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# A STUDY ON RESPONSE ANALYSIS OF PNEUMATIC VIBRATION ISOLATION TABLE LOADED BY TRANSIENT MOVEMENTS OF CARRIAGE ON IT

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**Abstract**

As environmental vibration requirements on precision equipments get more stringent, use of pneumatic vibration isolators becomes more crucial and, hence, their dynamic performance needs to be further improved. Dynamic behavior of those pneumatic vibration isolation tables is very important to both manufacturer and customer as performance specifications. Together with conventional transmissibility, transient response characteristics are another critical performance index especially when movements of components, e.g., xy-stage, of the precision equipments are very dynamic.

In this paper, analysis on transient response of a pneumatic vibration isolation table loaded by a mass moving on it is presented. This is a conventional dynamics problem on a rigid body with 6 degree of freedom and a mass on it with another degree of freedom. How to obtain transient responses of the isolation table is described when the movements of the mass are prescribed relative to the table.

## 1. INTRODUCTION

In the later 90's, as environmental vibration requirements on the precision equipments get more stringent, further improved pneumatic vibration isolators are needed to help the performance realization of precision equipments. Two of the biggest impediments towards realizing the performance goals are due to

- **Ground vibration** : At micro-levels of positioning accuracy, environmental effects become significant. These could be due to thermal effects, windage, turbulence in the ducts and so on. According to Gordon[1], the most significant environmental disturbance in a micro-electronics fabrication facility is due to seismic vibration.
- **Payload excitation** : Precision equipment on a pneumatic vibration isolation table could cause the transient movement of payload when it has moving parts such as a wafer stepper, xy-stage etc.

In case of ground vibration, a lot of regulations and researches have been developed and it could be settled by using pneumatic vibration isolation table. BBN and FHA Criteria are the most well known criteria in the world[1,2,3] and well-defined design guidelines on isolation table have been developed to isolate the ground vibration[3,4]. However, as the use of pneumatic vibration isolation table gets increasing, transient movement of isolation table caused by payload excitation becomes a more and more serious problem in real field due to the small stiffness of pneumatic spring. Therefore, there should be a trade-off between isolation of ground vibration and fast decay of payload excitation and we must consider both of them when pneumatic vibration isolation table is designed or installed. In this paper, prior to development of design procedure of the isolation table, analysis on transient response of pneumatic vibration isolation table loaded by masses moving on it is presented. Prediction of transient response of isolation table could help users to plan the length of the inspecting or manufacturing process and manufacturers to design the tables in an efficient way.

## 2. DERIVATION OF DYNAMIC EQUATIONS OF PNEUMATIC VIBRATION ISOLATION TABLE WITH MOVING MASSES ON IT

Although the whole system of a pneumatic vibration isolation table including precision equipment with moving parts is actually forced by an actuator like as a linear motor, the force of the actuator isn't a matter of concern when the moving parts have the deterministic motion profile. In such a case, the force imposed on pneumatic vibration isolation table could be described by inertial force of the moving parts according to Newton's 3$^{rd}$ law, action and reaction law. Therefore, movements of the components on isolation table are regarded as predicted inputs.

### 2.1 Assumptions in deriving equations of motion

In order to derive the equations of motion of a pneumatic vibration isolation table system including precision equipment with moving parts, some assumptions as follows are made:

- The system of pneumatic vibration isolation table consists of four pneumatic springs and two moving components on it as illustrated in Fig. 2.1.
- Granite of isolation table and fixed part of precision equipment on it are assumed as one rigid body and moving components as point masses.
- As shown in Fig. 2.1, each moving component is driven in x and y-axis relative to the granite with known profile of displacement, velocity and acceleration.
- The force generated by an actuator interact between granite and moving mass on it and are represented by the relative movement.
- The direction cosine for infinitesimal rotations is used to transform the coordinate because the maximum angular displacement of granite is assumed to be so small(< 3°): $\cos\theta \sim 1$ and $\sin\theta \sim \theta$.
- Rotational stiffness elements are ignored and each translational stiffness element exerts its force in corresponding direction only.

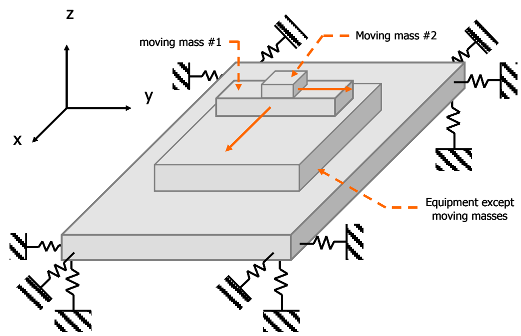



**Fig. 2.1** Scheme of the 6-DOF pneumatic vibration isolation table with moving masses on it

### 2.2 Derivation of dynamic equations of pneumatic vibration isolation table with moving masses on it

In order to explain and understand easily, a 3-DOF model of a pneumatic vibration isolation table with one moving mass is presented as illustrated in Fig. 2.2 and we will expand it to 6-DOF later. As shown in Fig.2.2, frame A is an inertial frame which consists of x and y-axis and frame B is a frame fixed to granite, x' and y'-axis, and originates at the mass center of the granite. The coordinate transformation between frame A and B can be done by eq.(1) below[5].

$$\begin{Bmatrix} \underset{\sim}{i}' \\ \underset{\sim}{j}' \end{Bmatrix} = \mathbf{C} \begin{Bmatrix} \underset{\sim}{i} \\ \underset{\sim}{j} \end{Bmatrix} = \begin{bmatrix} 1 & \theta \\ -\theta & 1 \end{bmatrix} \begin{Bmatrix} \underset{\sim}{i} \\ \underset{\sim}{j} \end{Bmatrix} \tag{1}$$

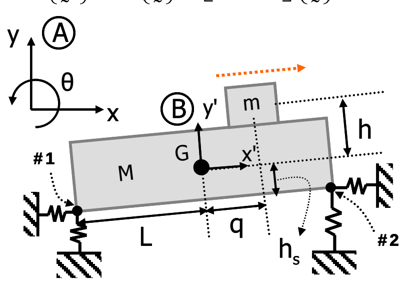



**Fig. 2.2** 3-DOF isolation table with a moving mass

Several vectors are defined below to describe movements in Fig. 2.2.

- Position vector of mass center of the granite including fixed base of precision equipment

$$\underset{\sim}{G} = X\underset{\sim}{i} + Y\underset{\sim}{j} \tag{2}$$

- Position vector of a moving mass in frame B

$$\underset{\sim}{d} = q\underset{\sim}{i}' + h\underset{\sim}{j}' \tag{3}$$

- Position vector of a moving mass in frame A

$$\underset{\sim}{r} = \underset{\sim}{G} + \underset{\sim}{d} = \left\{X\underset{\sim}{i} + Y\underset{\sim}{j}\right\} + \left\{q\underset{\sim}{i}' + h\underset{\sim}{j}'\right\} \tag{4}$$

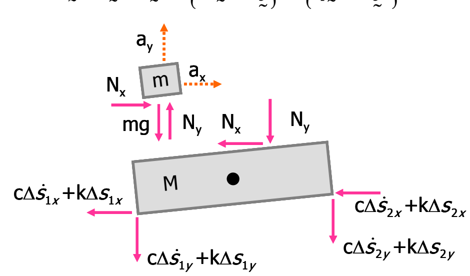


**Fig. 2.3** Free body diagram of 3-DOF isolation table with a moving mass

The force imposed on the isolation table consists of the inertial force computed from acceleration of a moving mass in frame A and spring and damping force in each direction as shown in Fig. 2.3. The acceleration of a moving mass in frame A is computed by differentiating position vector of it, eq. (4) as follows[6]:

$$\ddot{\underset{\sim}{r}} = \frac{{}^{A}d^2\underset{\sim}{r}}{dt^2} = \frac{{}^{A}d^2\underset{\sim}{G}}{dt^2} + \frac{{}^{B}d^2\underset{\sim}{d}}{dt^2} + 2\cdot{}^{A}\underset{\sim}{\omega}^{B}\times\frac{{}^{B}d\underset{\sim}{d}}{dt} + {}^{A}\underset{\sim}{\alpha}^{B}\times\underset{\sim}{d} + {}^{A}\underset{\sim}{\omega}^{B}\times\left({}^{A}\underset{\sim}{\omega}^{B}\times\underset{\sim}{d}\right), \tag{5}$$

where $\frac{{}^{A}d}{dt}$ and $\frac{{}^{B}d}{dt}$ denote differentiation in frame A and B respectively, ${}^{A}\underset{\sim}{\omega}^{B}$ angular velocity, ${}^{A}\underset{\sim}{\alpha}^{B}$ angular acceleration of the table denoted as follows:

$${}^{A}\underset{\sim}{\omega}^{B} = \dot{\theta}\underset{\sim}{k}, \qquad {}^{A}\underset{\sim}{\alpha}^{B} = \ddot{\theta}\underset{\sim}{k} \tag{6}$$

The interaction force between the isolation table and a moving component is derived from the free body diagram of a moving mass in Fig. 2.3 as follows:

$$\begin{aligned} N_x &= ma_x = m\left(\ddot{X} + \ddot{q} - 2\dot{q}\theta\dot{\theta} - q\theta\ddot{\theta} - q\dot{\theta}^2 - h\ddot{\theta} + h\dot{\theta}^2\right) \\ N_y &= ma_y + mg = m\left(\ddot{Y} + \ddot{q}\theta + 2\dot{q}\dot{\theta} + q\ddot{\theta} - q\theta\dot{\theta}^2 - h\theta\ddot{\theta} - h\dot{\theta}^2 + g\right), \end{aligned} \tag{7}$$

where $a_x$ and $a_y$ denote acceleration of the moving mass in x and y-direction, respectively. Inertial force derived by using acceleration of a moving mass is loaded to the isolation table in the opposite direction of acceleration by action and reaction law. The equation of motion of the isolation table, illustrated in Fig. 2.3, can be derived by Newton's 2$^{nd}$ and Euler law as follows:

$$M\ddot{X} + 2c(\dot{X} + h_s\dot{\theta}) + 2k(X + h_s\theta) = -N_x \tag{8}$$

$$M\ddot{Y} + 2c\dot{Y} + 2kY = -N_y \tag{9}$$

$$I\ddot{\theta} + 2c(L^2\dot{\theta} + h_s^2\dot{\theta} + h_s\dot{X}) + 2k(L^2\theta + h_s^2\theta + h_s X) = -(q - h\theta)N_y + (h + q\theta)N_x \tag{10}$$

Substituting eq. (7) into eq. (8)~(10) and neglecting the high order terms of X, Y and $\theta$ yield the equation of motion in matrix form.

$$[M]\{\ddot{x}\} + [C]\{\dot{x}\} + [K]\{x\} = \{F\}, \qquad \{x\} = \begin{Bmatrix} X \\ Y \\ \theta \end{Bmatrix} \tag{11}$$

$$[M] = \begin{bmatrix} M+m & 0 & -mh \\ 0 & M+m & mq \\ -mh & mq & I + mh^2 + mq^2 \end{bmatrix}, \quad [C] = \begin{bmatrix} 2c & 0 & 2ch_s \\ 0 & 2c & 2m\dot{q} \\ 2ch_s & 0 & 2c\left(L^2 + h_s^2\right) + 2mq\dot{q} \end{bmatrix},$$

$$[K] = \begin{bmatrix} 2k & 0 & 2kh_s \\ 0 & 2k & m\ddot{q} \\ 2kh_s & 0 & 2k\left(L^2 + h_s^2\right) - mgh \end{bmatrix}, \quad \{F\} = \begin{Bmatrix} -m\ddot{q} \\ -mg \\ mh\ddot{q} - mgq \end{Bmatrix}.$$

All of the above mass, damping and spring matrices have time variant terms, ($q$, $\dot{q}$, $\ddot{q}$) and the mass matrix is symmetric only. Therefore, the response of the isolation table can be calculated only numerically.

Derived equations of motion show that they should be treated as a dynamic or a vibration problem during each time interval. When a point mass on an isolation table move by a given profile, the equilibrium point can't be defined, and this problem should be regarded as a nonlinear dynamic problem and after a point mass stops, time variant terms go to zero and equilibrium point can be defined, this should be regarded as a linear vibration problem which has initial conditions. 6-DOF equation of motion of an isolation table with moving components on it could be expanded by employing the same procedure and the results of it have the same characteristics. They are mentioned in appendix.

# 3. PREDICTION OF TRANSIENT RESPONSE OF PNEUMATIC VIBRATION ISOLATION TABLE DUE TO MOVING MASSES ON IT

## 3.1 Verification of the derived dynamic equations by using ADAMS

The equations of motion of isolation table derived in section 2.2 are verified by using ADAMS, commercial multi-body dynamics software. $4^{th}$ order Runge-Kutta method is used to compute the nonlinear differential equation numerically. The conditions of simulation for comparison of the results from ADAMS and numerical computation with the derived equation of motion are drawn from Table 3.1. After modeling the granite as a rigid body in ADAMS, the point mass is positioned and its profile is designed as a sine curve, shown in Fig. 3.1 and 3.2. The results of simulation under those conditions are shown in Fig. 3.3. The results from ADAMS are plotted as solid line, the results from EOM as dotted line and the discrepancy between two results as dash-dotted line. The discrepancy between both results is negligible compared with response of isolation table in simulation, $10^{-3}$(discrepancy$_{rms}$/response$_{rms}$). From these results, complicated equations of motion of a pneumatic vibration isolation table with moving masses derived in section 2.2 are confirmed.

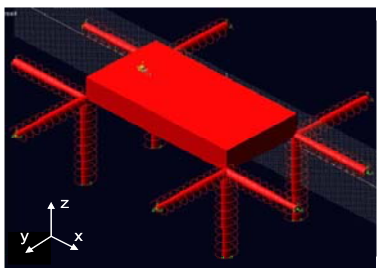



**Fig. 3.1** A model for simulation in ADAMS

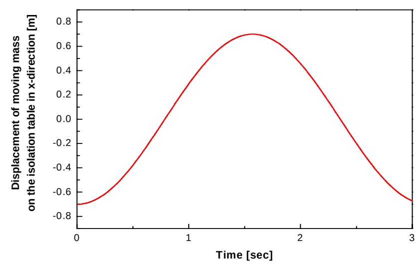



**Fig. 3.2** Displacement of a moving mass on the isolation table in $x$-direction

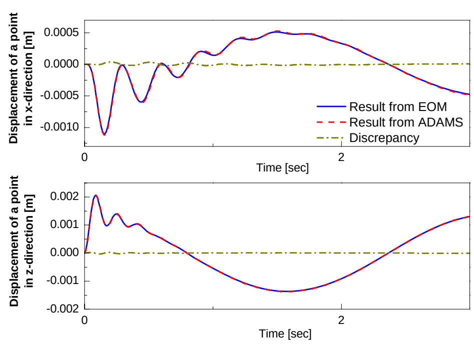



**Fig. 3.3** Comparison of simulation results from the derived EOM and ADAMS

**Table 3.1** Conditions of simulation for comparison of the results from ADAMS and the EOM

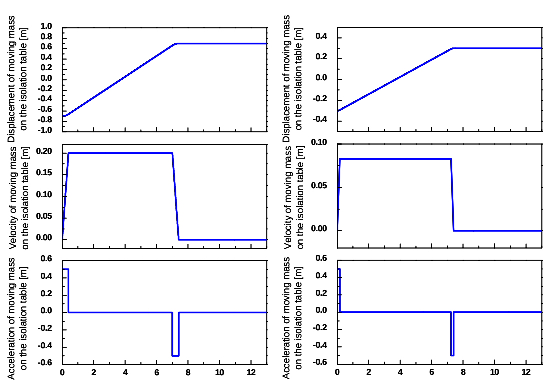
| | |
|---|---|
| Mass of the isolation table | $4400kg$ |
| Dimension of the table | Length : $2m$ Width : $1m$ Thickness : $0.3m$ |
| Stiffness of air spring | $5.7\times10^{5}\,N/m$ |
| Viscous damping of air spring | $5.7\times10^{3}\,N\cdot s/m$ |
| Mass of a moving part | $400kg$ |
| Distance of a moving mass from the mass center in z' direction | $0.17m$ |
| Input(displacement of a moving mass on the isolation table) | $x(t)=0.7\sin\left(2t-\frac{\pi}{2}\right)$ |

### 3.2 Simulation of transient movements

The driving conditions of LCD inspection system which is used in an industry field are taken as the conditions for simulation of an isolation table with two moving parts as shown in Table 3.2. The motion profile of moving masses, such as displacement, velocity and acceleration, is assumed to have three stages: firstly moving parts of equipment accelerate constantly from the initially rested condition to desired constant velocity, then have constant velocity near the desired position and finally decelerate to rest. The first mass moves according to x-axis on the isolation table and the second one according to y-axis on the first one. The behavior of the second one moves diagonally on isolation table from the starting point to the opposite point in frame B. The motion profiles of both masses are shown in Fig. 3.4 (a) and (b): displacement, velocity and acceleration, respectively.

**Table 3.2** Simulation conditions of pneumatic vibration isolation table with two moving masses

| | |
|---|---|
| Mass of the table | $4400kg$ |
| Dimension of the table | Length : $2m$ Width : $1m$<br>Thickness : $0.3m$ |
| Stiffness of air spring | $5.7\times10^5\,N/m$ |
| Viscous damping of air spring | $5.7\times10^3\,N\cdot s/m$ |
| Mass of moving parts | First mass : $400kg$<br>Second mass : $40kg$ |
| Distance of moving masses from the mass center in $z'$ direction | First mass : $0.35m$<br>Second mass : $0.5m$ |
| Input (displacement of moving masses on the isolation table) | First mass<br>Stroke : $1.4m$<br>Velocity : $0.2m/s$<br>Acceleration : $0.5m/s^2$ |
| | Second mass<br>Stroke : $0.6m$<br>Velocity : $0.083m/s$<br>Acceleration : $0.5m/s^2$ |




(a) (b)

**Fig. 3.4** Input profile: displacement, velocity and acceleration for two moving masses

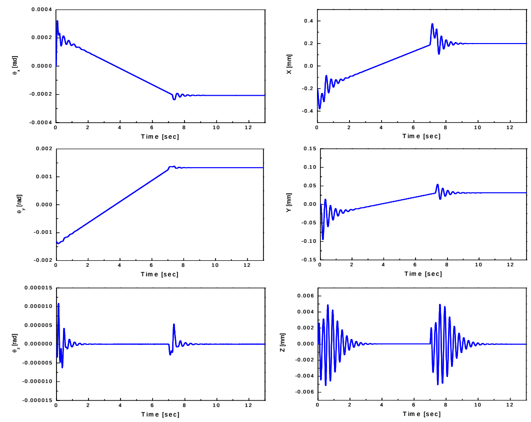



**Fig. 3.5** Simulation results of pneumatic vibration isolation table with two moving masses

Fig. 3.5 shows the simulation results: the left column shows rotational displacements and right one shows translational displacements in each direction. While moving masses have the constant acceleration, moving masses start or stop, isolation table has large vibration compared with other time interval and the motion for horizontal direction is larger than vertical direction. In aspect of settling time(for 5% of maximum displacement), we suggest that this precision machine should start to work after about 1.5 sec from the time when moving parts of that machine stop because of residual vibration of isolation table.

## 4. CONCLUSIONS

In this paper, it is pointed out that the transient movement of an isolation table can be caused by inertial force of moving parts when the precision equipment with moving components, like as XY-stage, works on it. In order to predict the transient movement, 6-DOF equation of motion was derived including the moving masses on it and examined the characteristics of it. The derived equation of motion is computed numerically for the given profiles of moving parts since it has nonlinear terms caused by moving masses and verified by using ADAMS.

## ACKNOWLEDGMEN

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# APPENDIX

$$[M]\{\ddot{x}\}+[C]\{\dot{x}\}+[K]\{x\}=\{F\}\ , \qquad \{x\}=\begin{Bmatrix}\theta_x\\ \theta_y\\ \theta_z\\ X\\ Y\\ Z\end{Bmatrix} \tag{A.1}$$

$$[M]=\begin{bmatrix}
\begin{matrix}I_x+m_1h_1^2\\ +m_2\left[q_2^2+(h_1+h_2)^2\right]\end{matrix} & -m_2q_1q_2 & \begin{matrix}-q_1\{m_1h_1\\ +m_2\left(h_1+h_2\right)\}\end{matrix} & 0 & \begin{matrix}-m_1h_1\\ -m_2\left(h_1+h_2\right)\end{matrix} & m_2q_2\\
-m_2q_1q_2 & \begin{matrix}I_y+m_1(q_1^2+h_1^2)\\ +m_2\left[q_1^2+(h_1+h_2)^2\right]\end{matrix} & -m_2\left(h_1+h_2\right)q_2 & \begin{matrix}m_1h_1\\ +m_2\left(h_1+h_2\right)\end{matrix} & 0 & -q_1\left(m_1+m_2\right)\\
\begin{matrix}-q_1\{m_1h_1\\ +m_2\left(h_1+h_2\right)\}\end{matrix} & -m_2\left(h_1+h_2\right)q_2 & \begin{matrix}I_z+m_1q_1^2\\ +m_2\left[q_1^2+q_2^2\right]\end{matrix} & -m_2q_2 & q_1\left(m_1+m_2\right) & 0\\
0 & \begin{matrix}m_1h_1\\ +m_2\left(h_1+h_2\right)\end{matrix} & -m_2q_2 & M+m_1+m_2 & 0 & 0\\
\begin{matrix}-m_1h_1\\ -m_2\left(h_1+h_2\right)\end{matrix} & 0 & q_1\left(m_1+m_2\right) & 0 & M+m_1+m_2 & 0\\
m_2q_2 & -q_1\left(m_1+m_2\right) & 0 & 0 & 0 & M+m_1+m_2
\end{bmatrix} \tag{A.2}$$

$$[C]=\begin{bmatrix}
\begin{matrix}4W^2c_v+4h_s^2c_h\\ +2m_2q_2\dot{q}_2\end{matrix} & -2m_2q_2\dot{q}_1 & -2\dot{q}_1\left[m_1h_1+m_2\left(h_1+h_2\right)\right] & 0 & 4h_sc_h & 0\\
-2m_2q_1\dot{q}_2 & \begin{matrix}4L^2c_v+4h_s^2c_h\\ +2q_1\dot{q}_1\left(m_1+m_2\right)\end{matrix} & -2m_2\left(h_1+h_2\right)\dot{q}_2 & -4h_sc_h & 0 & 0\\
0 & 0 & \begin{matrix}4c_h\left(W^2+L^2\right)\\ +2m_2q_2\dot{q}_2+2m_1\dot{q}_1\\ +2m_2q_1\dot{q}_1\end{matrix} & 0 & 0 & 0\\
0 & -4h_sc_h & -2m_2\dot{q}_2 & 4c_h & 0 & 0\\
4h_sc_h & 0 & 2\dot{q}_1\left(m_1+m_2\right) & 0 & 4c_h & 0\\
2m_2\dot{q}_2 & -2\dot{q}_1\left(m_1+m_2\right) & 0 & 0 & 0 & 4c_v
\end{bmatrix} \tag{A.3}$$

$$[K]=\begin{bmatrix}
\begin{matrix}4W^2k_v+4h_s^2k_h\\ +m_2q_2\ddot{q}_2\end{matrix} & -m_2q_2\ddot{q}_2 & -\ddot{q}_1\left[m_1h_1+m_2\left(h_1+h_2\right)\right] & 0 & 4h_sk_h & 0\\
-m_2q_1\ddot{q}_2 & \begin{matrix}4L^2k_v+4h_s^2k_h\\ +q_1\ddot{q}_1\left(m_1+m_2\right)\end{matrix} & -m_2\ddot{q}_2\left(h_1+h_2\right) & -4h_sk_h & 0 & 0\\
0 & 0 & \begin{matrix}4k_h\left(W^2+L^2\right)\\ +m_2q_2\ddot{q}_2+q_1\ddot{q}_1\left(m_1+m_2\right)\end{matrix} & 0 & 0 & 0\\
0 & -4h_sk_h & -m_2\ddot{q}_2 & 4k_h & 0 & 0\\
4h_sk_h & 0 & \ddot{q}_1\left(m_1+m_2\right) & 0 & 4k_h & 0\\
m_2\ddot{q}_2 & -\ddot{q}_1\left(m_1+m_2\right) & 0 & 0 & 0 & 4k_v
\end{bmatrix} \tag{A.4}$$

$$\{F\}=\begin{Bmatrix}
-m_2gq_2+m_2\left(h_1+h_2\right)\ddot{q}_2\\
q_1g\left(m_1+m_2\right)-\ddot{q}_1\left[m_1h_1+m_2\left(h_1+h_2\right)\right]\\
m_2\left(q_2-q_1\right)\ddot{q}_2\\
-\ddot{q}_1\left(m_1+m_2\right)\\
-m_2\ddot{q}_2\\
-g\left(m_1+m_2\right)
\end{Bmatrix} \tag{A.5}$$

$k_v$, $c_v$ : Stiffness and viscous damping in vertical direction, $k_h$, $c_h$ : Stiffness and viscous damping in horizontal direction, $m_1$, $m_2$ : The first and second moving mass

$q_1$, $q_2$ : Displacement of moving masses on the isolation table, $h_1$, $h_2$ : Height of each moving mass from the mass center of the isolation table,

W, L : Distance from the mass center of the isolation table to each pneumatic spring in x, y-direction, respectively