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SOME OF THE NEW APPROACHES TO THE THEORY OF DESIGNING ACOUSTIC THICKENERS

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Abstract

The algorithm of analytical calculation and way of planning of acoustic thickeners is considered. The types are determined by the function of the special kind containing substantial arbitrary permanent. As the ground of practical benefit of the offered algorithm, the chart of construction of acoustic thickener of energy is resulted with the detailed analysis of its main characteristics.

1. INTRODUCTION

Depending on the successful or unsuccessful choice of acoustic thickeners of energy of ultrasonic apparatus possibly consumption of electric energy with minimum expenses or its spending to excess. Dispersion of energy is the criterion of high output-input ratio of similar devices in thickener during his work, the value of which depends on many factors of such, in particular, as: type of material which thickener is made from; decrement of vibrations; weight of thickener; geometrical configuration of thickener.

Therefore the task of planning of acoustic thickeners of energy, possessing parameters for providing of which minimum consumable electric power is required, is actual. For the decision of this task the method of symmetries, based on the idea of groups approaches to the decision of differential equalizations, is used. Due to this method possibility of receipt of the reserved decisions of the proper equalizations appears with variable coefficients which can also contain freely selectable substantial permanent. Changing the value of these permanent, it is possible to explore properties and conduct of the real object – acoustic thickener type of which and will be will be determined by these variable coefficients of differential equalization.

Presently the thickeners ultrasonic energy found wide application in the industrial and technological systems and options. So, at the direct use of such devices effectively ultrasonic machine-tools, medical ultrasonic drills, ultrasonic surgical instruments, options for conducting of researches on the basis of properties of materials, for example for the study of fatigue etc. Important questions which touch dispersion of energy are not affected in the article, as this problem is the theme of separate scientific researches.

2. INITIAL EQUALIZATION

Differential equalization of longitudinal vibrations of bar of variable section, being the mathematical model of acoustic thickener, looks like [1]:

$$\frac{\partial^2 w}{\partial t^2} - c^2 \frac{\partial}{\partial X} \ln F \cdot \frac{\partial w}{\partial X} - c^2 \frac{\partial^2 w}{\partial X^2} = 0,$$

where $w(X,t)$ - moving of some transversal section at vibrations;

$c = \sqrt{\frac{E}{\rho}}$ - speed of distribution of longitudinal wave in a bar;

E - Young modulus;

ρ - closeness of material, from which a bar is made;

$F(x)$ - area of transversal section of bar.

Supposing on the method of Fourier $w = W(X)\cos(\omega \cdot t),$ i.e. examining harmonic vibrations, entering in place of coordinate X a relative coordinate $x = X/l$ (l-length of bar) and designating $F \approx D^2,$ we will get equalization of forms of vibrations [2]

$$W'' + 2\frac{D'}{D}W' + k^2 W = 0, \tag{1}$$

where $D(x) = \sqrt{F(x)}$ - parameter determining the form of transversal section;

$k = \frac{l\omega}{c}$ - own value (wave number);

$\omega = 2\pi f$ - circular frequency;

f - frequency of vibrations.

A stroke designates derivative on $x.$ Expressed through, equalization (1) will be written down in a kind

$$W'' + \frac{F'}{F}W' + k^2 W = 0 \tag{2}$$

Equalizations (1-2) are just on condition that the transversal sections at vibrations remain flat and that the transversal sizes of bar are small enough as compared to length (so-called thin bar). Scopes terms for thickener with free ends at $x = \alpha$ and $x = \beta$ look like:

$$W'(x = \alpha) = W'(x = \beta) = 0. \; (\beta - \alpha = 1) \tag{3}$$

If in general case law of change of transversal section of thickener, to define as

$$F(x) = x^n \quad (D(x) = x^{n/2}),$$

where $n -$ integer value. Then, from Eq. (2), it is possible to write down

$$W'' + \frac{1}{x^n} \cdot n \cdot x^{n-1} \cdot W' + k^2 W = 0$$

or

$$x^2 W'' + nxW' + k^2 x^2 W = 0. \tag{4}$$

From [3], the decision of equalization (4) looks like

$$W(x) = x^{\frac{1-n}{2}} Z_{\frac{1-n}{2}}(kx) \tag{5}$$

where $Z_{\frac{1-n}{2}}(x) = C_1 J_{\frac{1-n}{2}}(x) + C_2 N_{\frac{1-n}{2}}(x)$;
(C_1, C_2 - the arbitrary permanent; $J_\nu(x)$ - function of Bessel 1st genus of ν order; $N_\nu(x)$ - function of Bessel 2th genus (function of Neumann) ν order of argument x).
Differentiation for x Eq. (5) gives a next result

$$W'(x) = x^{\mu} Z_{\mu-1}(kx) \cdot k = k \cdot x^{\mu} \cdot \left[A J_{\mu-1}(kx) + B N_{\mu-1}(kx) \right]$$

where $\mu = \dfrac{1-n}{2}$. Using scopes terms Eq. (3), it is undifficult to get, that

$$\frac{B}{A} = -\frac{J_{\mu-1}(k\alpha)}{N_{\mu-1}(k\alpha)} = -\frac{J_{\mu-1}(k\beta)}{N_{\mu-1}(k\beta)} \; . \tag{6}$$

Then equalization of frequencies it is possible to write down in a general view

$$J_{\mu-1}(k\alpha) \cdot N_{\mu-1}(k\beta) - J_{\mu-1}(k\beta) \cdot N_{\mu-1}(k\alpha) = 0 \tag{7}$$

3. MAIN DESCRIPTIONS OF THICKENEROF THE KNOWN PROFILE

We will consider the decision of task for a row, i.e. at some types of function $D(x)$ (Fig. 1) which is included in Eq. (1).

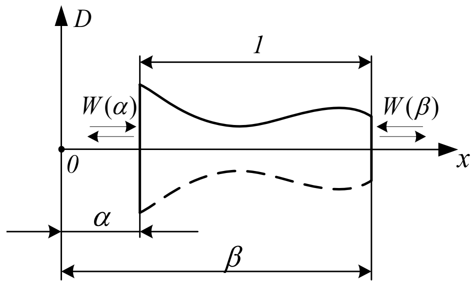

Fig. 1

We will accept, that $n = 1$. Type of acoustic transformer of energy at $D(x) = x^{1/2}$ ($n = 1$, $\mu = 0$) resulted on Fig. 2. Then expressions W, W' look like for this case

$$W(x) = A \left[J_0(kx) - \frac{J_{-1}(k\alpha)}{N_{-1}(k\alpha)} \cdot N_0(kx) \right] \tag{8}$$

$$W'(x) = A\left[J_{-1}(kx) - \frac{J_{-1}(k\alpha)}{N_{-1}(k\alpha)} \cdot N_{-1}(kx)\right] \cdot k.$$

Equalization of frequencies for the chosen example of thickener

$$J_1(k\alpha) \cdot N_1(k\beta) - J_1(k\beta) \cdot N_1(k\alpha) = 0\,. \tag{9}$$

Relation of scopes diameters of thickener $\delta = \dfrac{D(\beta)}{D(\alpha)}$ looks like

$$\delta = \left(\frac{\beta}{\alpha}\right)^{\frac{1-2\mu}{2}} = \left(\frac{\beta}{\alpha}\right)^{1/2} \tag{10}$$

Amplification factor of vibrations

$$M = \frac{W(x=\alpha)}{W(x=\beta)} = \left(\frac{\alpha}{\beta}\right)^{\mu-1} \cdot \frac{J_{\mu-1}(k\beta)}{J_{\mu-1}(k\alpha)} = \left(\frac{\beta}{\alpha}\right) \cdot \frac{J_1(k\beta)}{J_1(k\alpha)} \tag{11}$$

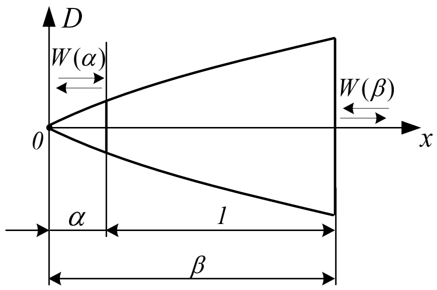

Fig. 2

4. RECEIPT OF DESCRIPTIONS FOR THICKENER OF MORE GENERAL VIEW AS FUNDAMENTAL PART OF ACTUATOR

We will consider a function $D(x) = 2 \cdot \sqrt{x}/(x^2 + C)$, containing arbitrary permanent C. This function is got on the basis of method of symmetry, realizing the idea of theoretical-groups approaches to find of the reserved decisions of differential equalizations. Possibility to synthesize the different types of thickeners with the in advance preset parameters appears due to this method. Decision of Eq. (2) and its derivative in this case assume an air

$$W_3(x) = -k^2 V_1(x) W(x) - W'(x) \cdot V_1'(x); \tag{12}$$

$$W_3'(x) = -k^2 V_1(x) \cdot W'(x), \tag{13}$$

where $V_1(x) = \frac{1}{2}(x^2 + C)$.

Because the derivative $W_3'(x)$ within a factor is equal derivative $W'(x)$ for thickener of kind $D(x) = x^{1/2}$, in obedience to scopes terms (3) frequency equalization in a kind (9) will be just and for this case $D(x) = 2 \cdot \sqrt{x}/(x^2 + C)$.

It is possible to assert at that rate, that own frequencies of bars of equal length, the type of which answers this function $D(x)$ at information α and β at first, accordant (9) do not depend on the value of coefficient C and, secondly, fully coincide with frequencies for thickener of type $D(x) = x^{1/2}$ at those α and β.

Changing the value of parameter α, it is possible to go out on cases, when $D(x=\beta) < D(x=\alpha)$, and then it is necessary to use correlations in a kind:

$$\delta_3^* = \frac{D(x=\alpha)}{D(x=\beta)} = \frac{1}{\delta} \cdot \left(\frac{\beta^2 + C}{\alpha^2 + C} \right) \tag{14}$$

$$M_3^* = \frac{W_3(x=\beta)}{W_3(\mathrm{x}=\alpha)} = \left(\frac{\delta}{M} \right) \cdot \delta_3^* \tag{15}$$

$$C^* = \frac{\delta_3^* \cdot \delta \cdot \alpha^2 - \beta^2}{1 - \delta_3^* \cdot \delta} \tag{16}$$

5. ANAYSIS OF RESULTS

Out of Eqs. (12) to (16) the pattern of choice of configuration of thickener of ultrasonic energy kind described by a function $D(x) = 2 \cdot \sqrt{x}/(x^2 + C)$ flows out directly, coming the required joining sizes from to actuator and his strengthening. In particular, for determination of strengthening (coefficient M_3) it is necessary to dispose by the proper values of sizes M and δ according to Eq. (15), and also own frequencies (numbers k) at set α or $\beta = \alpha + 1$. Choice of necessary size δ_3, using Eq. (14), is determined by the choice of permanent C, the values of which in the turn depend on one or another practical pre-conditions.

Lets $\alpha = 0.001$, then we will get on the module $M_3^* = \delta_3^* \cdot 12.744$ (Eq. (15)). If relation of joining sizes of thickener $\delta_3^* = 2$, his strengthening will be according to M_3^* =25.488. At other values of parameter δ_3^*=1; 4; 8 the values of strengthening are 12.744, 50.976, 101.952. Thus, comparing the got values, it is possible will make sure, that $M_3^* >> \delta_3^2$, and this is the turn considerably anymore, for example, by what at the use of thickener of catenoidal form. Using Eq. (16) for these values of parameter δ_3^*, it is possible to define the values of substantial permanent C^*. I.e. C^* = 0.033; 0.016; $7.98 \cdot 10^{-3}$; $3.973 \cdot 10^{-3}$ accordingly. Then, the types of thickener of ultrasonic energy, proper by found, are shown on Fig. 3.

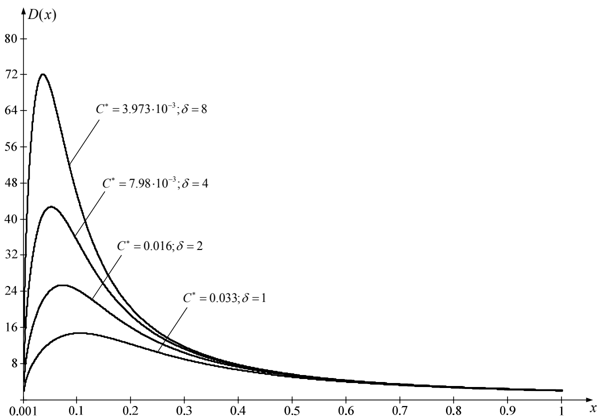

Fig. 3

Distributing of amplitudes of moving $W(x)$ and cyclic mechanical tensions $\sigma = EW'(x)$ is calculated according to (12) and (13) on an interval $x = \alpha \div \alpha + 1$ and represented on Figs. 4 and 5.

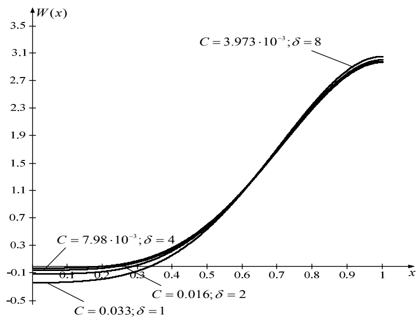

Fig. 4

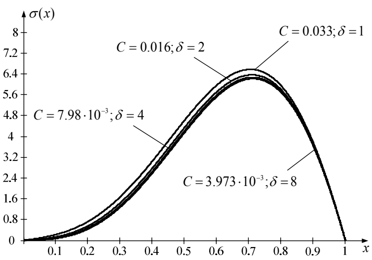

Fig. 5

According to Figs. 4 and 5 the maximum of mechanical tensions, are located approximately in the distance 0.3 lengths l from a thin end and does not coincide with knots, which are in the distance from 0.2 to 0.3l (depending on δ_3^*) counted off from the massive end of thickener of ultrasonic energy. This circumstance allows used without fears the key sections of considered acoustic thickeners for their functioning in composition electromechanical actuators.

6. CONCLUSION

The simple mathematical algorithm of providing of the required functional descriptions of thickener of ultrasonic energy is described. On the basis of algorithm results which allow estimating efficiency of the offered configurations of thickeners are got. The got new types of thickeners in the article, afterwards at experimental researches, allowed reducing the general power tricked into to the ultrasonic oscillating system on 46%, that underlines their efficiency during exploitation in the resonance mode.

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