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SHOCK ANALYSIS OF PROPULSION SHAFT OF THE SHIP WITH CONSIDERATION OF THE OIL FILM FORCE

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Abstract

The propulsion shaft of a ship is investigated. It is modelled by the Dynamics of an Elastic Body Method. Oil film forces are obtained by the Average Eigenvalue Method. The effect of rotation velocity and oil film force on the response of the shaft under shock is considered; the flexibility of the bearing housing is also considered. The equations of motion of the main propulsion shaft of the ship are derived; they are solved using classical theory. In the end, the Runge-Kutta Method is used to do the numerical simulation. Results show that oil film bearings have a good effect on the alleviation of the vibration of the shaft.
Keywords: Propulsion shaft, Oil film, Dynamics of an Elastic Body

1. INTRODUCTION

Large rotation shafts have been widely used in engineering for a long time. Such as propulsion shaft of the ship, rotor of hydraulic and stream turbo generator and so on. Because those shafts are key-components of the whole system, their vibration may affect the whole system and the devastation caused by the vibration may be disastrous [1]. So the research on the vibration of the shafts attracts the attention of scholars from home and abroad.

A difference in design between the propulsion shaft and other shafts is that, for propulsion shaft, it will inevitably encounter shock environment in its life. How to resist shock load generated by different excitation sources is a problem that must be considered in design of the propulsion shaft. But very few materials on the analysis of the propulsion shaft under dynamical shock load are available [7]. So how to guarantee the stability of the shaft under shock is a problem to which attention must be paid.

In engineering area, the coupling effect of the lateral vibration and the rotation velocity is traditionally neglected because the rotation velocity of the propulsion shaft is not very high. So in the analysis of vibration of the shaft, it is generally considered as rigid body or only the lateral vibration is considered without considering its coupling with the rotation [5][6]. How much the influence of the rotation velocity on the dynamic response of the shaft is, and whether it is neglectable will be discussed in this paper.

When considering the effect of the rotation velocity on the dynamic response of the shaft, the dynamics of an elastic body may be used to model the shaft [3]. The commonest modeling method is the Transform Matrix Method, other methods including Finite Element Method, Energy Method, and the Dynamics of Multi-Rigid or Elastic Body Method and so on [5]. The advantage of the dynamics of an elastic body method is that it can easily include the rotation effect and the oil film forces. Various numerical methods were used to analyze the trajectory of journal center motion. Such as Mode Superposition Method, Central Difference Method, Houbolt Method ,Wilson-θ Method and Newmark-β Method, etc [7].

At present, most research considered the bearing bush to be motionless; few materials that considered the flexibility of the bearing are available [4]. As all the bearings are placed on the ship which belongs to elastic body and the bearings themselves have stiffness, consideration of the stiffness of the support systems is desirable to predict the result more accurately. So, establishing the dynamic model considering both of the stiffness and carrying on some study on it is a focus of this paper.

2. MODELING OF THE OIL FILM BEARING

The schematic figure of the oil film bearing is shown in Fig. 1. For convenience, some simplifications have been made to the model:

a. The mass of the bearing bush is disregarded;
b. The elastic supports under the bearing bush are simplified as spring-damper systems;
c. For the supporting part， the torsion and the deflection effect are ignored.

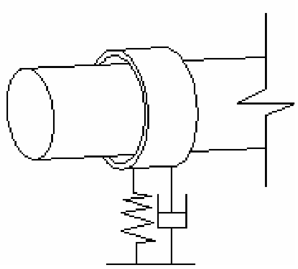


Fig.1 The oil film bearing

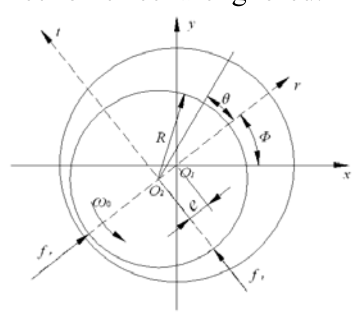


Fig.2 Coordinate of the oil film bearing

Fig. 2 is a sketch of the coordinate of the oil film bearing. The o_1xy coordinate system is fixed to the bearing bush; the o_2rt coordinate system is fixed to the shaft.

Here we use the average eigenvalue method to solve the Reynolds equation of the oil film. The basic principal of the average eigenvalue method is to divide the dynamical parts in Reynolds equation into squeeze effect and wedge effect and solve them separately to get the corresponding eigenvalue. The solution of Reynolds equation is the superposition of the two solutions. The detailed derivation is shown in [2], here only the result is listed.

The dimensionless oil film forces are

$$\left.\begin{aligned} f_r &= (1-\frac{2\dot{\phi}}{\omega})f_1(\varepsilon)+\frac{2\dot{\varepsilon}}{\omega}f_2(\varepsilon) \\ f_t &= (1-\frac{2\dot{\phi}}{\omega})f_3(\varepsilon) \end{aligned}\right\} \tag{1}$$

Where ω is the angular velocity of the rotor, ε is the eccentricity ratio, ϕ is the vortex angle (see Fig.2).

The functions $f_1(\varepsilon)$, $f_2(\varepsilon)$ and $f_3(\varepsilon)$ in equation (1) are

$$\left.\begin{aligned} f_1(\varepsilon) &= \frac{\varepsilon^2}{(1-\varepsilon^2)(2+\varepsilon^2)}[1-\frac{\tanh(B\sqrt{\lambda_1})}{B\sqrt{\lambda_1}}] \\ f_2(\varepsilon) &= \frac{1}{(1-\varepsilon^2)^{1.5}}(\varepsilon\sqrt{1-\varepsilon^2}+\arcsin\varepsilon+\frac{\pi}{2})[1-\frac{\tanh(B\sqrt{\lambda_2})}{B\sqrt{\lambda_2}}] \\ f_3(\varepsilon) &= -\frac{\pi\varepsilon}{2(2+\varepsilon^2)\sqrt{1-\varepsilon^2}}[1-\frac{\tanh(B\sqrt{\lambda_1})}{B\sqrt{\lambda_1}}] \end{aligned}\right\} \tag{2}$$

Where λ_1, λ_2 are average eigenvalues [2], B is the width-diameter ratio $B = L/2r$, L is the width of the bearing, r is the radius of rotor $r = R$.

The dimensionless oil film forces under the o_1xy coordinate system are

$$\left.\begin{aligned} f_x &= f_r\cos\phi - f_t\sin\phi \\ f_y &= f_r\sin\phi + f_t\cos\phi \end{aligned}\right\} \tag{3}$$

And the dimensional oil film forces under the same coordinate are

$$F_x = \frac{12\mu\omega Lr^3}{c^2}f_x\,,\quad F_y = \frac{12\mu\omega Lr^3}{c^2}f_y \tag{4}$$

Since the bearing bush is connected to the support systems, it must satisfy the following conditions

$$\begin{bmatrix} F_x \\ F_y \end{bmatrix} = k\begin{bmatrix} x_c \\ y_c \end{bmatrix} + c\begin{bmatrix} \dot{x}_c \\ \dot{y}_c \end{bmatrix} \tag{5}$$

Where： k and c are the stiffness and the damping coefficients of the support system respectively, x_c, y_c are the displacement of the center of the bearing bush.

3. MODELING OF THE SHAFT

As the real structure of the propulsion shaft is very complex, before the derivative of the dynamic equation, the following simplifications are introduced:

a. The shaft is modelled as elastic continual and symmetrical beam;

b. The stress and the deformation are linear correlated to the velocity of deformation;

c. The section of the shaft will keep as plane and perpendicular to the central line;

d. The unbalance due to deformation is small, a first order approximation is adopted.

In order to fully describe the position of the shaft in the derivation of the dynamic equation, three coordinates are adopted (see Fig.3):

a. inertia coordinates $o-{}_Ix_Iy_Iz$ (global coordinate)

b. rotation coordinates $o-{}_Rx_Ry_Rz$ (describe the rotation of the shaft)

c. sliding coordinates $o'-{}_Ex_Ey_Ez$ (for convenience of describing the position of the point)

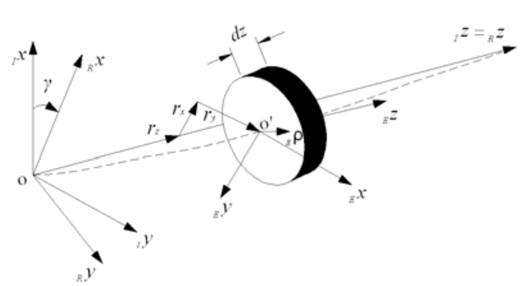


Fig. 3 Coordinate systems

As shown in Fig.3, at z position of the shaft, get a dz length element, and the radius vector of a random point in the element under the sliding coordinate system ${}_E\boldsymbol{r}$ may be described as

$$ {}_E\boldsymbol{r} = \boldsymbol{A}_{ER}({}_R\boldsymbol{r}_z + {}_R\bar{\boldsymbol{r}}) + {}_E\boldsymbol{\rho} \quad \text{where} \quad {}_R\bar{\boldsymbol{r}} = \begin{bmatrix} r_x & r_y & 0 \end{bmatrix}^T $$

Differentiate it yields the velocity of the point

$$ {}_E\boldsymbol{v} = \boldsymbol{A}_{EI}\frac{d}{dt}[\boldsymbol{A}_{IE}\boldsymbol{A}_{ER}({}_R\boldsymbol{r}_z + {}_R\bar{\boldsymbol{r}}) + \boldsymbol{A}_{IE}\,{}_E\boldsymbol{\rho}] = {}_E\boldsymbol{v}_s + {}_E\tilde{\boldsymbol{\omega}}_{IE}\,{}_E\boldsymbol{\rho} $$

Where $\boldsymbol{A}_{EI}$, $\boldsymbol{A}_{IE}$ and $\boldsymbol{A}_{ER}$ are the transform matrix, $\boldsymbol{\omega}_{IE}$ is the angular velocity of the global coordinate system relative to the sliding coordinate system, $\boldsymbol{v}_s$ is the velocity of the mass center of the element.

In global coordinate the momentum of the element may be expressed as

$$ \boldsymbol{P} = \int dm\ \boldsymbol{A}_{IE}({}_E\boldsymbol{v}_s + {}_E\tilde{\boldsymbol{\omega}}_{IE}\,{}_E\boldsymbol{\rho}) = dm\ \boldsymbol{A}_{IE}\,{}_E\boldsymbol{v}_s $$

(As the original of the sliding matrix is in the center of the mass, so the integration about ρ is zero)

Apply the momentum theorem to the element yields

$$ \boldsymbol{A}_{IE}({}_E\dot{\boldsymbol{v}}_s + {}_E\tilde{\boldsymbol{\omega}}_{IE}\,{}_E\boldsymbol{v}_s)dm - {}_I d\boldsymbol{f} = 0 $$

Where $d\boldsymbol{f}$ is the external applied load. Described in the rotation coordinate R we have

$$ {}_R d\dot{\boldsymbol{P}} + {}_R\tilde{\boldsymbol{\omega}}_{IR}\,{}_R d\boldsymbol{P} - {}_R d\boldsymbol{f} = 0 \tag{6} $$

Here, the following relation is applied

$$ \boldsymbol{A}_{AE}({}_E d\dot{\boldsymbol{p}} + {}_E\tilde{\boldsymbol{\omega}}_{IE}\,{}_E d\boldsymbol{p}) = \boldsymbol{A}_{AE}[\boldsymbol{A}_{EI}\frac{d}{dt}(\boldsymbol{A}_{IE}\,{}_E d\boldsymbol{p})] = \boldsymbol{A}_{AI}\frac{d}{dt}(\boldsymbol{A}_{IA}\,{}_A d\boldsymbol{p}) = {}_A d\dot{\boldsymbol{p}} + {}_A\tilde{\boldsymbol{\omega}}_{IA}\,{}_A d\boldsymbol{p} $$

4. THE STATE EQUATIONS AND THE MODAL FUNCTION

4.1 The state equations

As shown in Fig.3, let the deformation of the shaft at z position in R coordinate be ${}_R\bar{\boldsymbol{r}} = \begin{bmatrix} {}_R r_x & {}_R r_y & 0 \end{bmatrix}^T$. Express it in the global coordinate we have ${}_I\bar{\boldsymbol{r}} = \boldsymbol{A}_{IR}\,{}_R\bar{\boldsymbol{r}}$

Where $\boldsymbol{A}_{IR}$ is the transform matrix form R coordinate to global coordinate I.

$$ \boldsymbol{A}_{IR} = \begin{pmatrix} \cos\gamma & -\sin\gamma & 0 \\ \sin\gamma & \cos\gamma & 0 \\ 0 & 0 & 1 \end{pmatrix} $$

So ${}_I r_x = {}_R r_x \cos\gamma - {}_R r_y \sin\gamma$, ${}_I r_y = {}_R r_x \sin\gamma + {}_R r_y \cos\gamma$

For the i-th bearing, from the geometry of Fig.2 the displacements of the center of the bearing can be easily obtained to be

$$ x_{ci} = \delta(z - z_i)\,{}_I r_x + e_i\cos\phi = \delta(z - z_i)({}_R r_x\cos\gamma - {}_R r_y\sin\gamma) + e_i\cos\phi $$

$$ y_{ci} = \delta(z - z_i)\,{}_I r_y + e_i\sin\phi = \delta(z - z_i)({}_R r_x\sin\gamma + {}_R r_y\cos\gamma) + e_i\sin\phi $$

Differentiate it yields the velocity of the center of the bearing

$$ \dot{x}_{ci} = \delta(z - z_i)({}_R\dot{r}_x\cos\gamma - \omega_0\,{}_R r_x\sin\gamma - {}_R\dot{r}_y\sin\gamma - \omega_0\,{}_R r_y\cos\gamma) + \dot{e}_i\cos\phi - e_i\dot{\phi}\sin\phi $$

$$ \dot{y}_{ci} = \delta(z - z_i)({}_R\dot{r}_x\sin\gamma + \omega_0\,{}_R r_x\cos\gamma + {}_R\dot{r}_y\cos\gamma - \omega_0\,{}_R r_y\sin\gamma) + \dot{e}_i\sin\phi + e_i\dot{\phi}\cos\phi $$

Then equation (5) and (6) can be rewritten to be

$$ \begin{bmatrix} F_{xi} \\ F_{yi} \end{bmatrix} = k_i \begin{bmatrix} \delta(z - z_i)({}_R r_x\cos\gamma - {}_R r_y\sin\gamma) + e_i\cos\phi \\ \delta(z - z_i)({}_R r_x\sin\gamma + {}_R r_y\cos\gamma) + e_i\sin\phi \end{bmatrix} $$

$$+c_i\begin{bmatrix}\delta(z-z_i)({}_R\dot{r}_x\cos\gamma-\omega_0\,{}_Rr_x\sin\gamma-{}_R\dot{r}_y\sin\gamma-\omega_0\,{}_Rr_y\cos\gamma)+\dot{e}_i\cos\phi-e_i\dot{\phi}\sin\phi\\ \delta(z-z_i)({}_R\dot{r}_x\sin\gamma+\omega_0\,{}_Rr_x\cos\gamma+{}_R\dot{r}_y\cos\gamma-\omega_0\,{}_Rr_y\sin\gamma)+\dot{e}_i\sin\phi+e_i\dot{\phi}\cos\phi\end{bmatrix} \tag{7}$$

$$\int\begin{bmatrix}{}_R\ddot{r}_x\\ {}_R\ddot{r}_y\end{bmatrix}dm+2\omega\int\begin{bmatrix}-{}_R\dot{r}_y\\ {}_R\dot{r}_x\end{bmatrix}dm-\omega^2\int\begin{bmatrix}{}_Rr_x\\ {}_Rr_y\end{bmatrix}dm+M_j\delta(z-z_j)\begin{bmatrix}{}_R\ddot{r}_x\\ {}_R\ddot{r}_y\end{bmatrix}$$
$$+2\omega M_j\delta(z-z_j)\begin{bmatrix}-{}_R\dot{r}_y\\ {}_R\dot{r}_x\end{bmatrix}-\omega^2M_j\delta(z-z_j)\begin{bmatrix}{}_Rr_x\\ {}_Rr_y\end{bmatrix}=\sum_i\begin{bmatrix}F_{xi}\\ F_{yi}\end{bmatrix}+\int\begin{bmatrix}f_x\\ f_y\end{bmatrix}dz \tag{8}$$

Where $\begin{bmatrix}f_x & f_y\end{bmatrix}^T$ is the external applied load.

As r_x, r_y have the same boundary condition, we may have the following assumptions :

$$r_x=\sum u_i(z)q_{ui}(t) \qquad (\ i=0,1,2,\cdots n\ )$$
$$r_y=\sum u_i(z)q_{vi}(t)$$

Let state variables $x_{1i}=q_{ui}(t)$, $x_{2i}=q_{vi}(t)$, $x_{3i}=\dot{x}_{1i}$, $x_{4i}=\dot{x}_{2i}$, $x_{5i}=\varepsilon_i$, $x_{6i}=\phi_i$

From (7) and (8) the state equations may be expressed as

$$\begin{cases}\dot{x}_{1i}=x_{3i}\\ \dot{x}_{2i}=x_{4i}\\ \dot{x}_{3i}=(F_x+A_3a_x)/(A_1+A_2)+2\omega x_{4i}+\omega^2x_{1i}\\ \dot{x}_{4i}=(F_y+A_3a_y)/(A_1+A_2)-2\omega x_{3i}+\omega^2x_{2i}\\ \dot{x}_{5i}=(B_3B_4-B_1B_6)/(B_2B_4-B_1B_5)\\ \dot{x}_{6i}=(B_2B_6-B_3B_5)/(B_2B_4-B_1B_5)\end{cases} \tag{9}$$

Where A_1-A_3 and B_1-B_6 are coefficients.

4.2 Choose of modal function: (free-free boundary conditions) [8]

The free-free boundary conditions are

$$z=0,\quad u_i''(0)=u_i'''(0)=0$$
$$z=l,\quad u_i''(l)=u_i'''(l)=0$$

The characteristic root of the frequency equation is

$$\cos\lambda l\cosh\lambda l=1$$
$$\lambda_0 l=0,\quad \lambda_1 l=4.73,\quad \lambda_2 l=7.853,\quad \cdots\lambda_n l\approx(2n+1)\pi/2$$

The modal function

$$u_n(z)=\cosh\lambda_n z+\cos\lambda_n z-E_n(\sinh\lambda_n z+\sin\lambda_n z) \tag{10}$$

Where $\qquad E_n=\dfrac{\sinh\lambda_n l+\sin\lambda_n l}{\cosh\lambda_n l-\cos\lambda_n l}$

The rigid body translation modal is $\qquad u_0(z)=1\quad$ (m=0) (11)

The rigid body rotation modal is $\qquad u_1(z)=1-\dfrac{2z}{L}\quad$ (m=1) (12)

From the modal function (10), (11) and (12) we may get the coefficients, substitute them into the state equation (9) yields the solution of the state equation corresponding to every modal. And the numerical solution of the nonlinear equations (5) and (6) is the inverse transformation of the state variables to real variables.

5. ANALYSIS OF THE SHOCK RESPONSE OF THE MAIN PROPULSION SHAFT

5.1 figure of the propulsion shaft

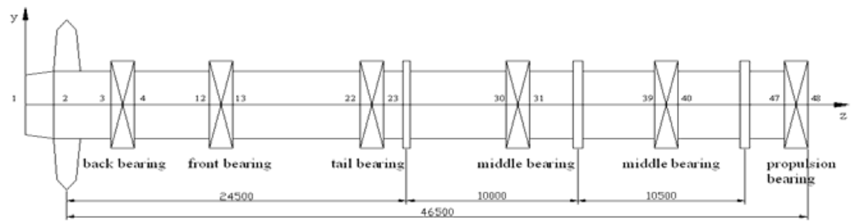
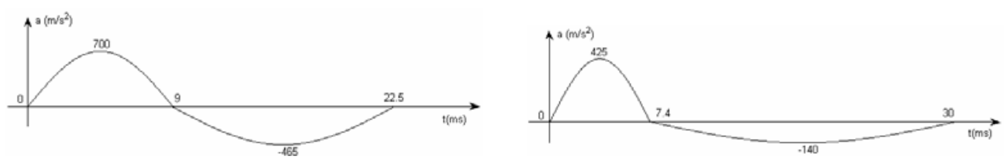


Fig.4 The propulsion shaft

5.2 shock function

Here the acceleration input is adopted， it is from the Germany navy criterion BV043/85, they are half-sine waves, see Fig. 5



Longitudinal shock acceleration　　Horizontal shock acceleration

Fig.5 Shock acceleration

5.3 Results and discussion

5.3.1 Comparison with the Finite Element Method

In order to verify the correctness of the model established by the Dynamics of an Elastic Body Method, we compare the results of a simple case with those obtained from the Finite Element Method. In the Finite Element Method, the propulsion shaft is treated as continuous beam. As the Finite Element Method for beam is well established in almost every Finite Element book, so the modelling process is ignored, here only the result is given. Because the Finite Element Method is difficult to compute the case $\omega > 0$, we choose $\omega = 0$. And the shock acceleration is simply chosen to be $a_x = 0$, $a_y = 500\sin(\pi t / 0.006)$, $a_z = 0$ (the duration of the shock is 0.006).

Fig.6 is the comparison of the shock response of the propeller where the left figure is the result by Finite Element Method; the right figure is the result by Dynamics of an Elastic Body Method.

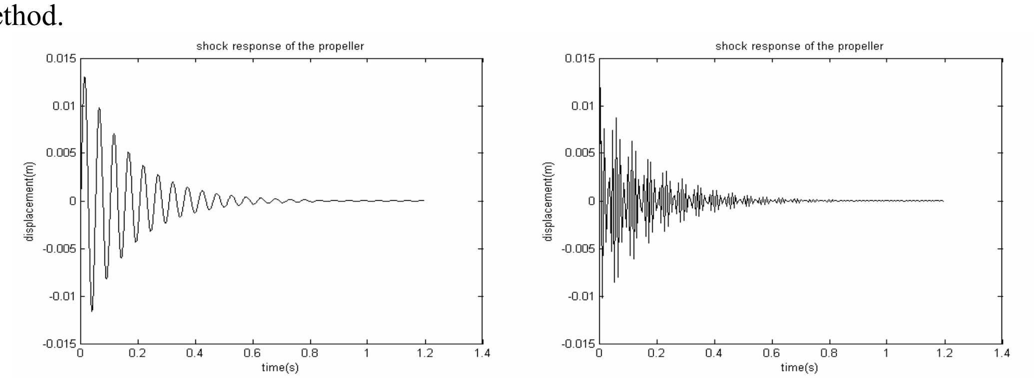


Fig.6 Comparison of the shock response of the propeller

From Fig.6 we can see that the relative error of the two methods is very small (within 8%) which means that the Dynamics of an Elastic Body Method is reliable.

5.3.2 The selection of the order of the modal

Table 1. Trial results of every modal

Maximum response	Number of modal(N) 0	1	2	3	4	5	6	7	8	9	10
x(m)	0.018	0.049	0.031	0.048	0.066	0.057	0.066	0.066	0.073	0.071	0.076
y(m)	0.039	0.080	0.065	0.091	0.107	0.100	0.115	0.106	0.129	0.129	0.122

In order to find a reasonable, simple model, we choose different order of modal to do the simulation. From Table 1 we can see that, from eight orders of modal, the effect of the order of modal to the displacement response is very small, so eight orders of modal is sufficient in calculation. Here we adopt the eight orders of modal.

5.3.3 Shock response of the propulsion shaft

The two horizontal axes are time and the position of the shaft respectively; the longitudinal axis is the shock response of the shaft. (The rotation velocity of the propulsion shaft is 100 n/min.)

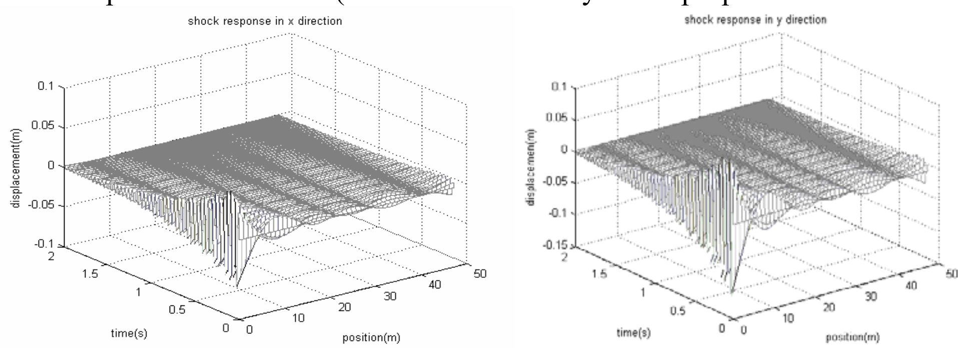


Fig.7 Shock response of the propulsion shaft

From Fig.7 we can see that, as there is a very large appending mass on the propeller (including the appending mass of the water), the response of the propeller is very large.

5.3.4 The effect of oil film force

Two cases are studied, one has oil film forces, and the other doesn't. The comparison of maximum shock response is shown in the following table.

Table 2. Comparison of maximum shock response

Maximum response	no oil film force	with oil film force	reduce by(%)
x	0.0681	0.0567	16%
y	0.1160	0.0999	14%

From Table 2 we can see that when oil film force exists, the maximum shock response of the shaft reduces both in x and y direction. So for propulsion shaft who always encounter shock environment, the installation of oil film bearing can protect the shaft from being destroyed.

5.3.5 The effect of rotation velocity on the response of the shaft

In order to study the effect of rotation velocity on the response of the shaft, we use different rotation velocity to do the simulation. The maximum response of the shaft under different rotation velocity is shown in the following figure.

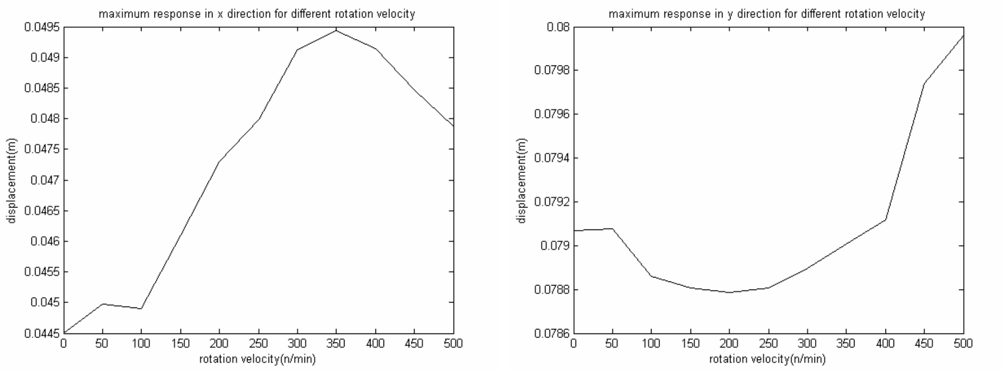


Fig.8 Shock response of the propulsion shaft for different rotation velocity

From Fig.8 we can see that rotation velocity has an important effect on the maximum response of the shaft. The classical static analysis may be deficient.

6. CONCLUSIONS

Based on the analysis and simulation, the following conclusions may be drawn:

(1) In the modelling of the nonlinear oil film model of the shaft, the vortex effect, the squeeze effect and the coupled effect of multi oil film bearings are considered, the flexibility of the bearing housing is also considered. Results show that the oil film bearings have a good effect on the alleviation of the vibration of the shaft.

(2) The Dynamics of an Elastic Body is used to model the shaft，The equations of motion of the main propulsion shaft of the ship are derived accordingly; the comparison with Finite Element Method proves that this model is reasonable.

(3) This paper uses classical theory to disperse the equations of the shaft, the result is sufficient enough with only the first eight orders of modal, so the disperse method is applicable.

(4) From the analysis of this paper, the rotation velocity of the shaft is important to the maximum response of the shaft. So the classical static analysis may be deficient. Results will be more accurate if rotation velocity of the shaft is considered.

(5) The model establish in this paper may be used as a basis for further analysis of control and reliability.

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