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Inharmonicity of Guitar String Vibration Influenced by Body Resonance and Fingering Position

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ABSTRACT

Sounds of stringed instrument has inharmonic spectrum due to the stiffness of strings. However, not only the string, but also the coupled vibration of string and body, cause on inharmonicity. Therefore, we focus on the coupled vibration of string and guitar body relates to inharmonicity of guitar sound. In addition, when we play a sound of the same pitch using a different string, we choose a different fingering position. Each fingering position to play the same pitch makes the string length unique; therefore changing the position influences on vibration spectrum. In this report, we construct simple model, in which a bridge, which is modeled as a spring-mass system, is connected to one end of a string with stifness to analyze the influences of stiffness and coupled vibration. By measuring parameter of string and guitar, we calculate each vibration mode frequency. Through this model, it is verified that we could express inharmonicity curve with a dip, which does not have reported in convensional theory.

INTRODUCTION

Sounds of stringed instrument has inharmonic spectrum. Inharmonicity, the deviation in frequency from the harmonic modes of vibration, is derived from bending stiffness of the string itself[1]. Bending stiffness is determined by Young's modulus, length, and diameter of string. This property of guitar sound is expected to apply to estimation of plucked string[2]. Jarvalainen *et al* have studied audibility of inharmonicity in acoustic guitar tones through formal listening experiments separately for steel- and nylon-stringed guitars[3]. Oonuki, *et al* have tried to estimate plucked string based on inharmonicity of guitar string vibration modes. They have reported that there are one or two dips among inharmonicity curve[2], which have not reported in the previous studies[4,5]. They considered that the characteristic was derived from resonance of guitar body.

One end of a string is attached to a turning peg, and other end is anchored at a piece attached to the body, therefore natural frequency of string vibration may be influenced by body resonance. In the case of piano, Takasawa *et al* have reported the theory of variation of spectrum and decay rate of piano string vibration[6]. Another study[5] have also reported the inharmonicity curve of guitar sound in the condition of that fundamental frequency of string vibration and resonance frequency of sound board are nearly equal, and there are large differences from the curve reported by Fletcher[1,5]. Previous report by Oonuki *et al*[2] does not consider the condition that the fundamental frequency of string vibration and resonance frequency of sound board are nearly equal. However, a higher mode of string vibration may be influenced by body resonance. Previous studies rarely have discussed about the situation that a higher mode of string vibration may be influenced by body resonance.

We discuss the variation of inharmonicity of guitar string vibration coupled with guitar body system. The body model is expressed by single spring-mass model. With this model, we consider about an influence of body resonance on inharmonicity curve of guitar string vibration.

THEORY

Vibration of string with stiffness

Figure 1 shows definition of coordinate system. Origin is the position of nut. The direction of the bridge along the string and the direction running perpendicular from the body are defined as *x* and *y*, respectively.

The partial differential equation governing the motion of a string with stiffness is given as,

$$EI\frac{\partial^4 y}{\partial x^4} - T\frac{\partial^2 y}{\partial x^2} + \rho\frac{\partial^2 y}{\partial t^2} = 0 \quad . \qquad (1)$$

where x, y, E, I, T, ρ, and l are string position, displacement of string in y direction, Young's modulus, Second moment of area, tension line density of string, and length of string, respectively. When the boundary condition of string is supported at its end, the condition is given as

$$y(x,t) = 0 \text{ at } x = 0, l \, , \qquad (2\text{-}1)$$

$$\frac{\partial^2 y(x,t)}{\partial x^2} = 0 \text{ at } x = 0, l \, , \qquad (2\text{-}2)$$

According to eqs. (1), (2-1), and (2-2), we can find the vibration frequency of n-th mode, f_n, as follows,

$$f_n = f_0 n\sqrt{1+Bn^2}, \quad (3)$$

where

$$B = \frac{\pi^2 EI}{l^2 T}, \quad (4)$$

Here B and f_0 are inharmonicity coefficient and fundamental frequency of string vibration, respectively[1]. I is given to assume the string as a rod of circular section[7],

$$I = \frac{\pi}{64} D^4. \quad (5)$$

Inharmonicity influenced by body resonance

To discuss the influence of body resonance, we construct simple model shown as **Figure 2**. In this model, end-point of string is connected to a spring-mass system. The spring-mass system is model of guitar body and has natural resonance. With this model, the boundary condition in $x = l$ is given as,

$$M \frac{\partial^2 y(l,t)}{\partial t^2} + ky(l,t) = -T \frac{\partial y(l,t)}{\partial x}, \quad (6\text{-}1)$$

$$\frac{\partial^2 y(l,t)}{\partial x^2} = 0, \quad (6\text{-}2)$$

where M and k are effective mass equivalent and elastic modulus of a vibrating mode of bridge including sound board, respectively. According to this boundary condition, we can find p_n, which has following relationship,

$$p_n - \left(\frac{f_{sp}}{f_0}\right)^2 \frac{1}{p_n} = \frac{m_{st}}{\pi M} \cot \pi p_n, \quad (7)$$

where m_{st} and f_{sp} are mass of string and resonance frequency of sound board, respectively. A partial number of influenced by body resonance (non-integer multiple) is expressed as p_n. With adapting p_n into the following equation, we can find vibration frequency of n-th mode influenced by body resonance[1]

$$f_n = f_0 p_n \sqrt{1+Bp_n^2}. \quad (8)$$

The elastic modulus of a vibrating mode, k, is given by adapting f_{sp} and M into the following equation,

$$k = \frac{4\pi^2 M}{f_{sp}}. \quad (9)$$

PROPRETIES OF TEST GUITAR

We compared our simulation result to the measurement result, reported by Oonuki, *et al*[2]. The devices they used are guitar of YAMAHA SLG100S and string of D'Addario EJ 16 (light gauge). Parameters of string are shown in Table 1.

To determine the resonance frequency of guitar body model, f_{sp}, we obtained impulse response of guitar body at the point of bridge with acceleration sensor (Kionix, KXM52-1050). As shown in **Figure 3**, the acceleration sensor was put on guitar bridge. We hit the position of near the sensor. The strings were muted by a towel. **Figure 4** shows measurement result of frequency characteristic of guitar body. The largest peak was observed at frequency of 341 Hz. To compose guitar body model, we set the peak frequency of 341 Hz as natural resonance frequency of the spring-mass system, f_{sp}.

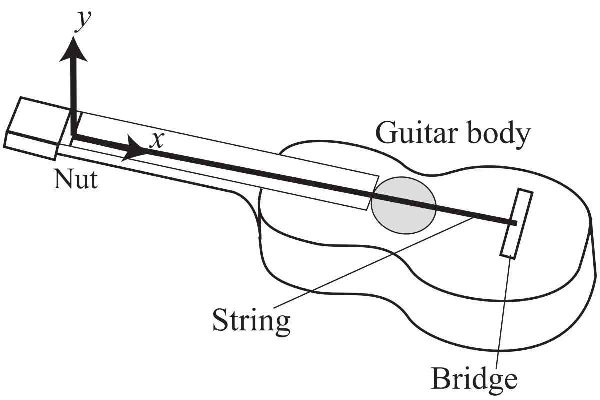


Figure 1. Definition of coordinate system

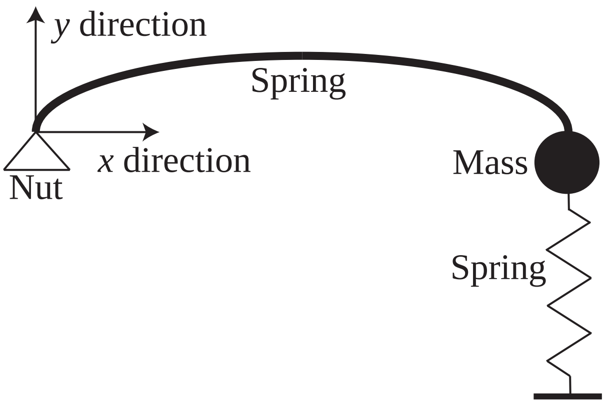


Figure 2. String-body coupled model

Table 1. Parameters of string

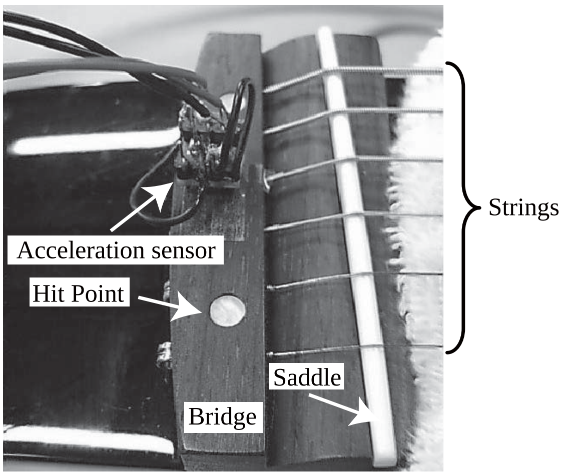
	Diameter (mm)	Tension (kg)
1st	0.30	10.57
2nd	0.41	10.57
3rd	0.61	13.70
4th	0.81	13.83
5th	1.07	13.56
6th	1.35	11.79
Material	Steel (1st and 2nd) Phosphor bronze (3rd, through)	
Young's modulus	220 GPa (1st and 2nd) 120 GPa (3rd, through)	

SIMULATION OF INHARMONICITY

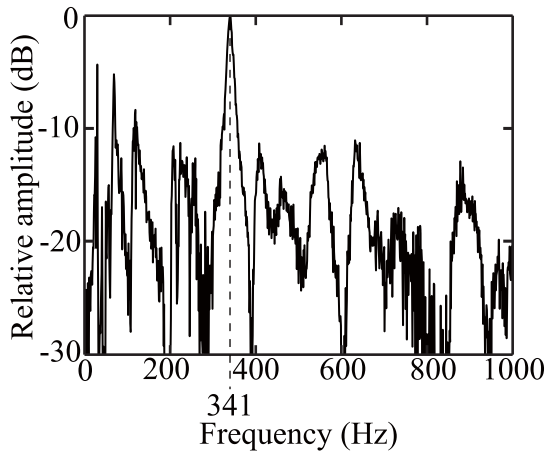
In this chapter, we compare inharmonicity curve based on proposed string-body coupled model and experimental result reported by Oonuki *et al*[2].To calculate inharmonicity curve, we use frequency difference normalized in f_0, d_n, which is defined as,

$$d_n = \frac{f_n - nf_0}{f_0} \quad . \qquad (10)$$

Figure 5 shows inharmonicity curves when the fundamental mode frequency was 82.4 Hz (E2 6th string). Vertical axis shows the frequency difference normalized in f_0, d_n. Horizontal axis shows partial number of string vibration mode. The curve of solid line with circles, ‘Experiment’, shows measurement result reported by Oonuki *et al*[2]. We cited one representative curve from ten curves they measured. Another curve of solid line with squares, ‘String coupled with body’, shows calculated curve based on the proposed string-body coupled model. The dashed line, ‘String only’, shows inharmonicity curve of string with stiffness based on the theory of Fletcher[5]. The curve of string-body coupled model shows a dip near the resonance peak of f_{sp}. This characteristic is not appeared in previous theory[5]. Therefore, with string-body coupled model, we can express the dip of inharmonicity curve, which is appeared in the real string vibration. However, the position and the width of dip observed in simulation and experiment were different. **Figure 6** shows inharmonicity curves when the fundamental frequency was 146.8 Hz (D3 4th string). The curve of ‘Experimental’ shows two dips. However, the curve of the proposed string-body coupled model has only one dip. With this model, we cannot express the characteristic obtained in the experiment. **Figures 7**, **8**, and **9** show results which share the same frequency of fundamental mode. The frequency is 246.9 Hz (B3). Figure 7 is the case of 4th string, Figure. 8 is 3rd string, and Figure. 9 is 2nd string, therefore each curve was different in the fingering position. In Figure 7, the curve of ‘Experimental’ shows two dips, and also in Figure. 8. However, the curve based on the model shows only one dip. In addition, the position of each dip was different. In Figure 9, ‘Experimental’ has a dip; however the position is different from the curve of ‘String coupled with body’.



(Oonuki, 2009)
Figure 3. Measurement of impulse response of guitar body



(Oonuki, 2009)
Figure 4. Frequency character of guitar body

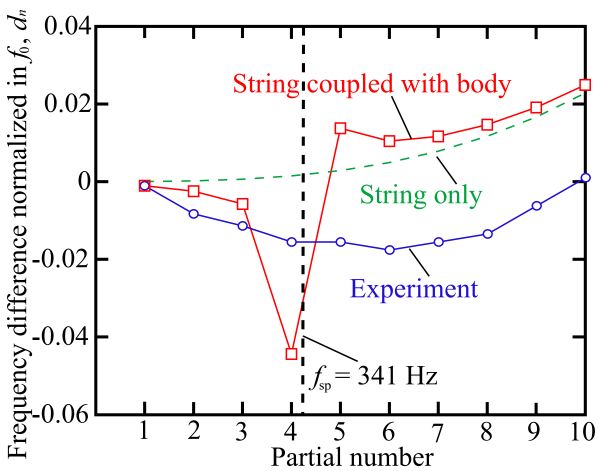


Figure 5. Inharmonicity of vibration modes (E2 82.4 Hz 6th)

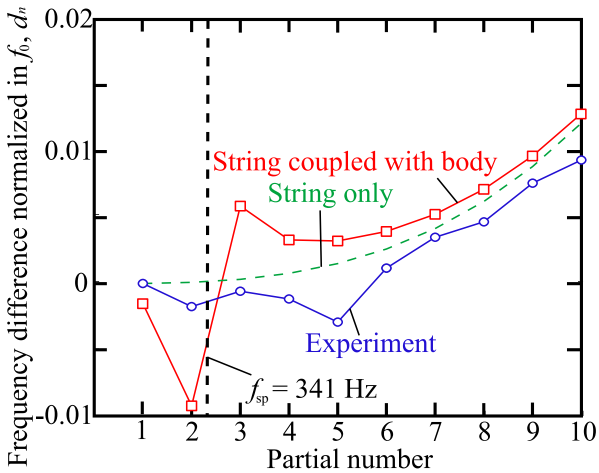


Figure 6. Inharmonicity of vibration modes (D3 146.8 Hz 4th)

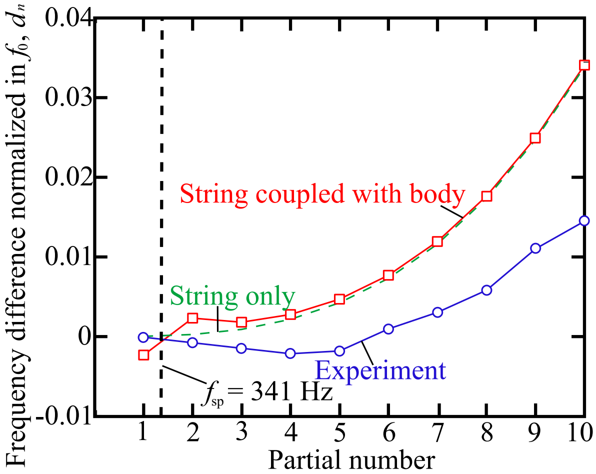


Figure 7. Inharmonicity of vibration modes (B3 246.9 Hz 2nd)

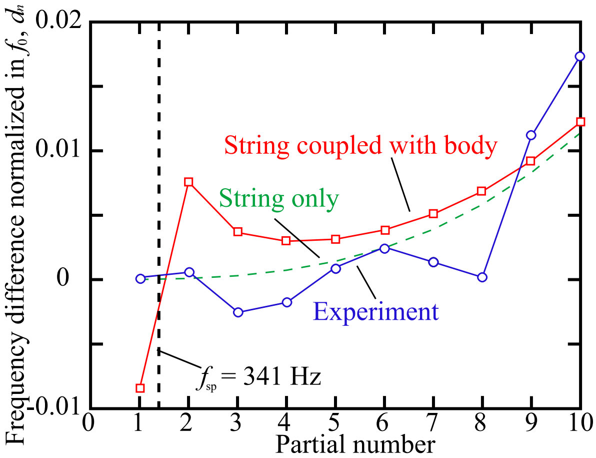


Figure 8. Inharmonicity of vibration modes (B3 246.9 Hz 3rd)

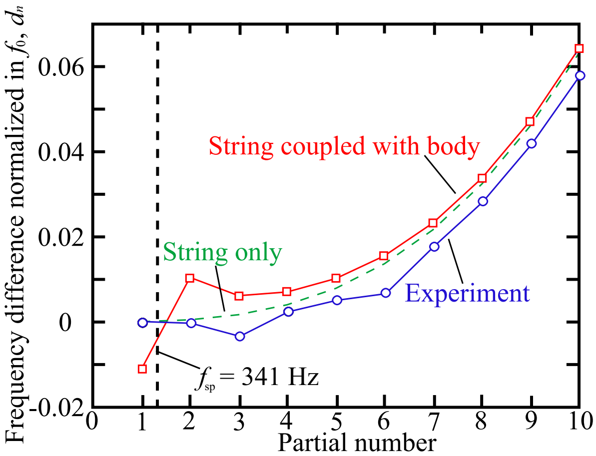


Figure 9. Inharmonicity of vibration modes (B3 246.9 Hz 4th)

DISCUSSION AND CONCLUSIONS

The differences in position and number of dip are expected due to other peaks of frequency character of guitar body in string-body coupled model. This characteristic indicates difference of driving-point admittance: in the case of a rectangle board, another driving-point excites another pattern of resonance peaks[8]. According to this report, we can expect the influence of driving-point admittance. When a string is attached in another position of guitar bridge, the driving-point admittance of guitar bridge is expected to change. For these reason, to make the inharmonicity curve more adequately, we need to measure driving-point admittance of guitar bridge at every string-attachement position. In addition, we also need to set M and k in each string.

We use overwound strings as 3rd, through 6th string. Chumnantas *et al* have reported influence of being wound on inharmonicity of string vibration[9]. To research more detail, we plan to measure string vibration of overwound string without be influenced by any mechanical resonance.

We showed that the characteristics of inharmonicity, which has a dip, can be expressed using a proposed model in which the coupled vibration with string and body is considered. Conventional theory cannot express the cause of the dip, however, with including resonance of guitar body, we can express it.

To make the curve more adequate, the driving-point impedances at many points on the guitar bridge on which the strings are attached. Furthermore, we plan to consider multiple spring-mass systems to reflect the influences of other resonance and expand the model to spring-mass-damper system.

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