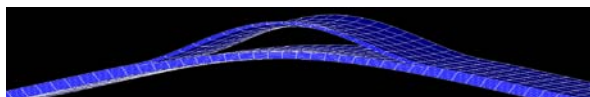


Figure 4 Delamination opening during vibration

The comparison in Figure 3 shows that the “free” FE model is capable of providing accurate predictions for natural frequencies of delaminated beams when the delamination is located close to the geometric mid-plane or if the delamination is small. This is attributed to there being no significant delamination opening and closing during vibration, which allows the nonlinear model to be represented by a linear one[13]. Therefore, we have used the free FE model to study how midplane delaminations (with different sizes and lengthwise locations) affect natural frequencies and compared the results with Tracy and Pardoen’s theoretical predictions for simply supported composite beams [15]. The first case is changing the length of delaminations (from zero, undamaged, to full length) which centres in the midplane of the beam. The present results using the free FE model for variation of natural frequencies with normalised delamination length are shown in Figure 5(a) and Tracy’s analytical predictions [15] are plotted in Figure 5(b). Figure 6(a) shows the variation of normalised frequencies for the first four modes against delamination location for a delamination size of 25% beam length located at mid-plane; corresponding results from Tracy’s paper are shown in Figure 6(b).

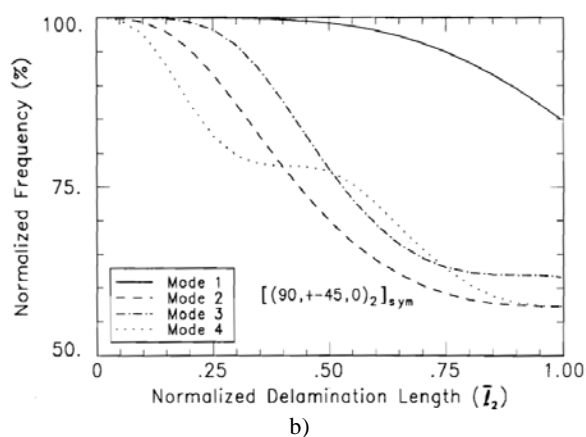
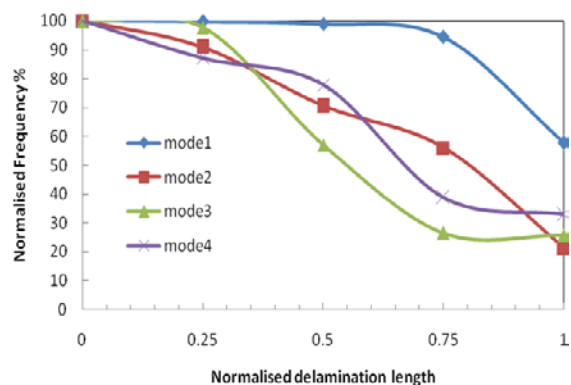


Figure 5 Normalised frequency versus delamination length for simply supported graphite epoxy beam with central midplane delamination (a) Present FE results (b) Tracy’s analytical predictions [15]

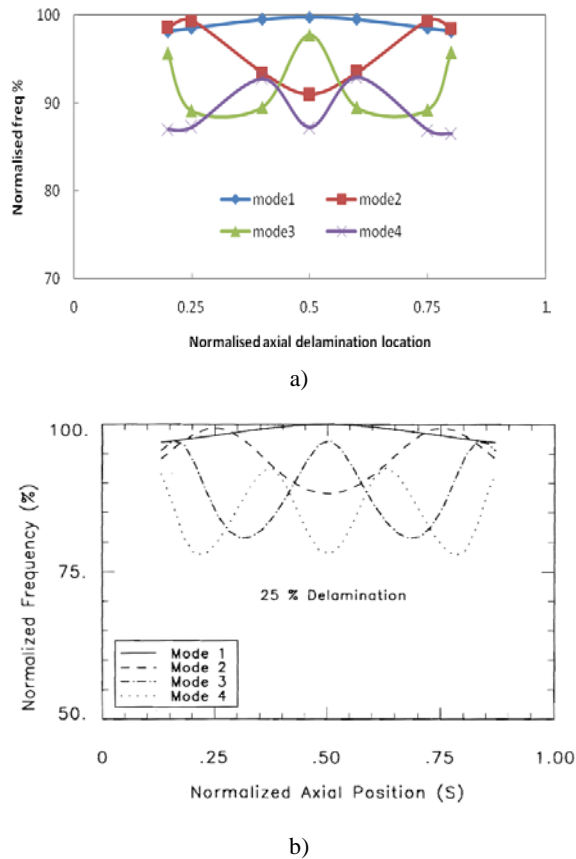
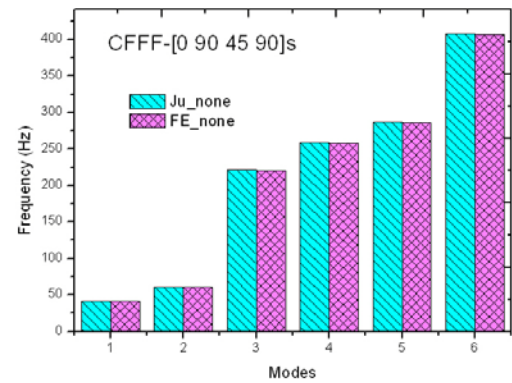


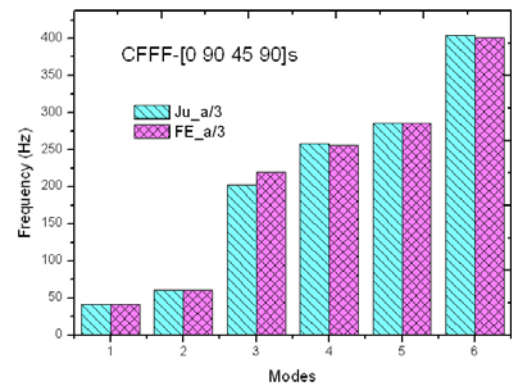
Figure 6 Normalised frequency versus axial delamination location for simply supported graphite/epoxy beam with 25% midplane delamination (a) Present FE results (b) Tracy's analytical predictions [15]

As can be seen, our FE model gives trends that are in good agreement with Tracy's analytical predictions in both cases. **Figure 5** indicates that at shorter delamination lengths all four modes approach the full values of an undelaminated beam. It can be deduced that different modes have different sensitivities to the presence of delamination. For instance, the natural frequency of mode 1 has almost no change when delamination length is 25%, whereas the natural frequency of mode 4 reduces by 15% of the original value. From **Figure 6** we can see that the shapes of the corresponding normalised frequencies versus the normalised axial positions are quite similar between our FE model and Tracy's analytical predictions and the variations of the normalised frequency with axial position are similar to the mode shapes of each mode. As the delamination moves from regions of high shear force to regions of high curvature, the effect of delamination is reduced.

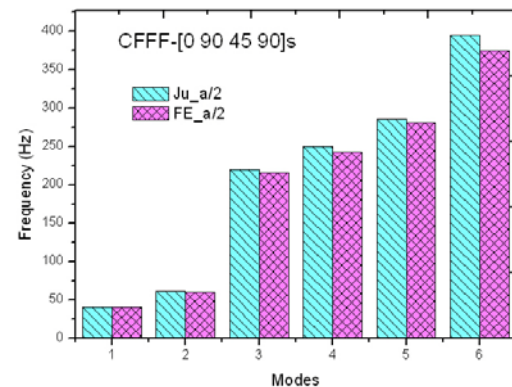
Modal analysis using the free FE model has also been performed on a square composite plate with layup [0 90 45 90]_s and the results compared to Ju et al.'s [16] FE results. Figures 7(a) to 7(c) show the comparison between the first six natural frequencies predicted by the present FE analysis and Ju's numerical results for a cantilever Clamped-Free-Free-Free (CFFF) boundary conditions for no delamination, a square delamination with a width of one-third that of the plate, and a square delamination with a width of one-half that of the plate. As can be seen, our models show very good agreement with Ju et al.'s results for both undamaged and delaminated cases.



a) No delamination



b) Square delamination (width 1/3 of plate width)

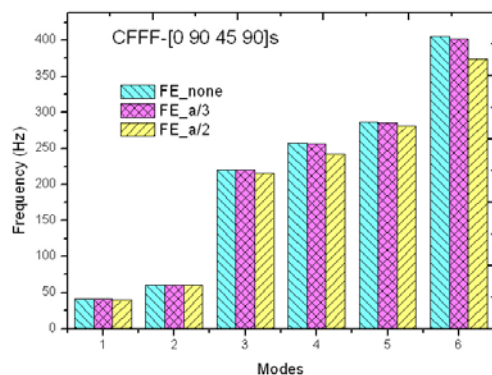


c) Square delamination (width 1/2 of plate width)

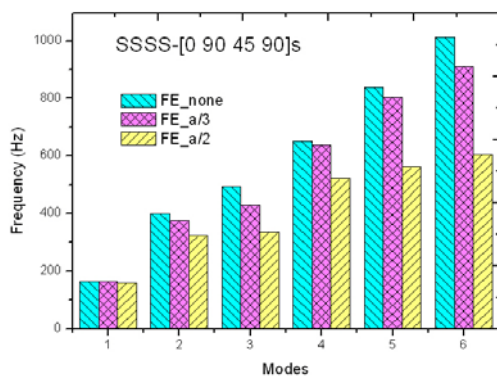
Figure 7 Comparison of present FE results with published numerical data [16] for CFFF square composite plate

Generally, a plate with delamination will experience reduction in natural frequencies due to the loss of stiffness. The extent of the stiffness loss depends on delamination characteristics and the boundary conditions. In the present work, we have studied the effect of increasing delamination size on natural frequencies under three different boundary conditions (**Figure 8**). The modal frequencies with delaminations of one third size and one-half size are compared to those of the un-

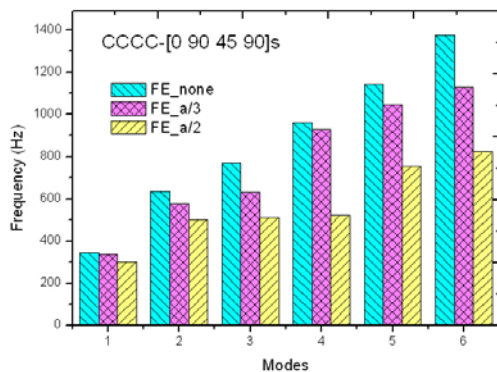
delaminated plate for the first six modes for the cantilever plate in Figure 8(a), for all sides simply supported (SSSS) in Figure 8(b) and for the plate with all sides clamped (CCCC) in Figure 8(c). It is clear that with increasing delamination size, the natural frequencies of composite plate decrease more, and this reduction is higher for the higher modes. By comparing Figure 8(a) to c), we may conclude that the effect of delamination on the natural frequencies is greatly dependent on the boundary conditions. The more strongly the plate is constrained the more reduction of the natural frequencies is experienced.



a) CFFF (cantilevered)



b) SSSS (four edges simply supported)



c) CCCC (four edges clamped)

Figure 8 Numerical frequencies from our plate models (free FE model) and Ju's FE model for delaminated composite plate

5.1.2 Transient analysis

As mentioned before, introducing contacts will introduce non-linearity in the model and modal analysis using solution of the eigen value problem can no longer be adopted. Therefore a transient dynamic analysis is run on the constrained FE model, and then the natural frequencies are extracted using Fast Fourier Transform (FFT) on the temporal nodal displacements.

Since transient analysis can be used for both linear and nonlinear models, we firstly applied transient analysis on the free FE model and compared the frequencies with those predicted by the modal analysis (Figure 9).

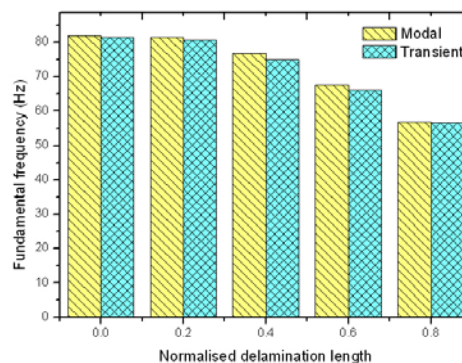


Figure 9 Natural frequencies of graphite epoxy cantilever beam of free FE model using modal analysis and transient analysis

As can be seen, both results agree well showing that transient analysis can be used successfully to extract natural frequencies. However, since transient analysis requires interactions with very small time steps, it takes a long time to get the results and hence computationally expensive. But due to the addition of contact elements (which are non-linear in nature) in our constrained FE model, transient analysis becomes the only solution to get natural frequencies. We have performed transient analysis with a constrained FE model to investigate the case of mid-plane delamination (interface 1) shown earlier in Figure 3(a). Figure 10 shows the comparison of natural frequencies from the transient analysis with those predicted by modal analysis of the free modal and the experimental results from [12]. The constrained model gives slightly higher natural frequencies than the free model which is in accordance with reported literature [13]. This is because we use "no separation" contact behaviour which forces the delaminated segments to vibrate together introducing additional constraints. In the case of delamination in a real laminate, the sub-laminates cannot penetrate each other but are free to separate; thus the real situation has less constraint than the contact model employed here. The modelling of contact which allows sliding and separation but prevents interpenetration is currently under investigation.

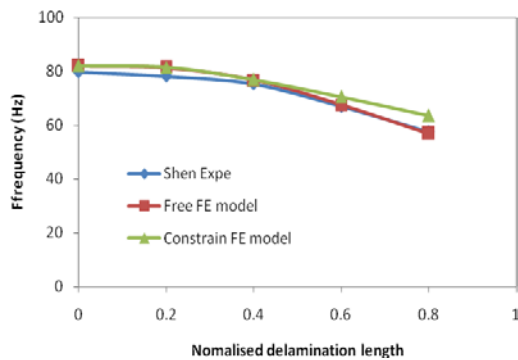


Figure 10 Results comparison among shen's experiment and our free & constrained FE models

5.2 Experimental results

We validated the results from our FE models with the experiments conducted on unidirectional and quasi-isotropic $[45, -45, 0, 90]_s$ carbon/epoxy beams in our laboratory. **Figure 11** shows the comparison of frequencies predicted for the first six modes using the free model for the quasi-isotropic beam and those measured experimentally on the intact beam. The average error between FE model and test results is less than 2%.

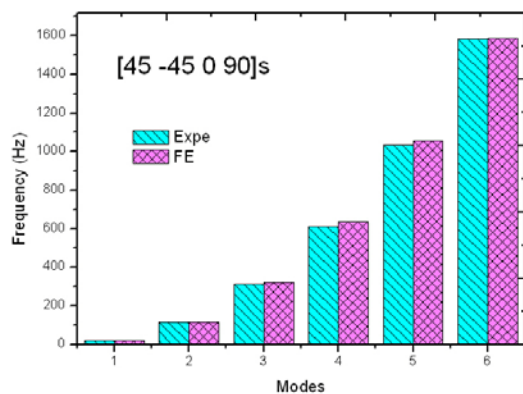


Figure 11 Experimental and numerically predicted frequencies for undamaged quasi-isotropic CFRP specimen

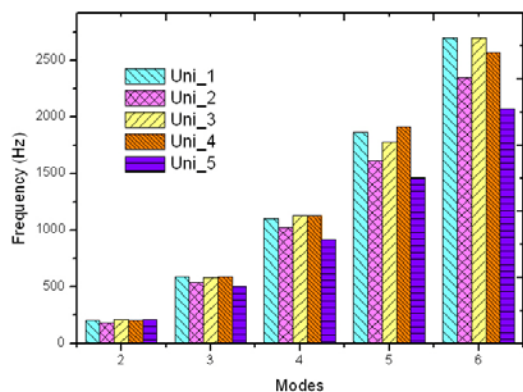


Figure 12 Experimental frequencies of uni-directional CFRP beams with and without delaminations

The experimental measured frequencies for the first six modes for the unidirectional CFRP beam without and with

delaminations (Uni_1 to Uni_5 in Figure1) are plotted in Figure 12. We can see that with increasing delamination size, the reduction of frequencies is larger (compare U3 and U5). For delamination closer to the clamped end, the frequency shift is more (compare U2 to U3 and U4). One more thing worthy to note is that the percentage reduction in frequency is different in different modes. This suggests that it may be possible to use frequency shifts in different modes to uniquely identify delamination size and location.

6 CONCLUSIONS

As a first step towards the goal of solving the inverse problem of detecting delamination in composite structures using natural frequency shifts, the application of finite element analysis using the commercial software ANSYS 12.0 for modelling the vibration behaviour of fibre reinforced composite laminates with delamination has been investigated. If contact between the delaminated sub-laminates can be ignored then a "free" model can be employed and the natural frequencies determined by solution of the linear eigen value problem using modal analysis. Comparison of the present free model results with published experimental and analytical data show that the free model can be employed without significant loss of accuracy if the delamination is located deeper, i.e. away from the free surfaces of the laminate or if the delamination is small. In other cases, a constrained model has to be employed, which requires a non-linear solution using transient analysis and Fourier transform to determine the natural frequencies. The transient analysis conducted shows that a fully constrained contact model, in which the sub-laminates are not permitted to separate, may be too constrained to represent reality and may over predict the frequencies. Our future work will include the building of appropriate contact models and generate a database of results to validate theoretical delamination detection methods.

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