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Real-Valued Amplitude Reflection Grating Designs for Sound Diffusion

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ABSTRACT

This paper presents the results of investigations into real-valued alternatives to binary or n-ary based amplitude reflection gratings for sound diffusion. The limitation of these gratings is discussed together with proposed methods of improving performance.

INTRODUCTION

Cost-effective means of achieving sound diffusion are a high priority in modern room acoustic design. Uniformity of listener experience and the reduction of colouration, and other acoustic defects, are goals that are assisted by the inclusion of sound diffusing treatments within critical listening spaces. Recent experiences with this form of diffusion treatment, such as the design shown in Figure 1, have been highly successful motivating further studies.

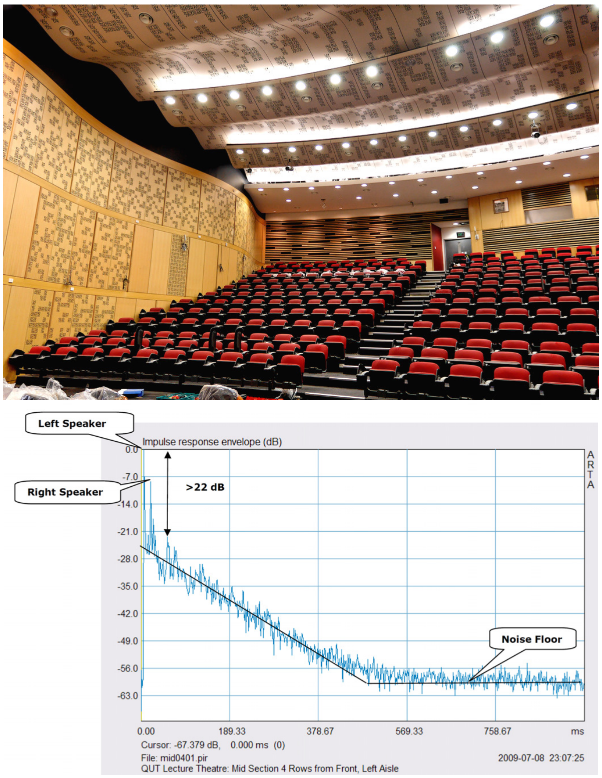


Figure 1. Lecture Theatre Design using Amplitude Reflection Gratings and a Measured Impulse Response Envelope of the Theatre

This paper describes the methods and results of investigations into amplitude reflection arrays expanded to allow real values for the array elements.

OUTLINE

An amplitude reflection grating comprises a surface pattern, or array, of reflective and absorptive zones. The pattern may be repeating or be considered as a finite single pattern.

If the pattern is to be repeated this allows a linear pattern, or sequence, to be converted to a planar (2D) array via the Chinese Remainder Theorem [1], provided it has coprime factors of its length. For example, a sequence with 15 elements may be converted into a 3 x 5 array. Essentially this converts the linear sequence into a diagonal 1D array, however, this construction is not valid unless the array is repeated.

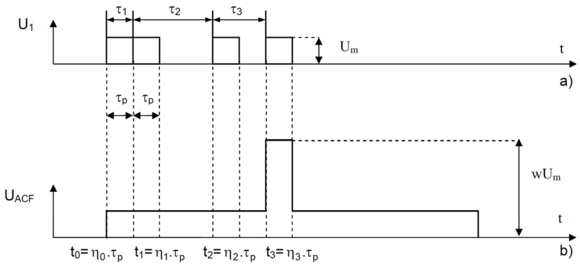
We are interested in the mathematical and physical behaviour of one and two dimensional arrays with the array values representing the sound reflection strength at their position within the array. The array will not have any negative values unless depth offsets relative to the main surface position are incorporated into the design. A negative value will occur at regular but intermittent frequencies where the offset will cause a delay derived 180 phase shift of the reflection relative to the phase at the main surface.

Because of the increased manufacturing complexity in implementing offsets into a surface reflection array this option will not be considered further in this discussion. Consequently, arrays with positive elements only, which will be described in the following as optical arrays or optical sequences [2], will be investigated. In addition to this we will normalize the reflection strength of an element to be in the range between 0, representing no reflection and 1, representing totally reflective.

The Wiener-Khinchin theorem states that the power spectrum of a sequence, or an array, is equal to the Fourier Transform of its autocorrelation function [3]. We are interested in patterns that result in a flat, or at least smooth, power spectrum which indicates that the reflected sound energy will be spread out in an even way.

In the case of optical arrays, it is not highly significant whether we use the Fourier Transform of the periodic or the aperiodic autocorrelation since the autocorrelation sidelobes of either are positive, with no opportunities for cancellation (the periodic autocorrelation function can be thought of as being derived from the addition of adjoining aperiodic autocorrelation functions). That is, reducing the sidelobe strength in the aperiodic autocorrelation will imply a reduction in the sidelobe strength in the periodic autocorrelation for a given sequence.

In the following one dimensional example [4] we have a linear optical binary sequence of length 7 depicted in the upper portion of the figure, a), namely 1,1,0,0,1,0,1 with the autocorrelation function of this sequence in the lower portion, b), derived from the number of similarities of a shifted version of the sequence with itself. The autocorrelation is symmetrical about the zero shift position.



Source:[4]
Figure 2. An Optical Binary Sequence and its Aperiodic Autocorrelation

More formally and allowing a planar array, the aperiodic autocorrelation function of an M x N array pattern a(m,n) is given by:

$$ACF(k,l) = \sum_m \sum_n a(m,n)a(m+k,n+l)$$

$$,\text{where } a(m,n) = 0 \text{ for } m,\ n< 0 \text{ or } m>M\ ,\ n>N$$

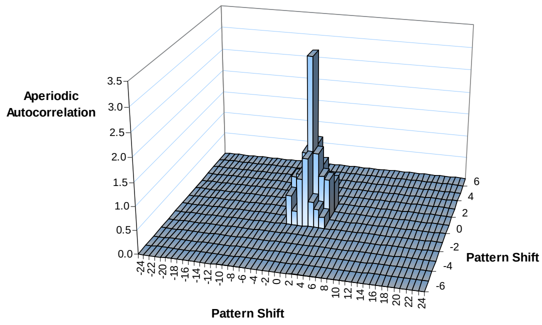
Patterns, their periodic and aperiodic autocorrelation, and FFTs of these to determine the array power spectrum response were determined in a Microsoft Excel 2003 spreadsheet using functions available in the Xnumbers Numerical Methods addin [5].

A genetic algorithm addin was used to optimize element values to a selected cost function. Generally a cost function directly related to maximizing the flatness and smoothness of the spectrum was found to provide better results than cost functions related to minimising autocorrelation sidelobes.

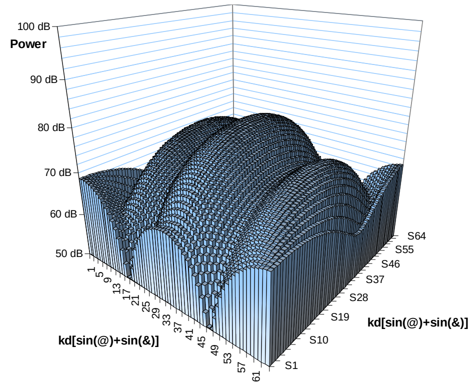
As an example, a 2 x 4 array with values as indicated in the Figure 3a below has an aperiodic autocorrelation function given in Figure 3b. The FFT of this, the power spectrum, is given in Figure 3c, which is not flat but is quite smooth. The specular response is that at the centre of the last graph.

0.695	0.268
0.000	0.147
1.000	0.445
0.910	0.920

a)



b)



c)

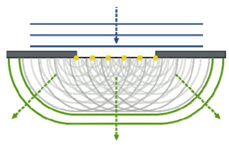
Figure 3. An Optical Real Array a), and its Aperiodic Autocorrelation b) and Power Spectrum c).

SIMULATION

The Huygens-Fresnel Principle

In the 17th Centaury Dutch physicist Christiaan Huygens proposed that every point on a propagating wavefront can be thought of as the source of secondary spherical wavelets. The envelope of these wavelets then approximates the wavefront some time later. Augustin Jean Fresnel modified this idea to include interference effects. The Huygens-Fresnel Principle states that [6]:

Every unobstructed point of a wavefront, at a given instant, serves as a source of spherical secondary wavelets (with the same frequency as that of the primary wave). The amplitude of the optical field at any point beyond is the superposition of all these wavelets (considering their amplitudes and relative phases).



Source: Wikipedia
Figure 4. Huygens- Fresnel Principle

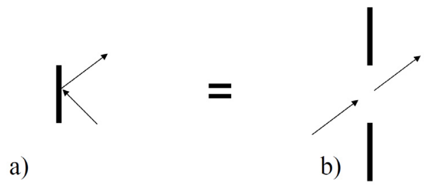
This approximation has been found to be adequate in many situations involving wave phenomena as it predicts in accordance with observations, particularly for large source and receiver distances and when the reflecting elements or apertures have dimensions that are large in comparison to the wavelength.

In 1882, Gustav Kirchhoff developed a mathematical theory of diffraction based on the solution of the differential wave equation. His analysis led to a more rigorous formulation of Huygens's Principle as a direct consequence of the wave equation. In this more refined version the secondary wavelets have cardioid directivity [0.5(1+cosφ)] normal to the wavefront, so there is no backward travelling component, and they radiate 90 degrees out of phase with the primary wavefront.

Babinet's Principle

Babinet's principle states that the reflection from a plane rigid surface is equivalent to the transmission through an opening, of the same geometrical shape, surrounded by an infinite rigid baffle [7]. Using these concepts, the problem of modelling a plane wave impacting a planar array of reflecting elements can be thought of as approximately equivalent to an incident plane wave propagating through an array of apertures in an infinite baffle.

A plane wave is sufficient to consider because an arbitrary sound field can be decomposed into plane waves.



Source: [6]

Figure 5. Babinet's Principle

The apertures/reflecting elements are modelled, as in the Figure 4, with omnidirectional sound sources acting as the source of secondary wavelets. We are only interested in the soundfield in the observable space on the exit side of the apertures and so as a first approximation we do not need to use cardioid secondary sources unless we want better information at more extreme off-axis receiver locations or if we also want to investigate the effect of small gradient curvature or modulation of the reflecting surface of the array. The Kirchhoff wavelet phase shift relative to the primary wavefront is not necessary to include in the modelling, since only the relative phases between points in the pressure field are of interest.

Spatial Aliasing

Frequency domain aliasing results from insufficient sampling of a signal in time. Spatial aliasing is the spatial domain equivalent, more commonly discussed in relation to loudspeaker arrays and wave field synthesis.

The far field power spectrum response of the array, the Fourier Transform of the autocorrelation of the array pattern, is a function of $kd_g\,[\sin(\theta_r) + \sin(\psi_{ipw})]$, where d_g is the array grid spacing, k is the wave number ($2\pi f/c$), c is the speed of sound, ψ_{ipw} is the angle of sound incidence (plane wave) and θ_r is the angle of reflection to a receiver location with respect to the normal to the surface of the diffuser.

For $kd_g[\sin(\theta_r) + \sin(\psi_{ipw})] = \pi$, the full power spectrum response is mapped into the observable space in front of the array. At values less than this only the initial portions of the response are mapped to the observable range. When this value is exceeded, more than one period of the response is mapped into the observable space and aliasing occurs.

The frequency above which aliasing commences, f_{alias}, is dependent on the array grid spacing and the angles of the incident sound and receiver with respect to the array:

$$f_{alias} = c \,/\, 2d_g|\sin(\theta_r) + \sin(\psi_{ipw})|$$

That is, aliasing occurs when the effective spacing of reflective elements is greater than half a wavelength.

In the example following, Figure 5, the smoothed power spectrum at a receiver for a 0° incident plane wave and receiver at 10°, aliasing begins at 3,300 Hz. Repeats of the power spectrum can be seen in the response as indicated between the triangular points and at frequencies above these.

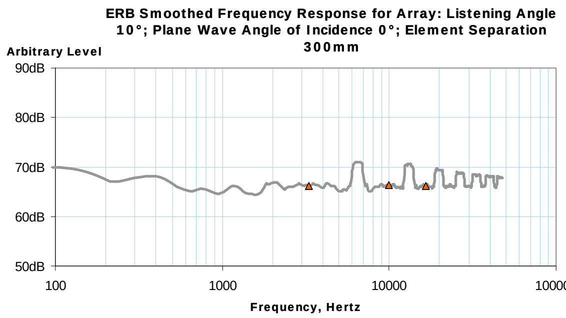


Figure 6. Aliasing Effects on Spectrum

A few points should be noted:

- Aliasing is independent of source and receiver distances.
- The optical array response is constant with frequency for a receiver in the specular direction. This is because the alias frequency becomes infinite ($\theta_r = -\psi_{ipw}$, so $\sin(\theta_r) + \sin(\psi_{pw}) = 0$) and only the start point of the power response, the constant dc component, is mapped to the observer. As the observer moves away from the specular direction, more of the power response is mapped to observable space and aliasing may start to occur.
- For optical arrays the specular response has maximum amplitude. The maximum amplitude increases directly in proportion to the number of array elements.
- The location at which aliasing will start first is at right angles to the array for source and receiver on the same side.

Soundfield Modelling

Acoustical prediction software developed by Meyer Sound Laboratories, Mapp (Multi-Purpose Acoustical Prediction Program) Online Pro [8] was used to approximate the behaviour of candidate arrays determined from the pattern optimizations. The arrays were modelled using an omnidirectional source for each element of the array. Each element was able to be adjusted for weight between an attenuation of 0dB and muted to represent a totally absorptive element.

Figure 6, below, shows an array soundfield and spectrum at right angle to on-axis generated by Mapp showing aliasing effects both spatially and in the spectrum response.

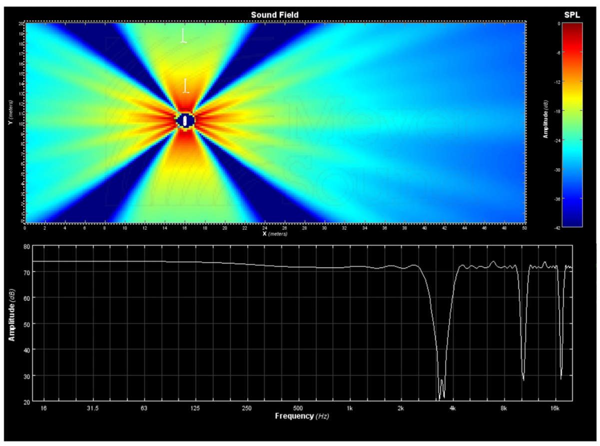


Figure 7. Array Soundfield and Off Axis Spectrum

The soundfield results from Mapp represent an idealised approximation to the array behaviour that does not take into account some of the physical aspects of element and array behaviour. These aspects, discussed further in [7] and [9], are the subject of ongoing work and refinement of models.

The main effect is high pass filtering of the array spectrum response that is a function of the source and receiver distances from the array, d_s and d_r, the size of the array and array elements, and the angles to the array. For design purposes this behaviour may be included in the modelling to some degree by incorporating lowpass functions and attenuations.

The array design considerations are summarised in the following figure.

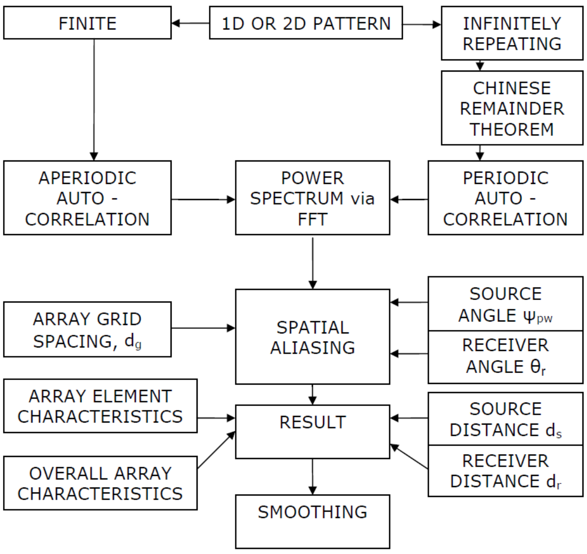


Figure 8. Outline of Amplitude Reflector Array Design

RESULTS

A large range of array sizes have been investigated. The results can be summarised as follows:

- Allowing intermediate values between 0 and 1 for array element weights expands the range of array possibilities and generally gave improvements in performance over optical binary behaviours.
- There are no simple means of determining array element weights other than by genetic algorithm or other optimization methods suitable for combinatorial problems. This approach is quite time consuming but does allow an optimized result within other constraint settings that may be desired.
- There are a range of cost functions that can be optimized however best results were obtained by costing the spectrum response aspects directly rather than cost functions of the autocorrelation sidelobes. Minimizing the variation and variability of the spectrum response was found to be a simple but effective approach.
- As the number of elements in the array increases it is increasingly difficult to obtain an even, or at least smoothly varying, spectrum response. Arrays based on a smaller number of elements have more scope to provide smoother spectrum responses.
- Best responses required sparse arrays which meant that the array was weakly reflecting.
- Best responses require low weight elements which conflicts with manufacturing feasibility.

IMPROVING DIFFUSER PERFORMANCE

Complementary Array Angles

Using two or more arrays with different tilt angles relative to the source and receiver positions can provide significant improvement in the resultant spectrum response at a receiver. The improvement can be greater than can be achieved by optimizing the array pattern. In the following example, the spectrum variation is reduced in the order of 10 dB (black line) by using two linear diffusers (green and blue lines) at complementary tilt angles.

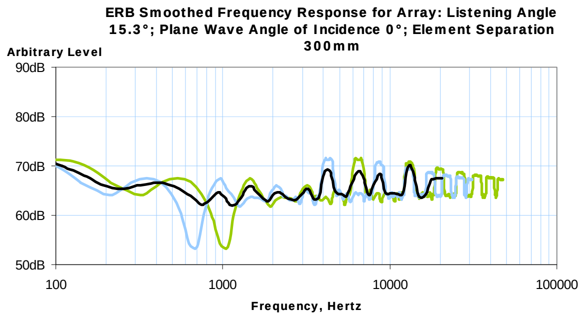


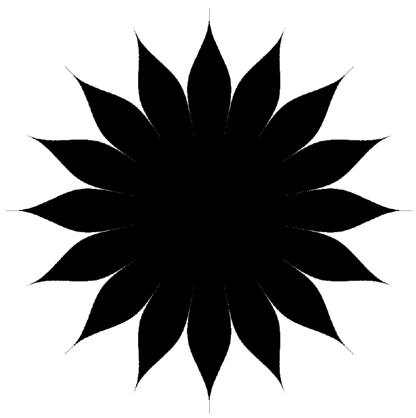
Figure 9. Outline of Amplitude Reflector Array Design

For the cases studied, it has been found that the improvement at the optimized angle is not markedly better than at other angles and some useful degree of improvement is generally achieved as long as there is a difference in angles, somewhere between 2° and 20°.

Aperture Shading

Considering the reflective elements of the array as apertures there is scope to reduce diffractive effects using amplitude shading. This diffraction is essentially lobing generated off the edges of each element and results in the degradation of diffusion. Amplitude shading is the tapering of the strength of the sound reflection between the centre and the edges of the reflective element.

Recent work on optical techniques for searching for extra-solar planets [10] has concentrated on occluders to block the light from a star but allow its surrounding planets to be discerned. To do this the occluder is tapered to minimise diffraction. The tapering was achieved by using petals around the perimeter of the occluder to transition the amplitude strength, as in the figure below.



Source: [10]
Figure 10. Aperture Shading by Tapered Form

This principle has been used in recent diffuser designs, as in the indicative example below for use in a periodic arrangement (light zones are reflective, dark zones are absorptive).

Figure 11. 4 x 4 Optical Real-Valued Amplitude Diffuser with Aperture Shading

CONCLUSION

Investigation of real-valued optical arrays has found that good performance is generally at the cost of sparseness, meaning the array is weakly reflecting. Optical arrays with less than ideal performance can be used in combination to achieve significantly improved performance by adjustment of relative tilts to the incident sound field. Aperture shading of array elements is also proposed as a means of improving diffusion performance.

ACKNOWLEDGEMENT

The author gratefully acknowledges Meyer Sound for permission to present array response results from the Mapp Online Pro acoustical prediction software used in the investigation.

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