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## ACTIVE CONTROL FOR VIBROPROTECTION OF OPERATORS OF TECHNOLOGICAL MACHINES

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### ABSTRACT

The electrohydraulic mechanism of a vibroprotection system with a digital regulator is considered. A mathematical model of the system is constructed. An optimal operational algorithm for the regulator is synthesized on the basis of the Riccati equation. This permits simulation of the system with various external perturbations and under the action of actual vibrational noise at the workstation of an SBSH-250 drilling machine.

### 1. INTRODUCTION

Vibrational-protection systems with an electrohydraulic mechanism are based on an idea proposed in [1]. A system with an electrohydraulic executive mechanism (Fig. 1) includes a hydraulic cylinder 7 between the object to be protected 1 and the base 2, electrohydraulic converter 9, accelerometers 3 and 4 at the object and the base, relative-displacement sensor 5, and regulator 6 [2]. Converter 9 consists of a choking hydraulic distributor with a controlling electric step motor, which adjusts the four-edged slide valve through a screw pair. Voltage pulses sent to the step motor turn its axle through a specified angle (the angular increment), usually  $1.5^\circ$ . Thus, the angle of rotation is determined by the number of pulses, while the speed of rotation is determined by their repetition frequency. The use of a step motor imposes certain constraints on the moment of inertia of the mechanism at the output axle and the motor pickup.

Essentially, the choking distributor is a hydraulic power amplifier 8 converting the mechanical input into the corresponding displacement of the output element. This changes the oil flow rate and direction of flow simultaneously in several lines as a function of the external control signal.

## 2. SYNTHESIS OF AN OPTIMUM DIGITAL ADJUSTER

We now formulate the mathematical model of a unidirectional hydraulic piston cylinder (Fig. 1; 11 is the sensor monitoring the flow rate of the working liquid). In constructing a model of the electric motor, the flow rate of the working liquid into the hydraulic cylinder is regarded as the input parameter, and the translational velocity of the piston rod as the output parameter. In fact, the hydraulic cylinder is described by nonlinear equations, but some of the nonlinearities may be neglected in constructing the motor model for control circuits. Accordingly, in constructing the model, we make the following assumptions: the leakage of the working liquid does not depend on the liquid flow rate; the elastic modulus of the working liquid remains constant; the cylinder operation passes beyond the bounds of the insensitivity zone. The object is assumed to be a rigid mass; there are no gaps in any elements of the system, except for the gap between the piston and the cylinder housing, which is taken into account.

Consider a high-speed unidirectional piston cylinder of low mass and volume per unit power and high efficiency. The hydrodynamic equation for the flow rates takes the form:  $Q = Q_{mo} + Q_{co} + Q_{le}$  where  $Q$  is the flow rate of working liquid to the cylinder;  $Q_{mo} = Az$  is the liquid flow rate responsible for the motion of the piston;  $Q_{co} = \beta p$  is the liquid flow rate characterizing the losses due to compressibility;  $Q_{le} = k_{le} p$  is the liquid flow rate characterizing the leakage losses;  $A$  is the area of piston action;  $\beta$  is the compressibility of the liquid;  $p$  is the pressure difference in the hydraulic cylinder;  $k_{le}$  is liquid leakage coefficient. The coefficients  $\beta$  and  $k_{le}$  may be determined from the formulae  $\beta = V(4E)^{-1}$ ,  $k_{le} = 9.2 \cdot 10^{-17} D \delta^3 (v l k_{sl})^{-1}$ , where  $V$  is the total volume of the piston and piston-rod cavities of the cylinder;  $E$  is the elastic modulus of the working liquid;  $D$  is the piston diameter, mm;  $\delta$  is the diametric gap,  $\mu\text{m}$ ;  $v$  is the kinematic viscosity of the working liquid,  $\text{mm}^2/\text{s}$ ;  $l$  is the piston thickness, mm; the coefficient  $k_{sl}$  takes account of the slot character ( $k_{sl} = 2.5$  for a concentric slot).

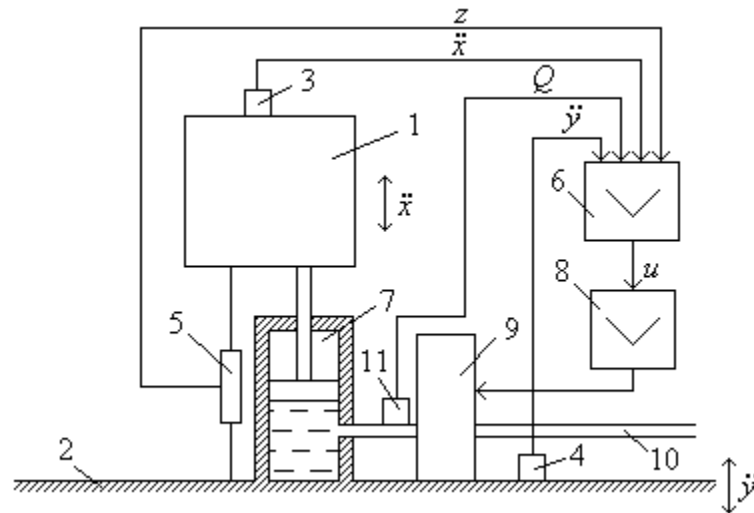


Figure 1.

The force displacing the piston is  $F = A p k_{fr}$ , where  $k_{fr}$  takes account of the frictional losses (usually,  $k_{sl} = 0.9-0.98$ ). On the other hand,  $F = F_c$ , where  $F_c$  is the drag force;  $F = m \ddot{z}$ .

Hence, we obtain the following system of equations

$$\ddot{z} = (A k_{fr} m^{-1}) p, \quad \dot{p} = -(4 A E V^{-1}) \dot{z} - (4 E k_{le} V^{-1}) p + (4 E V^{-1}) Q \quad (1)$$

completely describing the behavior of a hydraulic cylinder loaded by a body of mass  $m$ .

By appropriate calculations, an expression for the transfer function relating the influence of the liquid flow rate to the cylinder and the translational velocity of the piston rod  $z$  may be obtained:

$$W_{Q-z}(s) = 4AEk_{fr}(mVs^2 + 4mEk_{le}s + 4A^2Ek_{fr})^{-1}$$

We now construct a mathematical model of the electrohydraulic converter. The transfer function of the step motor  $W_{u-\varphi}(s) = \varphi(s)u^{-1}(s) = k_{em}s^{-1}$  which describes the relation between the armature voltage pulses  $u$  and the angle of motor-shaft rotation  $\varphi$ . All the control pulses are assumed to be of the same length. Each such pulse leads to rotation of the motor shaft by  $\Delta\varphi$ . The amplification factor  $k_{em}$  is given by the formula  $k_{em} = \Delta\varphi(A_p T_p)^{-1}$ , where  $A_p$  is the amplitude of the control voltage pulses;  $T_p$  is the length of each pulse.

The transfer function of the screw-nut coupling takes the form  $W_{\varphi-x}(s) = x_{gd}(s)\varphi^{-1}(s) = r_1$ , where  $x_{gd}$  is the slide-valve displacement in the choking gas distributor;  $r_1$ , is the transmission ratio of the screw pair in terms of the velocity parameters. The transfer function of the gas distributor relating  $x_{gd}$  and  $Q$  is:  $W_{x-Q}(s) = Q(s)x_{gd}^{-1} = k_{gd}$  where  $k_{gd}$  is the transmission ratio of the gas distributor in the  $x_{gd}$ - $Q$  channel. This parameter may be determined from the static characteristics of the choking gas distributor after linearization.

Consider the synthesis of optimal control for a system with an electrohydraulic executive mechanism. The whole system, including the executive mechanism, is described in state space. The state coordinates employed are the variables  $x_1 = x$ ,  $x_2 = z$ ,  $x_3 = p$ . Then Eq. (1) may be written in the form

$$\begin{aligned} \dot{x}_1 &= Ak_{fr}m^{-1}x_3 + \ddot{y}, & \dot{x}_2 &= x_1 - \dot{y} \\ \dot{x}_3 &= -4AEV^{-1}x_1 - 4Ek_{le}V^{-1}x_3 + 4EV^{-1}Q + 4AEV^{-1}\ddot{y} \end{aligned} \quad (2)$$

or in vector matrix form

$$\dot{X} = AX + BQ + GY \quad (3)$$

where  $X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$  is the state vector;  $A = \begin{bmatrix} 0 & 0 & Ak_{fr}m^{-1} \\ 1 & 0 & 0 \\ -4AEV^{-1} & 0 & -4Ek_{le}V^{-1} \end{bmatrix}$  is the coefficient

matrix of the system;  $B = \begin{bmatrix} 0 \\ 0 \\ 4EV^{-1} \end{bmatrix}$  is input matrix;  $G = \begin{bmatrix} 0 & 1 \\ -1 & 0 \\ 4AEV^{-1} & 0 \end{bmatrix}$  is the coefficient

matrix characterizing the influence of the perturbation;  $Y = \begin{bmatrix} \dot{y} \\ \ddot{y} \end{bmatrix}$  is the perturbation vector.

Equation (3) completely describes the behavior of the system with an electrohydraulic executive mechanism under the influence of perturbing and control signals. Each model of

the hydraulic cylinder corresponds to a particular coefficient matrix  $A$ , input matrix  $B$ , and perturbation-influence matrix  $G$ .

Note that the consumption of the working liquid  $Q$  is regarded as the control signal in the model in Eq. (3). This means that there are two loops in the system: the control loop for the kinematic parameters; and the auxiliary control loop for the working-liquid flow rate. An optimal state regulator is adopted as the regulator for the kinematic-parameter control loop.

In digital regulator synthesis, Eq. (3) is written in the form

$$X[i+1] = A_{\Delta} X[i] + B_{\Delta} u[i] + G_{\Delta} Y[i] \quad (4)$$

where

$$A_{\Delta} = \exp(AT) = I + \sum_{i=1}^{\infty} A^i T^i (i)^{-1}, \quad B_{\Delta} = \left( \int_0^T \exp(AT) dt \right) B = \left( IT + \sum_{i=1}^{\infty} A^i T^{i+1} [(i+1)]^{-1} \right) B$$

The structure of the optimal digital regulator is expressed as a matrix relation  $u_Q[i] = -FX[i]$ , where  $u_Q$  is the control signal, i.e., the desired working-liquid flow rate;  $F = [f_1 f_2 f_3]$  is the matrix of feedback coefficients with respect to the state variables. In fact, the optimal regulator is based on feedback with respect to the state of the system. Synthesis of the optimal regulator begins with formulation of the control-variable vector  $Z$

$$Z = \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} X$$

The general expression adopted for the quality criterion is an integral of quadratic forms of the control variables and the control signal, of the form

$$J = \int_0^{\infty} (q_1 z_1^2(t) + q_2 z_2^2(t) + ru^2(t)) dt \longrightarrow \min \quad (5)$$

where  $q_1, q_2, r$  are the weighting factors for the squares of the two control variables and the control signal and are selected experimentally. For the digital system, Eq. (5) may be written in the form

$$J = \sum_{i=0}^{\infty} (X^T[i] D^T Q D X[i] + ru^2[i]) \longrightarrow \min$$

where  $Q = \begin{bmatrix} q_1 & 0 \\ 0 & q_2 \end{bmatrix}$  is the weighting-factor matrix for the control variables. Thus, the

control problem may be stated as follows: to minimize the amplitude of the object's velocity (and hence the amplitude of its acceleration) and limit the displacement of the object relative to the base, thereby restricting the scope for control. The matrix of feedback coefficients is:

$$F = (r + B^T P B)^{-1} B^T P A$$

where  $P$  is a  $3 \times 3$  quadratic positive-definite auxiliary matrix and satisfies the discrete Riccati equation [3]

$$P = D^T Q D + A^T P A + A^T P B (r + B^T P B)^{-1} B^T P A \quad (6)$$

Equation (6) has a solution if the pair  $(A, B)$  is completely controllable, which is the case here. This equation has a unique positive-definite solution in the form of a symmetric matrix  $P$ . Such an optimal closed system is undoubtedly stable, since all the eigenvalues of the matrix  $(A - BF)$  are less than 1 in amplitude.

Solving Eq. (6) in accordance with the recommendations of [4], we obtain the control law of the optimal regulator in the form of feedback with respect to states of the system

$$u_Q[i] = -f_1 \dot{x}[i] - f_2 z[i] - f_3 p[i] \quad (7)$$

Note that it is difficult to measure the pressure difference in the cylinder  $p$ . Therefore, the state coordinate may expediently be expressed in terms of other quantities. From the first relation in Eq. (2), the pressure difference is expressed in terms of the acceleration at the object and at the base:  $p = m(Ak_{fr})^{-1} \ddot{x} - m(Ak_{fr})^{-1} \ddot{y}$ . Then Eq. (7) takes the form

$$u_Q[i] = -f_1 \dot{x}[i] - f_2 z[i] - f_3' \ddot{x}[i] - f_3' \ddot{y}[i] \quad (8)$$

where  $f_3' = f_3 m(Ak_{fr})^{-1}$  expression. In addition, the control signal must have upper and lower bounds corresponding to the expression

$$u_Q[i] = \begin{cases} u_Q[i], & |u_Q[i]| \leq Q_{\max} \\ \text{sign}(u_Q[i]) Q_{\max}, & |u_Q[i]| > Q_{\max} \end{cases}$$

where  $Q_{\max}$  is the maximum permissible working-liquid flow rate, determined by the parameters of the hydraulic line and the choking hydraulic distributor.

In the loop controlling the working-fluid flow rate, the a three-position relay regulator is employed, in accordance with the three possible control signals supplied to the step motor. The relay regulator sends a conventional control signal  $u_0$  and is tuned to the position  $\{-1, 0, 1\}$ . The relay regulator includes two hysteresis zones, in order to ensure normal operation of the step motor. The operational algorithm of the relay regulator is described by the expression

$$u_0[i] = \begin{cases} \text{sign}(\varepsilon[i]), & |\varepsilon[i]| > c_2 \\ 0, & |\varepsilon[i]| < c_1 \\ u_0[i-1], & c_1 \leq |\varepsilon[i]| \leq c_2 \end{cases}$$

where  $\varepsilon = (u_Q - Q)$  is the mismatch signal of the desired and actual working-liquid flow rates;  $c_1$ , and  $c_2$  are constants determined on the basis of the characteristics and operating conditions of the selected electric step motor ( $0 < c_1 \leq c_2$ )

The pulsed-control module (CPM) converts the conventional control signal  $u_0$  into voltage control pulses  $u$  for subsequent delivery to the step motor. The basic CPM parameters are the pulse-repetition period  $T_0$  and the pulse length  $T_p$ . The pulse-repetition

period is selected so that  $T_0 \geq f_{\max}^{-1}$ , where  $f_{\max}$  is the maximum pulse-repetition frequency for the chosen motor. The recommended pulse length is calculated from the formula:  $T_p = T_0 / 2$

The structure of a vibrational protection system with an optimal regulator is shown in Fig. 2, taking account of Eq. (8) and the constraints on the control signal that define the maximum permissible working-liquid flow rate.

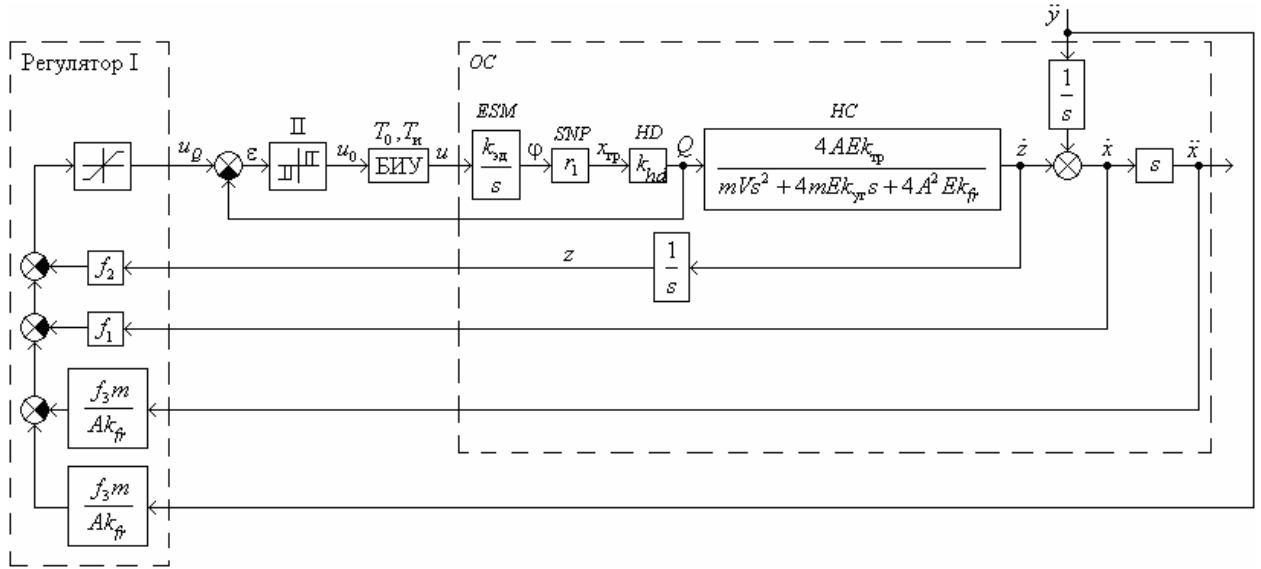


Figure 2. Structure of vibroprotection system with electrohydraulic executive mechanism and an optimal regulator: OC – object for control; ESM – electric step motor; SNP – screw-nut pair; HD – choking hydrodistributor; HC – hydraulic cylinder; I, II – regulators.

### 3. MODELLING RESULTS

We now synthesize the control signal for a system with a GTsP 70×50×400 hydraulic cylinder (Technical Specifications TU 2-0221050.004-88). The electrohydraulic converter consists of a ShD-5D1M electric step motor, a screw-nut pair, and a G61-41V choking hydraulic distributor (Technical Specifications TU 2-053-1477-80). The mass of the stabilization object  $m = 100$  kg. The parameters of the electrohydraulic converter for the selected system are:

$$T_0 = 1.25 \cdot 10^{-4} s, \quad T_p = 6.25 \cdot 10^{-5} s, \quad k_{em} = 8.727 V^{-1}, \quad k_{fr} = 0.938 m^2 / s,$$

In Eq. (3), the matrices A, B, and G take the form

$$A = \begin{bmatrix} 0 & 0 & 3.656 \cdot 10^{-5} \\ 1 & 0 & 0 \\ -2.014 \cdot 10^{10} & 0 & -9.680 \cdot 10^3 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 0 \\ 5.232 \cdot 10^{12} \end{bmatrix}, \quad G = \begin{bmatrix} 0 & 1 \\ -1 & 0 \\ 2.014 \cdot 10^{10} & 0 \end{bmatrix}$$

Switching to a digital problem, with a discretization period  $T=200 \mu s$ , we find the matrices  $A_\Delta$  and  $B_\Delta$  in Eq. (4)

$$A_{\Delta} = \begin{bmatrix} 0.992 & 0 & 3.217 \cdot 10^{-9} \\ 1.994 \cdot 10^{-4} & 1 & 4.206 \cdot 10^{-13} \\ -1.772 \cdot 10^3 & 0 & 0.140 \end{bmatrix}$$

The weighting-factor matrix  $Q = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ ;  $r=10^2$

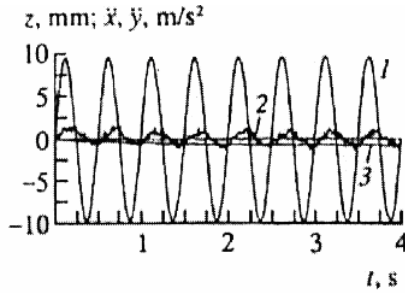


Figure 3.

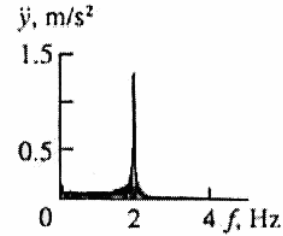


Figure 4.

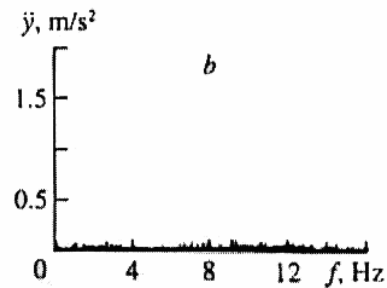
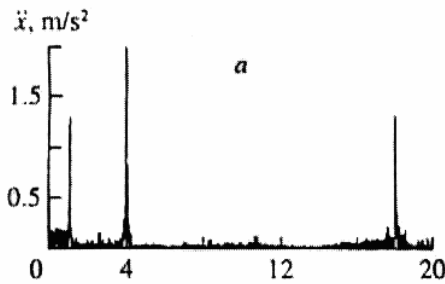


Figure 5.

After finding the feedback coefficients of the optimal regulator, we obtain the feedback-coefficient matrix  $F = [0.081 \ 0.086 \ 2.960 \cdot 10^{-10}]$ . In the regulator of the flow-rate control loop, the boundaries of the hysteresis zones are set:  $c_1=c_2 = 0$ . For matrix  $F$ , system operation with a perturbing signal ( $f=2$  Hz) of various types is modeled, as shown in Fig. 3: 1) acceleration at the base; 2) acceleration at the object; 3) displacement of the object relative to the base. The amplitude of the perturbing signal  $A_{\ddot{y}} = g$ . In that case, the coefficient of amplitude suppression of the perturbing signal is  $A_{\ddot{x}} \cdot A_{\ddot{y}}^{-1} = 0.1$ . Note that the relative displacement of the object is limited and may be held at an acceptable level. The steady relative displacement may be reduced by increasing  $q_2$  in Eq. (6) Note that the acceleration at the object increases correspondingly here. In any case, the solution will be a compromise.

The modeling results show that the system output (the acceleration at the object) looks more noisy, while the perturbing signal sent to the system is purely harmonic. In fact, spectral expansion of the output signal (Fig. 4) shows that, besides the primary 2 Hz component, there are small quantities of secondary frequencies that are not present in the perturbing signal. This may be attributed mainly to the discrete displacement of the step motor's axle, leading unavoidably to a small error in the working-liquid flow rate to the cylinder. The results are in qualitative agreement with [5, 6]. This finding is not of fundamental importance when considering the protection of the equipment and the human operator from vibration. In that case, the reduction in overall vibrational activity at the object is much more important.

#### 4. SUMMARY

Consider the vibrational background at the operator workstation of a SBSH-250 drilling mill. We know from measurements that the spectral characteristic of the vertical vibrational signal here has characteristic peaks at 1.1, 4, and 18 Hz and damped vibrational background at other frequencies. A signal with the spectral characteristic in Fig. 5a is sent to the system, as the perturbing signal (vibroacceleration) from the base. Mathematical modeling yields the spectral characteristic of the resultant acceleration at the object (Fig. 5b). Comparison indicates that the vibrational background is considerably reduced. All the characteristic peaks are practically completely damped.

This clearly shows the effectiveness of the system with an electrohydraulic executive mechanism and an optimal regulator in suppressing complex perturbations.

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